Optimization of Algorithms for Processing Flight Information in Flight Test Stage of Aerospace Objects

Genady Burov, Riga Technical University

Abstract - A specific feature of aircraft and spacecraft flight modes is related to difficulties of obtaining information about dynamic characteristics of vehicles and their onboard equipment. The reason is that any transients are undesirable from the flight safety point of view and consequently they are damped by special onboard devices. At the same time, there is a practical necessity of obtaining such information as it is necessary for indication of occurrence of emergency situations in flight conditions. This information can be derived by solving systems of equations formed from the results of signal measurement. But, because of weak dynamism, the equation systems, as a rule, are illconditioned and traditional algorithms cannot be used. Therefore, the problem of their optimization is particularly important in conditions of deficiency of dynamism in the flight information. The possibility of using new information technologies for signal processing at the aircraft flight test stage is investigated. Since there are restrictions imposed on flight modes, the identification of parameters of aircraft and its equipment is carried out in the conditions of bad observability of the dynamic characteristics. It complicates the problem of ensuring the usability of computing algorithms for solving the systems of equations formed from the results of measurements of flight parameters. Therefore, the possibility of application of new information technologies for creation of algorithms with parallel structure is investigated. It will allow applying parallel computers and increasing the speed of information processing. The new information technologies are offered to be realized on the basis of symbolical combinatory computing models.

Keywords – identification, flight test, symbolical combinatory model

I. INTRODUCTION

Aircraft flight tests are high-risk tasks; therefore, special attention is given to safety measures at all stages of preparation for carrying out these tests. The necessity of processing huge amounts of information over short time intervals during test flights demands using high-performance onboard computers and more advanced programs and algorithms [19], [20].

Wider introduction of mathematical modelling at the flight test phase will also allow increasing flight safety and gathering much more information about the characteristics of stability and controllability of an aircraft and also about the quality of functioning of new onboard equipment in real modes of flight [18], [19].

Special attention is given to the most dangerous modes of flight. The analysis shows that it is necessary to use new

higher performance onboard computers and more advanced control algorithms. Such problems can be solved by introducing information technologies constructed on novel principles.

It is necessary to take into account the fact that signals taken from measuring sensors have smooth-changing character and such signals are difficult to process using traditional computing algorithms. It is a general feature of all objects in aerospace systems and it is dictated by restrictions on flight modes, which are introduced to ensure the flight safety.

Therefore, the traditional algorithms related to the central problem of information processing: solving systems of equations, generated from such low-dynamic signals, turn out to be inapplicable, as they cannot preserve the needed accuracy in conditions when matrix of the system is close to singular. Such algorithms return false results that can lead to making wrong decisions both during the flight and during the inter-flight analysis. For this reason, theoretical models of identification suggested in numerous publications could not be used in practical applications.

The structure of these algorithms is based on principles of consecutive execution of computing operations. Therefore, they are not well suited for their realization in highperformance parallel computer. From here follows that for creation of more advanced systems of onboard measurements used in test flights, it is necessary to introduce new technologies, allowing realizing parallel principles of information processing. On the basis of such technologies, essentially new computing methods can be created that would allow to preserve high accuracy in conditions of illconditionality of equation system matrices. In this case, development of adaptive computing models with adjustable structure, which can be realized by software methods, is possible.

Symbolical combinatory (SC) models were constructed not on the basis of known classical methods of enumerative combinatorics, but on the basis of giving computing algorithms new properties possessed by determined combinatory operators. The validity of using such approach has been confirmed by solving the problem of finding the inverses of almost singular 20th order Hilbert matrices with 100% accuracy [21]. Earlier it was believed that solving such problem is impossible for matrices the order of which exceeds 10.

II. PROBLEM STATEMENT

A specific feature of flight information processing is that transients have weak dynamism as they are damped by special devices because of conditions of flight safety. Therefore partial coefficients C_i of transients:

$$y(kT) = \sum_{i=1}^{n} C_i q_i^{k}; q_i = \exp(-a_i T)$$

have small values and this leads to formation of illconditioned matrices of systems of identification equations. This is the reason why traditional algorithms for inversion of matrices of identification equation systems cannot be used. The vector of system's solution is used for diagnosis of condition of onboard equipment and, consequently, it is necessary to provide high accuracy of its estimate. As it is shown in [1], methods of identification based on various modifications of stochastic models [23], [24] are unsuitable for this purpose [1] because of introduced methodical mistakes and impossibility of decoding the obtained results that have abstract content that is not related to physical condition of the object. Traditional computing algorithms for solving illconditioned equation systems, which are based on Gaussian elimination, are inapplicable because in the recurrent procedure the results of the previous step are divided by small numbers that are comparable to the level of noise and that leads to unreliable results. Therefore, development of special algorithms intended for processing poorly dynamical flight information is necessary. For this purpose, symbolical combinatory models (SC models) are used. In this paper, methods for optimization of computing algorithms based on such models are examined.

III. OPTIMIZATION OF ALGORITHMS FOR IDENTIFICATION OF CONDITION OF AIRCRAFT ONBOARD SYSTEMS

Calculation of the vector of values of dynamic parameters is related to finding information about the coefficients of the transfer operator of identified object:

$$W(p) = \frac{R(p)}{Q(p)} = \frac{p^n + q_{n-1}p^{n-1} + \dots + q_1p + q_0}{b_m p^m + b_{m-1}p^{m-1} + \dots + b_1p + b_0} = \frac{y(p)}{x(p)} \quad (1)$$

which is analogous to its differential equation. However, as the process of transient processing has a discrete character, it is necessary to use parameters of the discrete operator:

$$D(z) = \varphi Z(T) * W(p) \Rightarrow \frac{A(z)}{B(z)} =$$
$$= \frac{\alpha_m z^m + \alpha_{m-1} z^{m-1} + \dots + \alpha_1 z + \alpha_0}{z^n + \beta_{n-1} z^{n-1} + \beta_{n-2} z^{n-2} + \dots + \beta_1 z + \beta_0} = \frac{y(z)}{x(z)} \quad (2)$$

which is an approximation of (1). On its basis, a system of difference equations is formed from the results of measurement of input signal x(iT) and output signal y(iT):

$$[X] \cdot \overline{\alpha} + [Y] \cdot \overline{\beta} = \overline{y}$$
(3)

