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# Scattering by a Layered Circular Cylindrical Post in a Rectangular Waveguide

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***Abstract.*** **In this paper we consider the problem of the dominant mode scattering by a layered circular cylindrical post in a rectangular waveguide. Both waveguide filter synthesis procedures and measurements of complex dielectric permittivity of materials require a solution to the inverse scattering problem. Since the computation time of an algorithm for solving the inverse scattering problem depends directly upon computation time of an algorithm for the forward scattering problem, our goal is to find an approach that provides both sufficient accuracy of results and rapid convergence. Over the last several decades, a number of approaches have appeared proposed by various authors for solving problems of scattering by a cylindrical post in a rectangular waveguide. The approach presented in this paper is a modified version of one of those approaches that provides sufficiently rapid convergence and results with a reasonable degree of accuracy. This modification allows a solution to the problem of scattering by a layered post. The approach divides the waveguide into separate regions by introducing imaginary cylindrical surface. In the outer regions the electromagnetic field is represented in terms of infinite series of rectangular waveguide modes. In the central region as well as in the layers of post, the field is represented in terms of cylindrical waves. By applying the point-matching method at the imaginary cylindrical surface and truncation of infinite series, a system of linear algebraic equations is obtained.**

***Keywords*: Scattering, layered post, composite, numerical study.**

## I. INTRODUCTION

The goal of this research is to discover the most appropriate approach for finding the reflection and transmission coefficients of the dominant mode scattered by a circular cylindrical post in a rectangular waveguide and to generalize this approach so that it can be applied to the case of the layered posts as well. This problem is of interest because the microwave waveguide post filters are still in use, and the procedure of filter synthesis requires a solution to the inverse scattering problem. Typically, an algorithm for solving the inverse problem requires a lot of computation time, because in each iteration it has to solve a number of forward problems. To speed up a process of a solution of the inverse problem we need to find an approach that provides the small computation time. Also the algorithm for solution of the inverse scattering problem may be used for measurements of the complex dielectric permittivity, since the plastic tube filled with liquid with the complex dielectric represents a two-layered post.

Of course, the problem can be solved by using some flexible methods, such as finite element method, finite difference method and finite difference method in the time domain [1] or software based on these methods such as HFSS or Ansys. Despite great flexibility of these methods, they, however, require a great amount of computation time. It is caused by the fact that these methods do not take into account some advantageous properties of geometry, such as capability of being divided into regions with simple geometry, such as cylindrical, spherical, elliptical, spheroidal and so on. These geometrical properties reduce computational effort by solving a part of problem by analytical means or at least by converting a problem to its simpler equivalent that requires less numerical effort.

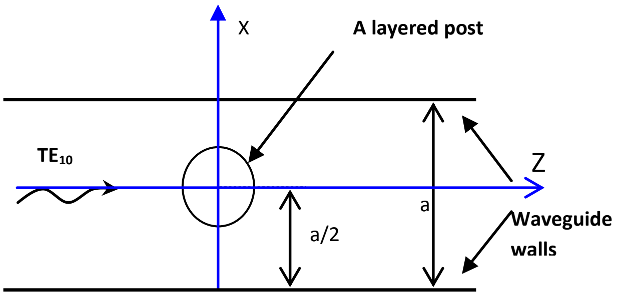

Fig. 1 scattering of the dominant mode TE$_{10}$ by cylindrical post

The first work devoted to discontinuities in waveguide was the Notes on Lectures by J. Schwinger [2]. This work presents solutions to a number of waveguide discontinuity problems, among which there is a solution to a problem of scattering by a circular post in rectangular waveguide obtained by using the variational method. The process of solving the problem begins with the construction of a functional to be minimized in terms of parameters of an equivalent network. Once the functional is found, a solution to the problem may be obtained by finding a function that minimizes the functional. A number of numerical experiments have showed that the solution is more accurate when the sought function is represented in terms of series of eigenfunctions of Laplace operator in coordinate system corresponding to geometry of the obstacle. This approach is applicable in case of shifted post, but, unfortunately, not applicable in case of layered post. Results presented in [3] have been obtained by using a first-order approximation, that is, series expansion of the sought function with only first two terms retained. Numerical experiments show that the approach gives accurate results only for posts of small radius and small permittivity. Also the greatest discrepancy with measured results is observed near resonance. Araneta [4] improved Schwinger’s variational solution by retaining two more cylindrical terms in series expansions, thereby obtaining the so-called second order approximation. Although the approach provides more accurate results, it is found that in case of a second-order approximation, expressions become quite tedious, that leads to inconvenience of using them in practice.