For this purpose, various modifications of algorithms [8], [9] can be used. In a test mode, when mathematical description of input signal is known, it is desirable to apply a system of smaller dimension:

$$\begin{bmatrix} Y \end{bmatrix} \cdot \overline{\hat{\beta}} = \overline{y}; \quad \begin{bmatrix} Y \end{bmatrix}_{i \ j} = y[t + (i + j - 1)T];$$
$$\begin{bmatrix} \overline{y} \end{bmatrix}_{i} = y[t + (n + i)T]$$
(4)

Its conditionality is better than that of the system (3). First, the system (4) is solved and an estimation of coefficients of characteristic polynomial of operator (2) is found. Then, an estimation of coefficients of polynomial-numerator of operator (2) is calculated [4]:

$$\overline{\hat{\beta}} \Longrightarrow H \cdot \overline{y}; \quad H = [Y]^{-1}$$
 (5)

The matrix H is formed as a form of Toeplitz matrix Y. Finding its inverse matrix H using numerical methods can possibly result in large errors, as Toeplitz matrices have extremely bad conditionality. This demands application of non-conventional methods for maintaining the numerical stability of algorithms for inversion of such matrices. Vectors of solutions (3) and (5) can be used as diagnostic attributes, and the technique of their calculation for regular modes of functioning is stated in [2], [25], [26]. They have a determined unequivocal character and are described by operations of direct and inverse discrete transform:

$$\varphi Z(T) * W(p) \Longrightarrow D(z); \ \varphi Z^{-1}(T) * D(z) \Longrightarrow W(p) \quad (6)$$

It has been proved that mathematical relation between the poles of operators (1) and (2) does not depend on the character of used interpolation approximation. It has the form of significantly nonlinear relations:

$$\left\{\xi_{i} = \exp(-a_{i}T)\right\} \Leftrightarrow \left\{a_{i} = -\frac{Ln(\xi_{i})}{T}\right\}$$
$$\left[\overline{\beta}\right]_{i} = \varphi Sum * \left(\varphi KC(i) * \bigcup_{j=1}^{n} \xi_{j}\right)$$
(7)

The expressions (7) are determined by the character of arrangement of analog and discrete poles on the complex plane. Mapping of analog poles into the discrete ones occurs by the principle of mapping the infinite left complex halfplane into the area of the unit right half-circle. Therefore,

2011 Volume 50

numerical values of distances between discrete poles, which are used in the algorithms, can become comparable in size with the methodical errors. Their negative influence shows in the solution of the systems of difference equations. The determined character of relations between the poles of operators (1) and (2) determines the same character of relations between the coefficients of their characteristic polynomials:

$$\left\{\varphi Z^{-1} * B(z)\right\} \Leftrightarrow \left\{Q(p) = \prod_{i=1}^{n} (p+a_i)\right\}$$
(8)

The processed transient, in general, can be described by expression:

$$y(t) = \sum_{i=1}^{n} C_i \exp(a_i \cdot t)$$
 (9)

which can be expressed as depending on the discrete poles of operator (2):

$$y(kT) = \sum_{i=1}^{n} C_{i}q_{i}^{k}; \ q_{i} = \exp(-a_{i}T)$$
(10)

In general, it is found from the analog transfer function of object W(p), taking into account the operator of interpolation filter F(p, z). This operator designates the mathematical operation of smoothing the mistakes of discrete approximation of input signal [25], [26]. Therefore:

$$D(z) = Fz(T) * \{W(p) \cdot F_{IIHT}(z, T)\} =$$

$$= \frac{\sum_{i=1}^{m} \alpha_{i} \cdot z^{-(k+i)}}{\sum_{i=1}^{n} \xi_{i} \cdot z^{-n}} = \frac{y(z)}{x(z)}$$
(11)

Let's consider a mode of test identification when a system of smaller dimension (5) is solved:

$$\overline{\xi}^{(n)} = H \cdot \overline{y}^{(n)} \quad H = Y^{-1} \tag{12}$$

$$\overline{\xi} = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \cdots \\ \xi_n \end{bmatrix} \quad \overline{y} = \begin{bmatrix} y_1 \\ y_1 \\ \cdots \\ y_n \end{bmatrix}$$
(13)

Computing difficulties arise at finding of the inverse matrix $Y^{-1}(12)$. Let's denote in the form of attached matrix \tilde{H} the elements of which are expressed by values of minors of matrix *Y* of the transient (5):

$$\widetilde{H} = \begin{bmatrix} H_{11} & H_{21} & \dots & H_{n1} \\ H_{12} & H_{22} & \dots & H_{n2} \\ \dots & \dots & \dots & \dots \\ H_{1n} & H_{2n} & \dots & H_{nn} \end{bmatrix}$$
(14)

$$\tilde{H} \cdot \frac{1}{\Delta} \cdot H = E \tag{15}$$

$$\Delta = \det H \tag{16}$$

The elements of *H* can be found by division of elements of attached matrix \tilde{H} (14) by the determinant *Y*:

$$H = \tilde{H} \cdot \frac{1}{\Delta} \tag{17}$$

$$\left[H\right]_{ij} = \frac{H_{ji}}{\Delta} \tag{18}$$

So, the vector of solution of system (12) can be represented by expression:

$$\overline{\xi} = \widetilde{H} \cdot \frac{1}{\Delta} \tag{19}$$

Let's notice that algebraic complements of elements of rows are located in \tilde{H} in corresponding columns, that is, the operation of transposing is done.

Here $\Delta = \det H$ is the determinant of initial matrix Y. Like the minors in (18), it is found using the formula:

$$\det H = \sum_{P} (-1)^{\chi} \cdot (a_{1\alpha_1} \cdot a_{2\alpha_2} \cdot, \dots \cdot a_{n\alpha_n})$$
(20)

Here, the sum is distributed over all possible permutations $P = (\alpha_1, \alpha_2, ..., \alpha_n)$ of elements Y: 1, 2, ..., n, not containing repetitions of elements. Hence, equation (20) contains n! addends, while $\chi = 0$ for even permutations and $\chi = 1$ for odd permutations.

Elements of vector $\overline{\xi}$ can also be determined using Cramer's rule:

$$\xi_1 = \frac{\Delta_1}{\Delta}; \ \xi_1 = \frac{\Delta_2}{\Delta}; \dots \xi_n = \frac{\Delta_n}{\Delta}; \tag{21}$$

Operation of calculation of minors is used here as well, and unfolding of matrix is done based on elements of free members of system (5):

$$B = \begin{bmatrix} h_{11} & \dots & u_{1,i-1} & y_1 & h_{1,i+1} & \dots & h_{1n} \\ h_{21} & \dots & h_{2,i-1} & y_2 & h_{2,i+1} & \dots & h_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ h_{n1} & \dots & h_{n,i-1} & y_n & h_{n,i+1} & \dots & h_{nn} \end{bmatrix}$$
(22)

2011 Volume 50

where:

$$\Delta_i = \sum_{j=1} H_{ji} \cdot y_j \tag{23}$$

IV. ADDRESS SC MODEL OF IDENTIFICATION ALGORITHM

An advantage of this algorithm is that elements of H (18) are calculated without application of recurrent procedure, therefore, equation (20) is used, which ensures that optimization of algorithm is achieved.

However, use of full set of permutations in (20) increases the amount of calculations, which requires special methods for their calculation that are developed in [6] on the basis of SC models. They allow creating parallel algorithms, which increases the performance of identification algorithms.

Such SC models have a form of a branching graph [4]:

$$\varphi Gr^{*}(n, \overline{v}) \Rightarrow \left[\varphi KC(v_{1})^{*}(\overline{1.n}) \right] \times \left[\varphi KC(v_{1})^{*}(\overline{1.n}) / \widetilde{a}_{1} \right] \times \circ$$
$$\times \left[\varphi KC(v_{1})^{*}(\overline{1.n}) / \widetilde{a}_{12} \right] \dots \times \left[\varphi KC(v_{1})^{*}(\overline{1.n}) / \widetilde{a}_{12\dots n-1} \right]$$
$$\overline{v}^{T} = \left[v_{1} v_{2} \dots v_{k} \right]; \quad \sum_{i=1}^{k} v_{i} = n$$
(24)

Sections of graph (24) are formed according to differenceresidual principles: combinative operators $\varphi KC(v_i)$ act on components of difference sets $\tilde{a}_{12...n-1}$ which are left after formation of previous sections. Relation between sections is denoted by operation of lexicographic product. Graph (24) is a symbolical image of the index matrix grid made from numbers of rows and columns $ims(\bar{r} \times \circ \bar{L})$ of submatrix from Y (4) for which the minor is found, and consists of its addresses.

Thus, expressions of elements of the initial matrix Y(10)can be replaced with numbers of coordinates of the index matrix grid $ims(r \times \circ L)$, an associative submatrix which are substituted in (24). From them, permutations are formed in branches (24) and realization of the base formula (20) is provided. We shall denote the algorithm for formation of graph (24) with the operator $\varphi Graf(n,v)$. Here, the dimension of associative submatrix in (15) and the vector V, the elements of which specify the width of graph sections, are given in brackets. Assembly of the algorithm is carried out by calculation of products of elements in graph branches and their summation which are designated by the operator ρDpv acting on the graph. The architecture of computing algorithm is determined by the argument of operator (24). Since the graph has the form of a branching tree, it is suitable for optimization of algorithms providing a parallel mode of processing.

Operations with graph fragments that are realized in arithmetic space can be used for optimization of algorithms because part of arithmetic operations can be done in space of symbolical descriptions with the help of operator $\phi Dvp(arg)$,

which realizes the operation of multiplication of expressions located in graph vertices and their summation. It allows to minimize and simplify the SC model (24). The result_of optimization can be written down as $\varphi Dvp(arg) * \varphi Graf(n, v)$. For this optimization, local SC model of product of transient (10) values is used:

$$D(n,m) \Longrightarrow (1.n) \times \circ (1.n) \times \circ \dots \times \circ (1.n)$$
 (25)

We use the principle of synchronization of multiplier positions in (25) with positions of elements in components of formed numerical sequences. Components in (25) we shall form on the basis of operation of lexicographic multiplication of intervals of numerical series. Factors of (25) designate the sums (10) and have dimension n of the sums of partial components of the transient. Product (25) can be represented in the form of numerical series Ich(n,m), which is formed on the basis of recurrent procedure in which a rectangular matrix formed in the previous step is unfolded in a transposed vector:

$$\overline{a_{m-1}}^T U_m \Rightarrow \overline{a_{m-1}}^T \times \circ (\overline{1.n}); \quad U_{m-1} \Rightarrow \overline{a_{m-1}}^T$$
(26)

As an ordering of multipliers in (3) is imposed and multipliers themselves are ordered, the components of Ich(n,m) will possess the property of partial order:

$$\begin{bmatrix} U_m \end{bmatrix}_j \Rightarrow \begin{pmatrix} u_{j1}; u_{j2}; \cdots u_{jm} \end{pmatrix}; \quad u_{jk} \ge u_{j,k-1}$$
(27)
$$D(n,m) \Rightarrow (\overline{1n}) \times \circ (\overline{1n}) \times \circ \ldots \times \circ (\overline{1n}) \Rightarrow$$
$$\Rightarrow \prod_{i=1}^m (\overline{1n})_i \Rightarrow Ich(n,m)$$
(28)

In [5, 6], it has been shown that (28) can be transformed into strictly ordered sequence NumSec(n,m) formed by combinative operator:

$$\varphi KC(\nu)^*(\overline{1.n}) \Longrightarrow NumSec(n,\nu)$$
(29)