A multifilament moment solution proposed in [5] allows finding the reflection and transmission coefficients of the dominant mode scattered by a metallic post. The scattered field produced by a post is simulated by a set of filamentary current sources with amplitudes to be determined. These sources are positioned on a closed surface enclosing the post. Numerical study shows that the most accurate results may be obtained when the closed surface has a similar shape to that of the cross section of the post. This approach was subsequently generalized to cases of dielectric post [6] and composite post [7]. Other authors have developed an approach that allows a solution to the scattering problem for structures consisting of a number of posts in the waveguide [8]. The total field inside the post is represented as a superposition of an incident wave and an unknown scattered field produced by the polarization current inside the post. Then by applying the waveguide Green's function, the inhomogeneous wave equation is converted to a Fredholm integral equation of the second kind that contains unknown function both under integral sign and explicitly outside it. In turn, this integral equation can be solved using volume discretization procedure, that is, by dividing cross section of the posts into a number of elements of simple shape and applying numerical integration (numerical quadrate) [9]. Despite the flexibility of the approach, it requires a lot of computation time in case of a post with large enough value of the dielectric permittivity, because the length of the greatest side of elements (simplex) should be at least ten times shorter than the wavelength of the electromagnetic wave in the post's material. Also latter two approaches involve waveguide Green's function that is represented in terms of infinite series of waveguide modes. For computation of each entry of the generalized impedance matrix the waveguide Green's function must be truncated and summed up that leads to a considerable increase in computation time.

The first approach based on division of the geometry of a problem into regions of simple geometry was proposed by Nielsen in [10]. Since the fields in regular regions are expressed in terms of series of eigenfunctions of the Laplace operator, it allows solving a boundary problem on the interfaces between layers of the post analytically, that in turn considerably reduces computational burden and provides sufficiently high resolution of field distribution. The last step of the approach is the derivation of a system of linear equations by application of the point matching procedure and truncation of infinite series. This approach is applicable only to the centered posts, namely, the posts with its axis placed at equal distances from the walls of the waveguide. Experiments showed that this approach gives accurate results only for posts of sufficiently small radius. This latter limitation can be overcome by introducing a small modification, namely, by replacing the rectangular interaction region with the circular one as proposed in [11]. The center of the circular interaction region coincides with the axis of the post and its radius equal to half the width of the broad wall of the waveguide. It has been found that the modified approach works fine even in case when values of the radius of the post and the complex dielectric permittivity are large. Also this approach is faster than aforementioned approaches in terms of the computation time and provides a sufficiently high degree of accuracy of results.

The major advantage is that the boundary problem on interfaces between layers of the posts can be solved analytically. This allows us to apply this approach also for the case of the layered post with an arbitrary number of layers by introducing modification only in its analytical part. Such a modification is discussed in this paper.

## II. FORMULATION

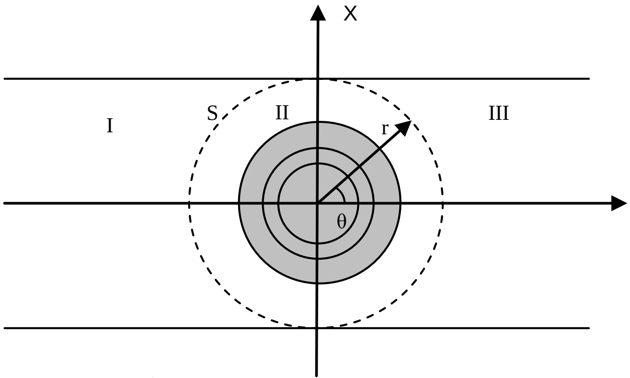

Fig. 2. Geometry of the problem.

Consider the dominant mode $TE_{10}$ propagating in a rectangular waveguide in direction of z axis. The width of the broad wall of the waveguide *a* is chosen so that only the dominant mode of the waveguide can propagate. Since higher order modes decay exponentially with distance from the post, the reflected and transmitted waves consist only of dominant modes. Nerveless, solving the boundary problem on the surface of the post, all existing higher order modes have to be taken into account, since they essentially affect the field distribution in immediate vicinity of the post. Since the geometry of the problem as well as the distribution of fields are uniform along the *y* axis, the problem can be reduced to an equivalent two-dimensional one, thus considerably simplifying the solution.

Throughout this paper we use the notation shown in Fig. 2. Total field in region I can be expressed as the superposition of incident and reflected dominant modes as well as higher order modes that exponentially decay in the negative z direction.

The incident wave is represented as follows:

$$E_y^i = \cos\left(\frac{\pi}{a}x\right)e^{-jk_1 z}. \tag{1}$$

where $k_1 = \sqrt{k_o^2 - \left(\frac{\pi}{a}\right)^2}$ is the waveguide wavenumber for the dominant $TE_{10}$; $a$ – the width of the broad wall of the waveguide; $k_o = \frac{\omega}{c}\sqrt{\mu_r^{(0)}\varepsilon_r^{(0)}}$ is the wavenumber of the waveguide filling material; $\varepsilon_r^{(0)}$ is the complex dielectric permittivity of the waveguide filling material; $\mu_r^{(0)}$ is the complex permeability of the waveguide filling material.

Due to linear dependence of the reflection and transmission coefficients upon amplitude of the incident wave, the normalization can be accomplished so that unknown coefficients $A_1$ and $B_1$ are identical to the reflection and transmission coefficients, respectively. The reflected part of the dominant mode together with higher order modes can be expressed as follows:

$$E_y^I = \sum_{m=1}^{\infty} A_m \cos\left(\gamma_m x\right) e^{jk_m z}. \tag{2}$$

where $k_m = \sqrt{k_0^2 - \left(\frac{m\pi}{a}\right)^2}$ is the waveguide wavenumber for different modes; $m$ – a positive integer; $\gamma_m = \frac{m\pi}{a}$.