We use operation of digit-by-digit subtraction of fixed components 0.(m-1) from components (28) and have:

$$Ich(n,m) - \left[\overline{0.(m-1)}\right] \Rightarrow NumSec(n,m)$$
(30)

Using (7), in [3, 5] it has been proved that sequence (28) can be represented as canonical decomposition in the basis of vectors formed by operators $\varphi KC(v)$; $v \in \overline{1.m}$. Repetitions of elements in components (28) are formed by the operator of partitioning the number *m* into *v* digits $\varphi Part(v)*m$. The proof uses the principle of equivalent representation realized by the operator of permutation $\varphi Perm*[\varphi Part(v)*m]$ acting on components of partitions. The obtained result is placed in the components of base vectors $\varphi KC(v)$; $v \in \overline{1.m}$. Therefore, we can write:

2011 Volume 50

$$\prod_{i=1}^{m} (\overline{1.n})_{i} \Rightarrow \sum_{\nu=1}^{m} \left[\varphi KC(\nu) * \widetilde{q}^{(n)} \otimes \circ \widetilde{C}^{(n)} \right] *$$
$$* \varphi Arang(\varphi Perm * \left[\varphi Part(\nu) * m \right]$$
(32)

Here the notation of direct lexicographic product of set of discrete poles with a set of the same dimension of coefficients of decomposition of transient is used. The set of powers is placed by the operator $\varphi Arang$ over discrete poles $\tilde{q}^{(n)}$. The set $\tilde{C}^{(n)}$ does not depend on powers and, consequently, can be taken out of brackets of local SC model as additional multiplier, so we get:

$$\prod_{i=1}^{m} (\overline{1.n})_{i} \Rightarrow \sum_{\nu=1}^{m} \left[\varphi KC(\nu) * \widetilde{q}^{(n)} \otimes \circ \widetilde{C}^{(n)} \right] *$$
$$* \varphi Arang(\varphi Perm * \left[\varphi Part(\nu) * m \right] \Rightarrow$$
$$\Rightarrow \sum_{\nu=1}^{m} \left[\varphi KC(\nu) * \widetilde{C}^{(n)} \right] \otimes \circ \left\{ \varphi KC(\nu) * \widetilde{q}^{(n)} \right\}$$
(33)

V.RESTORATION OF COMPUTING ALGORITHM FROM ITS ADDRESS SC MODEL

Partitioning of graph (24) into separate sections corresponds to partitioning of matrix into separate poles from the set of columns, each of which has its own set of powers of discrete poles. To set of minors used in (18), there corresponds a set $ims(r_i \times \circ L_i)$ of associative submatrices from Y. The indices of elements of submatrices in rows and columns can be expressed in the form of the ordered intervals (1.*n*) forming a product $\tilde{\omega}^{(n \times n)} \Rightarrow (1.n) \times \circ (1.n)$. Elements of submatrices are formed according to expression:

$$[Y]_{r,l} \Rightarrow \left[\overline{q}^{(n)} * \varphi Arang(r) \right]^{T} \cdot Diag(\overline{C}^{(n)}) \cdot \left[\overline{q}^{(n)} * \varphi Arang(l) \right]$$
(34)

From here we get:

$$[Y]_{r,l} \Rightarrow \begin{bmatrix} q_1^r & q_2^r & \cdots & q_n^r \end{bmatrix} \times \begin{pmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{bmatrix} \end{pmatrix} \times \begin{bmatrix} q_1^l \\ q_2^l \\ \vdots \\ \vdots \\ q_n^l \end{bmatrix}$$
$$(r,l) \in \widetilde{\omega}^{(n \times n)}$$
(35)

Index matrix grid of associative matrix related to the element $[Y]_{i,i}$ is determined from difference sets:

$$ims(i, j) \Rightarrow \left[\overline{(1.n)} / i \right] \times \circ \left[\overline{(1.n)} / j \right]$$
 (36)

$$\begin{bmatrix} q_1^r & q_2^r & \cdots & q_n^r \end{bmatrix} \cdot \begin{bmatrix} C_1 \\ C_2 \\ \cdots \\ C_n \end{bmatrix} \cdot \begin{bmatrix} q_1^l \\ q_2^l \\ \cdots \\ q_n^l \end{bmatrix}$$

$$\overline{r} \in (\overline{1.r_1}) \bigcup [(r_1+2)n]; \overline{l} \in (\overline{1.l_1}) \bigcup [(l_1+2)n] \qquad (37)$$

Coordinates *ims* in the form difference sets define the breaks in the regularity of following of power indices, which can be represented by a set of two poles from row and column indices:

$$Y\left\{\widetilde{\omega}^{(n\times n)}/(i,j)\right\} \Rightarrow \begin{bmatrix} \widetilde{q}^{(n)} * \varphi Arang(\overline{1,i}) \\ \widetilde{q}^{(n)} * \varphi Arang[(i+2)n] \end{bmatrix} \times \circ$$
$$\times \circ \left\{ Diag \begin{bmatrix} C_1 \\ C_2 \\ \cdots \\ C_n \end{bmatrix} \right\} \times \circ \begin{bmatrix} \widetilde{q}^{(n)} * \varphi Arang(\overline{1,j}) \\ \widetilde{q}^{(n)} * \varphi Arang[(j+2)n] \end{bmatrix} \right\}$$
(38)

Here, SC model for the first multiplier can be expressed in the form of graph (24), consisting of two sections. The numerical series for the first section is formed as:

$$\widetilde{L}_{1} = (\varphi KC(\nu = i) * \left[\widetilde{L} = (\overline{1.n}) / j\right]$$
(39)

In the second section, there will be the residual difference set, which will have the form of complementary index vector:

$$\widetilde{L}_2 = \left\langle \left(\overline{1.n} \right) / \widetilde{L}_1 \right\rangle \tag{40}$$