Let us now consider expressions for fields in region III. In this region summary field is made up of transmitted wave and higher order modes that exponentially decay in z direction:

$$E_y^{III} = \sum_{m=1}^{\infty} B_m \cos\left(\gamma_m x\right) e^{-jk_m z}. \tag{3}$$

Since the distribution of the electric field of incident wave and geometry of the problem is symmetric about x = 0 plane, the distribution of electric component of scattered field must also have the same symmetry. In other words, only higher order modes, which have the symmetric distribution of electric field about x = 0 plane, are present in the waveguide, that is, the scattered wave consists only of modes with odd values of the index m=1,3,5...

In all cylindrical regions, except for the innermost layer, the electrical fields are expressed in terms of superposition of all possible solution of Laplace equation in cylindrical coordinate system, satisfying azimuthal periodicity condition. Cylindrical waves can be subdivided into two categories, namely, waves which have even distribution of electrical field about x=0 plane and those, which have odd distribution of the electrical field. Due to the symmetry of our problem, only even waves can propagate.

Expression for the electrical field component in region II:

$$E_y^{II} = \sum_{n=0}^{\infty} \left(C_n^0 J_n\left(k_0 r\right) + D_n^0 Y_n\left(k_0 r\right)\right) \cos\left(n\Theta\right). \tag{4}$$

Expression for the electrical field component in the k-th layer of the post:

$$E_y^k = \sum_{n=0}^{\infty} \left(C_n^k J_n\left(k_k r\right) + D_n^k Y_n\left(k_k r\right)\right) \cos\left(n\Theta\right). \tag{5}$$

where n is a positive integer;

$k_k = \frac{\omega}{c}\sqrt{\mu_r^k \varepsilon_r^k}$ is the wavenumber of material of k-th layer; $\varepsilon_r^k$ is the complex dielectric permittivity of the material of k-th layer; $\mu_r^k$ - the complex permeability of the material of k-th layer; *k* – the index of layer.

Let us now consider representation of fields in the innermost layer of the post. In contrast to the previous case, where dependence on the radial coordinate was represented by both cylindrical functions, in this case the coefficient in front of the Neumann function must be equal to zero. That is because the value of the Neumann function tends to minus infinity as the value of the argument approaches zero. However, field intensities cannot be infinitely great, in accordance with the energy conservation law. As a result, in the innermost layer of the post, distribution of electrical field is represented by the following expression:

$$E_y^p = \sum_{n=0}^{\infty} \left(C_n^p J_n\left(k_p r\right)\right) \cos\left(n\Theta\right). \tag{6}$$

where $k_p = \frac{\omega}{c}\sqrt{\mu_r^p \varepsilon_r^p}$ is the wavenumber of material of the post; $\varepsilon_r^p$ is the complex permittivity of the innermost layer of the post; $\mu_r^p$ is the complex permeability of the innermost layer of the post; $p$ is index value for the innermost layer.

Components of the magnetic field may be found by using expressions (1) through (6) and the second Maxwell's equation. After some mathematical manipulations, we obtain the following expressions for the azimuthal component of the magnetic field in different regions.

Expression for the incident wave:

$$\dot{H}_\varphi^i = \frac{j}{\omega\mu_0\mu_r^{(0)}}\Big(\gamma_1 \sin(\Theta)\sin\left(\gamma_1 x\right) + + jk_1 \cos(\Theta)\cos\left(\gamma_1 x\right)\Big)e^{-jk_1 z}. \tag{7}$$

Expression for the reflected wave:

$$\dot{H}_\varphi^I = \frac{j}{\omega\mu_0\mu_r^{(0)}} \sum_{m=1}^{\infty} A_m \Big(\gamma_m \sin(\Theta)\sin\left(\gamma_m x\right) - - jk_m \cos(\Theta)\cos\left(\gamma_m x\right)\Big)e^{jk_m z}. \tag{8}$$

Expression for the transmitted wave:

$$\dot{H}_\varphi^{III} = \frac{j}{\omega\mu_0\mu_r^{(0)}} \sum_{m=1}^{\infty} B_m \Big(\gamma_m \sin(\Theta)\sin\left(\gamma_m x\right) + +jk_m \cos(\Theta)\cos\left(\gamma_m x\right)\Big)e^{-jk_m z}. \tag{9}$$

Expression for region II:

$$H_\varphi^{II} = = \frac{-j}{\omega\mu_0\mu_r^{(0)}} \sum_{n=0}^{\infty} k_0 \left(C_n^0 J_n'\left(k_0 r\right) + D_n^0 Y_n'\left(k_0 r\right)\right) \cos\left(n\Theta\right). \tag{10}$$

Expression for the kth layer of the post:

$$H_\varphi^k = = \frac{-j}{\omega\mu_0\mu_r^k} \sum_{n=0}^{\infty} k_k \left(C_n^k J_n'\left(k_k r\right) + D_n^k Y_n'\left(k_k r\right)\right) \cos\left(n\Theta\right). \tag{11}$$

Expression for the innermost layer of the post:

$$H_\varphi^p = \frac{-j}{\omega\mu_0\mu_r^p} \sum_{n=0}^{\infty} k_p \left(C_n^p J_n'\left(k_p r\right)\right) \cos\left(n\Theta\right). \tag{12}$$