To partitioning of the matrix into two poles there corresponds partitioning the graph into two sections:

$$\left[\underbrace{\overline{(1\,i)}}_{\times} \circ \widetilde{L}_2 \right] \circ \left[\underbrace{(i+2)n}_{\times} \circ \widetilde{L}_2 \right]$$

$$\tag{41}$$

In the branches of (24), there will be permutations from components of set of rows:

$$Graf \Rightarrow \left\{ \varphi Perm * \left(\overline{1.i} \right) \right\} \times \circ \widetilde{L} \right\} \times \circ \left\{ \varphi Perm * \left(\overline{i+2.n} \right) \right\} \times \circ \widetilde{L} \right\}$$
(42)

As it can be seen from (4), powers of discrete poles in SC model are defined by parameters $ims(\bar{r} \times \circ \bar{L})$ of associative matrices. The values of row and column indices are arguments of the operator $\varphi Dvp(\bar{r}^{(n)} \times \bar{L}^{(n)})$ that forms the graph (24). The index matrix grid $ims(\bar{r} \times \circ \bar{L})$ determines the algorithm for sampling the measurements of transients from recorded

flight information and the powers of poles placed by the operator ϕ Arang over discrete poles:

$$\varphi Dvp(\operatorname{Im} s) * \tilde{q}^{(n)} \Rightarrow \tilde{q}^{(n)} * Arang\left[\varphi Dvp(\bar{r}^{(n)} \times \bar{L}^{(n)})\right]$$
(43)

Then the minor of associative matrix is determined using operator ϕDpv according to formula:

$$\varphi Dpv * Graf \Rightarrow \varphi Dpv * \left\{ \varphi KC(i) * \widetilde{L} \right\} \times \circ \left(\overline{1,i}\right) \otimes \\ \otimes \varphi Dpv * \left[\varphi KC(n-i-1) * \widetilde{L} \right] \times \circ \left(\overline{i+2.n}\right) \right\}$$
(44)

In the first multiplier, local associative matrices formed from row indices of the first column are determined and the following local SC models are used:

$$ims_i \Rightarrow (\overline{1.i}) \times \circ \left[\varphi KC(i) * \widetilde{L} \right]$$
 (45)

Influence of operator φDpv on the set of submatrices having a regular character of succession of powers of discrete poles generates vector of values of minors which are calculated using the operator:

$$\varphi Fg * ims_i \Longrightarrow [(q_1 - q_2) \cdot (q_1 - q_3) \cdots (q_1 - q_m)] \dots$$

$$\cdots [(q_{m-1} - q_m)] \qquad (46)$$

Algorithm of its formation has the following form:

$$\varphi KC(\nu = 2) * \widetilde{q}^{(m)} \Rightarrow \begin{bmatrix} 12\\13\\\dots\\(m-1),m \end{bmatrix} (\widetilde{q}^{(m)}) \Rightarrow$$
$$\Rightarrow \begin{bmatrix} q_1 - q_2\\q_1 - q_3\\\dots\\q_m - q_{m-1} \end{bmatrix} \Rightarrow \prod_{i,j \in (1,n)} (q_i - q_j) \tag{47}$$

Vector of minors for submatrix in the second columns also can be determined using the operator Fg (13). For this purpose, from the submatrix it is necessary to extract a diagonal matrix from elements of column's first row and transformed it into a regular form. The result of optimization (33) shows that the computing algorithm will contain fragments formed by operator (Fg^*).

Components of *Ims* containing the coordinates of elements of allocated submatrix are used as arguments of operator φDpv . Therefore, all operations connected to inversion of matrix *Y*, first of all, the re-addressing operation, can be executed in index symbolical space:

$$\Rightarrow \tilde{q}^{(n)} * Arang \left[\varphi Dvp(\bar{r}^{(n)} \times \bar{L}^{(n)}) \right]$$
(48)

 $\varphi Dvp(\operatorname{Im} s) * \widetilde{q}^{(n)} \Rightarrow$

Then we shall have:

$$\varphi Dvp \left\{ (\varphi Perm * \overline{r}^{(n)}) \times \overline{L}^{(n)} \right\} * \widetilde{q}^{(n)} \Rightarrow$$
$$\Rightarrow \widetilde{q}^{(n)} * Arang \left[\left(\varphi Dvp * \overline{r}^{(n)} \right) \oplus \overline{L}^{(n)} \right]$$
(49)

On the basis of Im $s = \lfloor (\overline{0.n-1}) \times \circ (\overline{0.n-1}) \rfloor$ in the SC model the difference operators φFg (*) can be introduced:

$$\varphi Fg * \tilde{q}^{(n)} \Rightarrow \prod_{i,j(i \neq j)} q_i - q_j \tag{50}$$

Then we have:

$$\varphi Dvp \left\{ (\varphi Perm^* \overline{r}^{(n)}) \times \overline{L}^{(n)} \right\}^* \widetilde{q}^{(n)} \Rightarrow$$
$$\Rightarrow \left[\varphi Fg(\overline{r}^{(n)})^* \widetilde{q}^{(n)} \right] \cdot \left[\widetilde{q}^{(n)} * Arang\left(\left(\overline{L}^{(n)} \right) \right) \right] \qquad (51)$$
$$\varphi Dvp \left\{ \overline{r}^{(n)} \times (\varphi Perm^* \overline{L}^{(n)}) \right\}^* \widetilde{q}^{(n)} \Rightarrow$$

$$\Rightarrow \left[\varphi Fg(\overline{L}^{(n)})^* \widetilde{q}^{(n)} \right] \cdot \left[\widetilde{q}^{(n)} * Arang(r^{(n)}) \right]$$
(52)

$$\varphi Dvp \left\{ (\varphi Perm * \overline{r}^{(n)}) \times \overline{(\varphi Perm * \overline{L}^{(n)})} \right\} * \widetilde{q}^{(n)} \Rightarrow$$
$$\Rightarrow \left[\varphi Fg(\overline{r}^{(n)}) * \widetilde{q}^{(n)} \right] \times \left[\varphi Fg(\overline{L}^{(n)}) * \widetilde{q}^{(n)} \right] \qquad (53)$$