By enforcing boundary conditions on the interface between the innermost layer of the post and the next layer as well as taking the advantage of the mutual orthogonality of cylindrical waves with respect to coordinate $\Theta$, the $D_n^{p-1}$ constants may be expressed through corresponding $C_n^{p-1}$ constants. The relation between constants for the layer next to the innermost one may be expressed as follows:

$$D_n^{p-1} = \alpha_n^{p-1} \cdot C_n^{p-1}. \tag{13}$$

where

$$\alpha_n^{p-1} = \\ = \frac{\left( \frac{k_p}{\mu_r^p} J_n\left(k_{p-1} r_p\right) J_n'\left(k_p r_p\right) - \frac{k_{p-1}}{\mu_r^{p-1}} J_n'\left(k_{p-1} r_p\right) J_n\left(k_p r_p\right) \right)}{\left( \frac{k_{p-1}}{\mu_r^{p-1}} Y_n'\left(k_{p-1} r_p\right) J_n\left(k_p r_p\right) - \frac{k_p}{\mu_r^p} Y_n\left(k_{p-1} r_p\right) J_n'\left(k_p r_p\right) \right)}$$

Here, the prime denotes the derivative with respect to the radial coordinate. By iteratively applying boundary conditions for each interface between layers of the post we obtain the following expression for every k value, from 0 trougth p-2:

$$D_n^k = \alpha_n^k \cdot C_n^k \tag{14}$$

where

$$\alpha_n^k = \frac{\left( \frac{k_{k+1}}{\mu_r^{k+1}} J_n\left(k_k r_{k+1}\right) \beta_n'^k - \frac{k_k}{\mu_r^k} J_n'\left(k_k r_{k+1}\right) \beta_n^k \right)}{\left( \frac{k_k}{\mu_r^k} Y_n'\left(k_k r_{k+1}\right) \beta_n^k - \frac{k_{k+1}}{\mu_r^{k+1}} Y_n\left(k_k r_{k+1}\right) \beta_n'^k \right)};$$

$$\beta_n^k = J_n\left(k_{k+1} r_{k+1}\right) + \alpha_n^{k+1} Y_n\left(k_{k+1} r_{k+1}\right);$$

$$\beta_n'^k = J_n'\left(k_{k+1} r_{k+1}\right) + \alpha_n^{k+1} Y_n'\left(k_{k+1} r_{k+1}\right).$$

Expression for the coefficient $\alpha_n^0$ is similar to that in [11] except that in our case the constant $D_n$ is expressed through the constant $C_n$, not otherwise. Numerical experiments show that our choice is more appropriate, because it gives a significant decrease in condition number of the resultant matrix that in turn leads to more accurate results. Also it is found that the further decrease in the condition number of the matrix may be achieved by dividing the second and the fourth equations of the system (18) by wavenumber $k_0$.

Since the value of $\alpha_n^0$ coefficient is dependent only on the radial component, it follows that its value does not have to be recalculated for every point on the surface of interaction region when applying point matching procedure for fixed values of frequency and the complex dielectric permittivity that in turn results in the reduction of the computation time. Therefore, it is advantageous to use the cylindrical surface not only because it ensures convergence of the approach, but also because of smaller amount of the computation time.

By inserting expression for coefficient $\alpha_n^0$ into expressions (4) and (10), we obtain the following compact representations of fields in region II.

For the component of the electric field:

$$E_y^{II} = \sum_{n=0}^{\infty} C_n^0 Q_n\left(k_o r\right) \cos\left(n\Theta\right). \tag{15}$$

For the component of the magnetic field:

$$H_\varphi^{II} = \frac{-j}{\omega \mu_0 \mu_r^{(0)}} \sum_{n=0}^{\infty} k_o C_n^0 Q_n'\left(k_0 r\right) \cos\left(n\Theta\right). \tag{16}$$

where $Q_n\left(k_0 r\right) = J_n\left(k_0 r\right) + \alpha_n^0 \cdot Y_n\left(k_0 r\right),$
$Q_n'\left(k_0 r\right) = J_n'\left(k_0 r\right) + \alpha_n^0 \cdot Y_n'\left(k_0 r\right).$

By enforcing boundary conditions on surface S, we obtain the set of four equations (17). As it can be seen, this set contains infinite series of cylindrical waves and waveguide modes. Since cylindrical waves and waveguide modes are not mutually orthogonal, it means that it is impossible to solve this system by some analytical manipulations. The only way to solve this system is to convert it into the system of linear algebraic equations. This can be accomplished by means of the so-called series truncation procedure that retains only first *M* terms in series of waveguide modes and only *N* first terms in series of cylindrical waves. Once truncation procedure has been applied, the next step is to apply another method, which is referred to as a point matching method. The pre-requisite of this method is to meet boundary conditions only at a set of the selected points on the boundary surface.

By performing the above-mentioned procedures, we obtain a system of linear algebraic equations (18) with respect to unknown expansion coefficients $A_n$, $B_n$ and $C_n^0$. Values of these coefficients can be found by applying some of the commonly used methods for solving a system of linear algebraic equations, such as Gaussian elimination method. It is possible to reduce by half the computation time by using block decomposition methods, but we plan to consider it in a detail in forthcoming works.

There is a problem that arises when the post is metallic or there is a post with a metallic layer that has a high value of conductivity, namely, the value of the Bessel function becomes so large that it causes an overflow of double precision floating point number used in MATLAB. This problem may be avoided in either of two ways. One way is to enforce approximate boundary conditions on the surface of the metallic layer of the post, such as those presented in [12]. However, this approximation is valid only when the wavelength in the metal is much smaller than the thickness of the layer. Another way to avoid this problem is to employ approximations for the modified Bessel functions for large argument values. For more information on these approximations the interested reader may refer to [13].

$$\begin{cases} \sum\limits_{n=0}^{\infty} C_n^0 Q_n\left(k_0\frac{a}{2}\right)\cos n\Theta - \sum\limits_{m=1}^{\infty} A_m \cos(\gamma_m x_\varphi) e^{jk_m z_\varphi} = \cos(\gamma_1 x_\varphi) e^{-jk_1 z_\varphi} \\ \sum\limits_{n=0}^{\infty} k_0 C_n^0 Q'_n\left(k_0\frac{a}{2}\right)\cos n\Theta - \sum\limits_{m=1}^{\infty} A_m \Big(jk_m \cos(\gamma_m x_\varphi)\cos(\Theta) - \\ \qquad -\gamma_m \sin(\gamma_m x_\varphi)\sin(\Theta)\Big) e^{jk_m z_\varphi} = -\Big(\gamma_1 \sin(\gamma_1 x_\varphi)\sin(\Theta) + jk_1 \cos(\gamma_1 x_\varphi)\cos(\Theta)\Big) e^{-jk_1 z_\varphi} \\ \sum\limits_{n=0}^{\infty} C_n^0 Q_n\left(k_0\frac{a}{2}\right)\cos n\Theta - \sum\limits_{m=1}^{\infty} B_m \cos(\gamma_m x_\varphi) e^{-jk_m z_\varphi} = 0 \\ \sum\limits_{n=0}^{\infty} k_0 C_n^0 Q'_n\left(k_0\frac{a}{2}\right)\cos n\Theta + \sum\limits_{m=1}^{\infty} B_m \Big(jk_m \cos(\gamma_m x_\varphi)\cos(\Theta) + \gamma_m \sin(\gamma_m x_\varphi)\sin(\Theta)\Big) e^{-jk_m z_\varphi} = 0 \end{cases} \tag{17}$$

where $x_\varphi = \frac{a}{2}\sin(\Theta)$; $z_\varphi = \frac{a}{2}\cos(\Theta)$.

$$\begin{cases} \sum\limits_{n=0}^{N} C_n^0 Q_n\left(k_0\frac{a}{2}\right)\cos n\Theta_a - \sum\limits_{m=1}^{M} A_m \cos(\gamma_m x_a) e^{jk_m z_a} = \cos(\gamma_1 x_a) e^{-jk_1 z_a} \\ \sum\limits_{n=0}^{N} k_0 C_n^0 Q'_n\left(k_0\frac{a}{2}\right)\cos n\Theta_a - \sum\limits_{m=1}^{M} A_m \Big(jk_m \cos(\gamma_m x_a)\cos(\Theta_a) - \\ \qquad -\gamma_m \sin(\gamma_m x_a)\sin(\Theta_a)\Big) e^{jk_m z_a} = -\Big(\gamma_1 \sin(\gamma_1 x_a)\sin(\Theta_a) + jk_1 \cos(\gamma_1 x_a)\cos(\Theta_a)\Big) e^{-jk_1 z_a} \\ \sum\limits_{n=0}^{N} C_n^0 Q_n\left(k_0\frac{a}{2}\right)\cos n\Theta_b - \sum\limits_{m=1}^{M} B_m \cos(\gamma_m x_b) e^{-jk_m z_b} = 0 \\ \sum\limits_{n=0}^{N} k_0 C_n^0 Q'_n\left(k_0\frac{a}{2}\right)\cos n\Theta_b + \sum\limits_{m=1}^{M} B_m \Big(jk_m \cos(\gamma_m x_b)\cos(\Theta_b) + \gamma_m \sin(\gamma_m x_b)\sin(\Theta_b)\Big) e^{-jk_m z_b} = 0 \end{cases} \tag{18}$$

where $x_a = \frac{a}{2}\sin(\Theta_a)$; $x_b = \frac{a}{2}\sin(\Theta_b)$;$z_a = \frac{a}{2}\cos(\Theta_a)$;$z_b = \frac{a}{2}\cos(\Theta_b)$;$\Theta_a = \pi - \Theta_b$;

$\Theta_k = \frac{\pi k}{2K}$, where$k = 0, 1, 2, ..., K-1$.