If $\overline{r}^{(n)} = \overline{m \cdot m + (n-1)}$ then the common multiplier is allocated:

$$\varphi Dvp \left\{ \varphi Perm * \left[\left(\overline{0.n - 1} \right) \oplus m^{[n]} \right] \right\} * \widetilde{q}^{(n)} \Rightarrow$$
$$\Rightarrow \left[\widetilde{q}^{(n)} * Arang(m^{[n]}) \right] \cdot \left[\varphi Fg * \widetilde{q}^{(n)} \right]$$
(54)

The conjugate property of decomposition is observed:

$$\varphi Dvp(\varphi Perm * \bar{r}^{(n)}) * \left[\tilde{q}^{(n)} * Arang(\bar{r}^{(n)} \oplus \bar{L}^{(n)}) \right] \Rightarrow$$

 $\Rightarrow \left[\tilde{q}^{(n)} * Arang\left(\varphi Dvp * \bar{r}^{(n)} \right) \right] \cdot \left[\tilde{q}^{(n)} * Arang\left(\bar{L}^{(n)} \right) \right]$ (55)

$$\varphi Dvp(\varphi Perm * \overline{L}^{(n)}) * \left[\widetilde{q}^{(n)} * Arang(\overline{r}^{(n)} \oplus \overline{L}^{(n)}) \right] \Rightarrow$$

$$\Rightarrow \left[\tilde{q}^{(n)} * Arang\left(\varphi Dvp * \tilde{L}^{(n)} \right) \right] \cdot \left[\tilde{q}^{(n)} * Arang\left(\tilde{r}^{(n)} \right) \right]$$
(56)

The SC model possesses recursive properties that allow to apply the methods of reduction of algorithm complexity. For this purpose, we shall represent the product in graph branches in the form of ordered numerical sequence [4, 6]:

$$R(m,n) \Rightarrow \sum_{\nu=1}^{m} \varphi Perm * \left\{ \varphi KC(\nu) * \overline{1.n} \right\} * \varphi Arng(Z_{\nu})$$
$$Z_{\nu} \Rightarrow \varphi Perm * \left[\varphi Part(\nu) * m \right]$$
(57)

1...

We use local SC models expressions of products in branches of graph (24) as numerical series G(m, n) (30).

Using the above described properties, we have:

$$\varphi Dvp(\varphi Perm^* \bar{r}^{(n)})^* G(m,n) \Rightarrow$$
$$\Rightarrow \varphi Dpv(\varphi Perm^* \bar{r}^{(n)})^* \left[\varphi KC(m)^* (\overline{1.n})\right]$$
(58)

$$\varphi Dvp \left\{ \varphi Perm * \overline{r}^{(n)} \right\} \times \overline{L}^{(n)} \left\} * G(m, n) \Longrightarrow$$
$$\Rightarrow \widetilde{q}^{(n)} * Arang \left\{ \varphi Dpv(\varphi Perm * \overline{r}^{(n)}) \right\} \\ * \left[\varphi KC(m) * (\overline{1.n}) \right] \right\}$$
(59)

Using the property of decomposition of the operator ϕDpv , we shall find:

$$\varphi Dvp * \left[Q^{(n \times m)} \cdot P^{(m \times n)} \right] \Rightarrow \bar{u}^{T} \cdot \bar{h}$$
(60)

$$\begin{bmatrix} \bar{u} \end{bmatrix}_i \Rightarrow \varphi Dvp * Q_i^{(n \times n)} \begin{bmatrix} \bar{h} \end{bmatrix}_i \Rightarrow \varphi Dvp * P_i^{(n \times n)}$$
(61)

The coordinates of submatrices are used as arguments of the operator φDpr . As such, we use the vectors made from the components of ordered numerical sequences $\overline{G(n,m)} = (\overline{0.n}) \oplus \overline{R(n,m)}$ (30):

$$\begin{bmatrix} \overline{u} \end{bmatrix}_{i} \Rightarrow \varphi Dvp \left[\left(\overline{1.n} \times \circ G(n,m)_{i} \right) \right]^{*} Q; \begin{bmatrix} \overline{h} \end{bmatrix}_{i} \Rightarrow$$
$$\Rightarrow \varphi Dvp \left(G(n,m)_{i} \times \overline{1.n} \right)^{*} P \tag{62}$$

Then the expression (33) can be written down in the following form:

$$\varphi Dvp * \left[Q^{(n \times m)} \times P^{(m \times n)} \right] \Rightarrow$$
$$\Rightarrow \varphi Sum * \left\{ \varphi Dvp(\overline{\operatorname{arg}}_{1}) * Q \right] \otimes \otimes \left[\varphi Dvp(\overline{\operatorname{arg}}_{2}) * P \right] \right\} (63)$$
$$\overline{\operatorname{arg}}_{1} \Rightarrow \left[\overline{1.n} \right] \times \circ \overline{G(n,m)}^{T} \quad \overline{\operatorname{arg}}_{2} \Rightarrow \overline{G(n,m)} \times \circ \left[\overline{1.n} \right] (64)$$

Let's find the argument set for φDpv , acting on the product $\left[\underbrace{Q}^{(n \times m)} \cdot \underbrace{M^{(m \times m)}}_{i \times o} \underbrace{W^{(m \times n)}}_{G(n,m)} \right]$ With the help of the vector $ims_i \Rightarrow \overline{G(n,m)}_i \times \circ \overline{G(n,m)}$, we shall allocate a submatrix $S^{(n \times m)}_i \in M^{(m \times m)}$. Using (4), we find:

$$\overline{\arg} \left[\varphi D p v^* (S_i \cdot W) \right] \Longrightarrow \left[\overline{G(n,m)}_i \times \circ \overline{G(n,m)}^T \right]_M \otimes \\ \otimes \circ \left[\overline{G(n,m)} \times \circ \left(\overline{1.n} \right) \right]_W$$
(65)

$$Arg\left(\varphi Dpv^*M^{(m\times m)}\right) \Rightarrow$$
 (66)