## III. NUMERICAL RESULTS

To exanimate the convergence of the approach we may use the point where S surface touches the wall of the waveguide, namely, the point, which has the following coordinates (r=a/2, θ = π/2). Since we consider that the walls of the waveguide are made of a perfect electric conductor, the value of the intensity of the electric field must vanish at this point. Let us now examine the convergence of the approach, with the following values of parameters: ε = 2, λ/a =1.4, f = 9.373828 GHz, r/a=0.05 and provided that $2M = N$ and $K = M$. The results of this examination are presented in the data table below. Another way to verify the convergence of the algorithm, although valid only for the case of a lossless material, is to estimate the value of the absorption coefficient. The value of the absorption coefficient may be obtained by the following expression:

$$A = \sqrt{1 - |R|^2 - |T|^2}. \tag{19}$$

where *R* is the reflection coefficient; *T* - the transmission coefficient.

As it can be seen from the table, the error decreases as the N value increases, where *N* is number of retained terms in cylindrical series expansion of field. It is seen that initially as the value of *N* increases, the deference between results decreases monotonically. However, as a value of *N* becomes very large there is no more monotonic decrease in the value of the error. At small values of *N*, the major source of error is caused by the insufficient number of terms that leads to the inaccurate representation of fields. In turn, at high *N* values error arises due to the round-off error that is caused by the ill-conditioned matrix.

TABLE I

EXAMINATION OF THE CONVERGENCE OF THE ALGORITHM

| N | Error in intensity of the electric field | Absolute value of the reflection coefficient | Phase of the reflection coefficient |
|---|---|---|---|
| 4 | 6.117604e-001 | 0.50243539072841 | 118.8327 |
| 6 | 6.751014e-003 | 0.05274591850723 | 93.0655 |
| 8 | 6.570297e-004 | 0.04924808825854 | 92.8592 |
| 10 | 9.613224e-005 | 0.04952066532204 | 92.8748 |
| 12 | 2.311151e-005 | 0.04949766995303 | 92.8735 |
| 14 | 6.105177e-006 | 0.04949324542010 | 92.8732 |
| 16 | 1.881289e-006 | 0.04949227904848 | 92.8732 |
| 18 | 7.769263e-007 | 0.04949204975343 | 92.8732 |
| 20 | 3.362675e-007 | 0.04949195921715 | 92.8732 |
| 22 | 1.350975e-007 | 0.04949191917442 | 92.8732 |
| 24 | 4.691545e-008 | 0.04949190226793 | 92.8732 |
| 26 | 1.584562e-008 | 0.04949189660296 | 92.8732 |
| 28 | 5.535497e-009 | 0.04949189478046 | 92.8732 |
| 30 | 2.265794e-009 | 0.04949189421693 | 92.8732 |
| 32 | 1.088017e-009 | 0.04949189400668 | 92.8732 |
| 34 | 6.339962e-010 | 0.04949189392193 | 92.8732 |

The point matching procedure ensures that field intensities are equal only at the set of selected points on the surface of interaction region, but it does not necessarily mean that the intensities given by different expansions will be the same at other points of this surface. Absolute values of errors in electric and magnetic field intensities may be obtained by the following formulae similar to those proposed in [7].

For the S surface part connecting regions II and I:

$$\Delta E = \frac{\left|E_y^{II}\left(\frac{a}{2},\Theta\right)\right| - \left|E_y^{I}\left(\frac{a}{2},\Theta\right)\right|}{\left|E_y^{II}\left(\frac{a}{2},\Theta\right)\right|} \quad (20)$$

$$\Delta H = \frac{\left|H_\varphi^{II}\left(\frac{a}{2},\Theta\right)\right| - \left|H_\varphi^{I}\left(\frac{a}{2},\Theta\right)\right|}{\left|H_\varphi^{II}\left(\frac{a}{2},\Theta\right)\right|} \quad (21)$$

For the S surface part connecting regions II and III:

$$\Delta E = \frac{\left|E_y^{II}\left(\frac{a}{2},\Theta\right)\right| - \left|E_y^{III}\left(\frac{a}{2},\Theta\right)\right|}{\left|E_y^{II}\left(\frac{a}{2},\Theta\right)\right|} \quad (22)$$

$$\Delta H = \frac{\left|H_\varphi^{II}\left(\frac{a}{2},\Theta\right)\right| - \left|H_\varphi^{III}\left(\frac{a}{2},\Theta\right)\right|}{\left|H_\varphi^{II}\left(\frac{a}{2},\Theta\right)\right|} \quad (23)$$

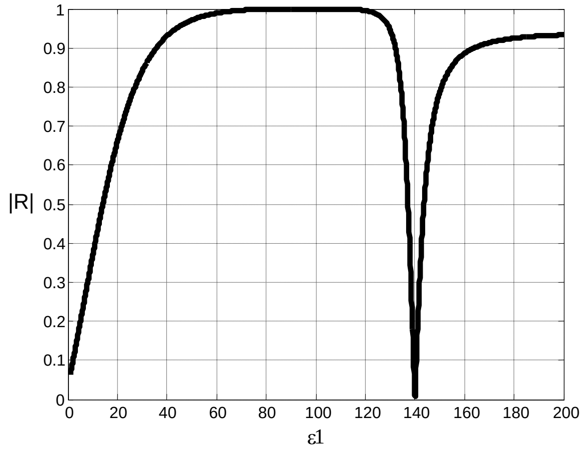

Fig. 3. Modulus of reflection coefficient as a function of dielectric permittivity (λ/a =1.4, f = 10 GHz, N = 12, r1/a=0.05, r2/a=0.03, r3/a=0.02, ε1 = 1 – 200 with step size 0.1, ε2=4, ε3=5-j0.05).

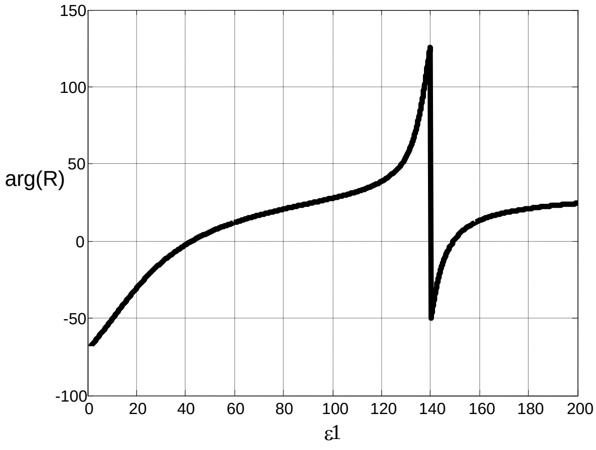

Fig. 4 Phase angle of the reflection coefficient as a function of dielectric permittivity (λ/a =1.4, f = 10 GHz, N = 12, r1/a=0.05, r2/a=0.03, r3/a=0.02, ε1= 1 – 200 with step size 0.1, ε2=4, ε3=5-j0.05).

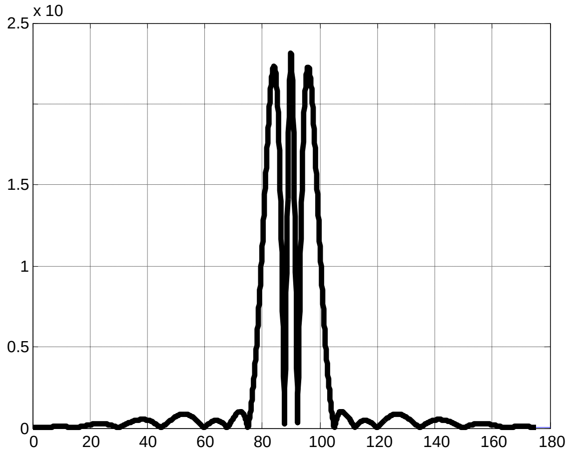

Fig. 5. Absolute value of error of electric field intensity at the surface r=a/2 as a function of azimuthal coordinate (ε = 2, λ/a =1.4, f = 9.373 GHz, r1/a=0.05, θ = 0 - 180 with step size 0.09004).

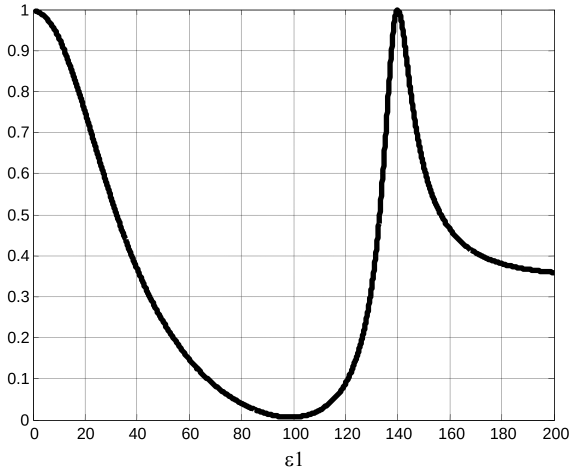

Fig. 6 Modulus of the trransmission coefficient as a function of dielectric permittivity (λ/a =1.4, f = 10 GHz, N = 12, r1/a=0.05, r2/a=0.03, r3/a=0.02, ε1 = 1 – 200 with step size 0.1, ε2=4, ε3=5-j0.05).

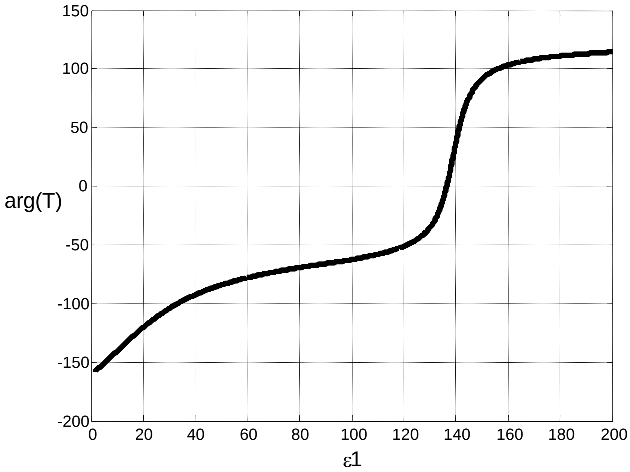

Fig. 7 Phase angle of the transmission coefficient as a function of dielectric permittivity (λ/a =1.4, f = 10 GHz, N = 12, r1/a=0.05, r2/a=0.03, r3/a=0.02, ε1 = 1 – 200 with step size 0.1, ε2=4, ε3=5-j0.05).