The result of the influence of the operator ϕDpv we shall write down as:

$$\varphi Dpv * \left[Q^{(n \times m)} \cdot M^{(m \times m)} \cdot W^{(m \times n)} \right] \Rightarrow \overline{d}^{T} \cdot Z \cdot \overline{w}
\Rightarrow \left\{ \overline{G(n,m)} \times \circ \overline{G(n,m)}^{T} \right\}_{M}$$

$$\overline{d}^{T} \Rightarrow \varphi Dpv \left\{ \overline{(1,n)} \times \circ \overline{G(n,m)} \right\} * Q
\overline{w} \Rightarrow \varphi Dpv \left\{ \overline{(1,n)} \times \circ \overline{G(n,m)} \right\} * W
\left[Z \right]_{i j} \Rightarrow \varphi Dpv \left\{ \overline{G(n,m)}_{i} \times \circ \overline{G(n,m)}^{T}_{j} \right\} * M^{(m \times m)}$$
(68)

Using the result (33) and the expression (18) [1], we find the SC model for the inverse matrix of the dynamic process:

$$Y^{-1} \Rightarrow \varphi Dpv \left\{ (\overline{G(n,m)} \times \circ \overline{G(n,m)}) \right\} * Y \Rightarrow$$
$$\Rightarrow \left[\varphi Dpv * W(\overline{G(n,m)}) \right] \cdot$$
$$\cdot \left[\varphi Dpv \left\{ (\overline{G(n,m)} \times \circ \overline{G(n,m)}) \right\} * M^{(k \times k)} \right] \cdot$$
$$\cdot \left[\varphi Dpv * W(\overline{G(n,m)}) \right]$$
(69)

Thus, from symbolical description of SC model (24), we have derived the description of computing algorithm in the matrix form. First and last multipliers can be realized in the form of diagonal matrices. They are related among themselves by an average weight matrix.

VI. CONCLUSIONS

For processing poorly dynamical transients measured in flight modes, it is offered to use new computing algorithms for processing of flight information that does not require inversion of ill-conditioned matrices of systems of identification equations. They were developed on the basis of their symbolical images in the form of address symbolical combinatory models (SC models) that allow effective optimization of algorithms. For this purpose, the method of filtration of SC model's components has been developed and the form of algorithm for inversion of ill-conditioned matrices with minimized complexity has been obtained. It has allowed to derive for the first time a symbolical description of the analytical expression for the solution of identification equations systems. The developed method of transformation of symbolical descriptions of computing algorithms in their real forms can be effectively used for creation of software for onboard computers that process the flight information in real time of test flight. Such method allows deriving analytical expressions for vectors of solutions of identification equations that can be used for imitation modelling of processes of information processing for various flight modes. In particular, on their basis, key parameters for parallel calculation organization in problems of identification, control and diagnosis can be chosen. The developed method for mapping the symbolical descriptions of computing algorithms allows creating effective software for onboard computers which are carrying out tasks of control and diagnosis of aircraft onboard equipment, which can be used not only at the test stage, but also at the stage of their normal operation.

Reference

- [1] Г. Буров, "Символьные комбинаторные модели в задачах идентификации аналоговых технических объектов," Сборник статей XI конференции "Фундаментальные и прикладные исследования, разработка и применение высоких технологий в промышленности," Санкт- Петербург, Россия, 2011, том №3.
- [2] G. Burov, "Information technologies for increasing the usability of algorithms used during aircraft flight test stage," in *Scientific Proceedings of Riga Technical University*, Series: Computer Science, Technologies of Computer Control, vol. 39. Riga: RTU Publishing House, 2009.
- [3] G. Burov, "Principles of automation of identification processes of aerospace object characteristics at the flight test stage," in *Scientific Proceedings of Riga Technical University*, Series: Computer Science, Boundary Field Problems and Computer Simulation, vol. 41. Riga: RTU Publishing House, 2009.
- [4] G. Burov, "Formation of computing algorithms on the basis of graph address structures," in *Scientific Proceedings of Riga Technical University*, Series: Computer Science, Applied Computer Systems, vol. 22. Riga: RTU Publishing House, 2005.
- [5] G. Burov, "Address computing models for tasks of identification," in *Scientific Proceedings of Riga Technical University*, Series: Computer Science, Technologies of Computer Control, vol. 19. Riga: RTU Publishing House, 2004.
- [6] G. Burov, "Parallel architecture of algorithms of dynamic objects identification," in *Scientific Proceedings of Riga Technical University*, Series: Computer Science, Technologies of Computer Control, vol. 3. Riga: RTU Publishing House, 2003.
- [7] G. Burov, "Combinatorial structure of parallel algorithms of linear algebra," in *Scientific Proceedings of Riga Technical University*, Series: Computer Science, Boundary Field Problems and Computer Simulation, vol. 45. Riga: RTU Publishing House, 2003.