## IV. CONCLUSIONS

In this paper we have chosen from a number of approaches proposed by various authors the one that at the same time provides accurate results and requires small amount of the computation time. Then this approach has been generalized to the case of scattering by a layered post. Such an approach is of interest because it provides a faster solution to the problem than other approaches and software employing flexible methods, such as a finite element method and finite difference method.

In forthcoming papers we plan to investigate application of metamerials with an objective to improve some characteristics of narrowband microwave waveguide post filters. Also we plan to verify a hypothesis that the accuracy of measuring of the complex dielectric permittivity of liquids by a short-circuited waveguide method may be improved by choosing some optimal combinations of values of inner and outer radius of the platic tube.

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**Romāns Kušņins, Jānis Semeņako. Difrakcija uz slāņaina cilindra taisnstūra viļņvadā**
Šajā darbā tiek apskatīts pamattipa viļņa difrakcijas uz slāņaina cilindra taisnstūra viļņvadā. Gan mikroviļņu filtru sintēzes procedūra, gan kompleksas dielektriskās caurlaidības mērīšana prasa inversā difrakcijas uzdevuma atrisināšanu, tādēļ ka inversā uzdevuma algoritma skaitļošanas laiks it tieši atkarīgs no atbilstoša tiešā uzdevuma algoritma skaitļošanas laika, un mūsu mērķis ir atrast tādu tiešā uzdevuma risināšanas metodi, kura nodrošina augstu rezultātu precizitāti un strauju konverģenci. Dažu pēdējo desmitgadu laikā parādījās dažādu autoru piedāvātas metodes difrakcijas uz cilindra taisnstūra viļņvadā uzdevuma risināšanai. Šajā darbā apskatītā metode ir viena no šīm piedāvātajām metodēm, proti, tās metodes, kura nodrošina strauju konverģenci un pietiekami augstu rezultātu precizitāti, modifikācija. Šī modifikācija ļauj risināt uzdevumu slāņaina cilindra gadījumā. Metodes princips balstās uz viļņvada sadalīšanu atsevišķos apgabalos ar vienkāršu ģeometriju, ar iedomātas cilindriskas virsmas palīdzību. Ārējos apgabalos lauks tiek aprakstīts bezgalīgu viļņvada viļņu tipu rindu formā. Vidējā apgabalā lauks tiek aprakstīts bezgalīgu cilindrisku viļņu rindu formā. Pielietojot kolokāciju metodi uz iedomātās virsmas un arī atmetot augtākas kārtas rindu locekļus, rezultātā iegūstam lineāru algebrisku vienādojumu sistēmu, kura raksturojas ar lielu nosacītības skaitli. Šo nosacītības skaitli var samazināt ar vienkāršu matemātisku manipulāciju palīdzību. Algoritma konverģences pārbaudīšanai šajā darbā tiek izmantoti divi kritēriji. Pirmais kritērijs ir absorbcijas koefcienta vērtība, kurai ir jābūt vienādai ar nulli gadījumos, kad slāņains cilindrs ir izveidots no materiāliem bez zudumiem. Otrais kritērijs ir elektriskā lauka vērtība punktā, kurā cilindriskā virsma pieskaras pie viļņvada sienas. Otrais kritērijs ir spēkā arī gadījumos, kad cilindrs sastāv no materiāliem ar zudumiem.

**Роман Кушнин, Янис Семеняко. Дифракция на многослойном цилиндре в прямоугольном волноводе.**
В работе рассмотренна задача рассеяния основной моды на многослойном цилиндре в прямоугольном волноводе. Как процедура синтеза фильтров, так и измерение комплексной диэлектрической проницаемости требуют решения обратной задачи рассеяния. Так как время, затрачиваемое алгоритмом на решение обратной задачи, напрямую зависит от времени, затрачиваемого алгоритмом, для решения прямой задачи, нашей целью является отискание такого метода, который обеспечивает достаточно высокую точность расчетов и обладает быстрой сходимостью. За последние несколько десятилетий различными авторами был предложен ряд методов решения задачи рассеяния на цилиндре в прямоугольном волноводе. Метод, рассмотренный в данной работе, является модификацией одного из этих методов, а именно того, который обеспечивает достаточный уровень

точности результатов, а также быструю сходимость. Модифицированный метод позволяет решать задачи рассеяния на многослойном цилиндре. Суть метода заключается в разделении волновода на отдельные области, посредством мнимой цилиндрической поверхности. Во внешних областях поле представленно в виде разложения в бесконечный ряд по волноводным модам. В свою очередь, в средней области поле представлено в виде разложения в бесконечный ряд по цилиндрическим волнам. Применяя метод коллокаций на мнимой цилиндрической поверхности, а также отбрасывая члены высших порядков бесконечных рядов, в результате получаем систему линейных алгебраических уравнений, которая характеризуется большим значением числа обусловленности. Число обусловленности можно уменьшить с помощью простых математических манипуляций. Для проверки сходимости в данной работе использованы два критерия. Первым критерием является значение коэффициента поглощения, которое должно равняться нулю в случае, когда цилиндр состоит из материалов без потерь. Вторым критерием является значение элетрического поля в точке, в которой цилиндрическая поверхность соприкасается со стенкой волновода. Второй критерий применим также и в случае, когда цилиндр состоит из материалов с потерями.