- [8] G. Burov, "Combinatorial methods of formation of parallel algorithms of the signals processing," in *Scientific Proceedings of Riga Technical University*, Series: Computer Science, Boundary Field Problems and Computer Simulation, vol. 45. Riga: RTU Publishing House, 2003.
- [9] G. Burov, "Principles of formation of parallel algorithms of the information processing in dynamic objects," in *Scientific Proceedings of Riga Technical University*, Series: Computer Science, Technologies of Computer Control, vol. 3. Riga: RTU Publishing House, 2003.
- [10] G. Burov, "Parallel architecture of algorithms of dynamic objects identification," in *Scientific Proceedings of Riga Technical University*, Series: Computer Science, Technologies of Computer Control, vol. 15. Riga: RTU Publishing House, 2003.
- [11] G. Burov, "Principles of creating parallel algorithms for solving identification problems," in *Scientific Journal of Riga Technical University*, Series: Computer Science, Boundary Field Problems and Computer Simulation, vol. 45. Riga: RTU Publishing House, 2010.
- [12] G. Burov, "Combinatorial methods of formation of parallel algorithms of the signals processing," in *Scientific Proceedings of Riga Technical University*, Series: Computer Science, Boundary Field Problems and Computer Simulation, vol. 45. Riga: RTU Publishing House, 2003.
- [13] G. Burov, "Optimized programming of dynamic object identification problems," in *Scientific Journal of Riga Technical University*, Series: Computer Science, Boundary Field Problems and Computer Simulation, vol. 42. Riga: RTU Publishing House, 2010.
- [14] G. Burov, "Symbolical combinatory model for solving the problem of eigenvalues in tasks of identification of dynamic objects," in *Scientific Proceedings of Riga Technical University*, Series: Computer Science, Technologies of Computer Control, vol. 32. Riga: RTU Publishing House, 2008.
- [15] G. Burov, "Symbolical combinatory model of parallel algorithm of identification using the method of least squares," in Scientific Proceedings of Riga Technical University, Series: Computer Science, Boundary Field Problems and Computer Simulation, vol. 50. Riga: RTU Publishing House, 2008.
- [16] G. Burov, "Numerically stable symbolical combinatory model of polynomial approximation for problems of identification and imitation modeling," in Scientific Proceedings of Riga Technical University, Series: Computer Science, Boundary Field Problems and Computer Simulation, vol. 50. Riga: RTU Publishing House, 2008.
- [17] Р. Г. Ярмарков, Летные испытания первых опытных образцов самолетов. Москва: Машиностроение, 1987.
- [18] Автоматизированное управление самолетами и вертолетами, под. ред. С. М. Федорова. Москва: Транспорт, 1977.
- [19] Ю. Е. Махоньков и другие, Автоматизированная обработка результатов измерений при летных испытаниях. Москва: Машиностроение, 1983.
- [20] А. М. Знаменская и другие, Информационно-измерительные системы для летных испытаний самолетов и вертолетов. Москва: Машиностроение, 1984.
- [21] G. Burov, "Combinatory models of inversion of special type matrixes," in Scientific Proceedings of Riga Technical University, Series: Computer Science, Boundary Field Problems and Computer Simulation, vol. 47. Riga: RTU Publishing House, 2005.
- [22] Г. Буров, "Символьное комбинаторное исчисление и его применения в задачах линейной алгебры и идентификации динамических систем," *Латвийский математический ежегодник*, Nr. 34. Рига: Зинатне, 1993.
- [23] Я. Цыпкин, Основы информационной теории идентификации. Москва: Наука, 1984.
- [24] Л. Льюнг. Идентификация систем. Теория для пользователя. Москва: Наука, 1991.
- [25] Г. Буров, "Применение интерполяционных методов в задачах идентификации," Сборник научных сообщений семинара «Методы и средства технической кибернетики», выпуск 7. Рига: Рижский политехнический институт, 1970.
- [26] Г. Буров, "Алгоритмическое соответствие между непрерывными и дискретными изображениями," Сборник научных трудов "Методы и модели управления", выпуск 8. Рига: Рижский политехнический институт, 1973.
- [27] Г. Буров, "Метод разложения дробно рациональных функций и его использование для расчета переходных процессов," Известия Высших учебных заведений. Приборостроение №12, 1979.

Genady Burov was born 1937 in St. Petersburg, Russia. He received the Doctoral Degree in 1966.

Currently Genady Burov is a lead researcher at the Environment Modelling Centre of Riga Technical University.

Genādijs Burovs. Lidojumu informācijas apstrādes algoritmu optimizācija aerokosmisko objektu izmēģinājumu lidojumu etapam

Informāciju par lidaparātu un to aptīkojuma dinamiskajiem raksturojumiem ir grūti iegūst mērāmo pārejas procesu lēno izmaiņu dēļ, kas tiek slāpētas ar speciālās iekārtām lidojumu drošības nodrošināšanai. Tomēr šādas informācijas iegūšana ļauj koriģēt izmēģinājumu lidojumu programmu, samazināt tās ilgumu un izmaksas. Šādi novērtējumi ir jāiegūst arī lidaparāta ekspluatācijas laikā, lai varētu diagnosticēt aprīkojuma stāvokli un savlaicīgi birīdināt par ārkārtas situāciju rašanos tā funkcionēšanā. Tradicionālie algoritmi nav piemēroti darbam šādos apstākļos, jo tajos rodas deģenerētas situācijas, kā rezultātā iegūtie rezultāti nav droši. Ņemot vērā lidaparātiem raksturīgo pārejas procesu dinamisma deficītu to slāpēšanas dēļ, ir piedāvāts izmantot jaunus algoritmus lidojuma laikā. Tika izstrādāta metode skaitļošanas algoritmu aprakstīšanai simbolisko kombinatorisko modeļu veidā. Tie var tikt izmantoti algoritmu optimizācijai un formalizētu matemātisko metožu radīšanai lidaparātu datoru programmatūras izstrādei, kas ļauj realizēt paralēlus signālu mērījumu rezultātu apstrādes režīmus dinamisma deficīta apstākļos.

Геннадий Буров. Оптимизация алгоритмов обработки полетной информации на этапе летных испытаний аэрокосмических объектов

Информация о динамических характеристиках летательных аппаратов (ЛА) и их бортового оборудования труднодоступна по причине слабой изменчивости измеряемых переходных процессов, которые демпфируются специальными устройствами в целях обеспечения безопасности полетов ЛА. Однако получение такой информации позволяет скорректировать программу летных испытаний, сократить их продолжительность и материальные затраты. Такие оценки требуется получать также в период эксплуатации ЛА с целью диагностирования состояния бортового оборудования и своевременной индикации возникновения нештатных ситуаций его функционирования. Традиционные алгоритмы не приспособлены для работы в таких условиях, поскольку в них возникновения нештатных ситуаций его функционирования. Традиционные алгоритмы не приспособлены для работы в таких условиях, поскольку в них возникают вырожденные ситуации, нарушающие их работоспособность и ведущие к получению недостоверных результатов. Учитывая специфику ЛА, связанную с дефицитом динамичности переходных процессов из-за их демпфирования, было предложено использовать новые алгоритмы для обработки полетной информации, способные работать в реальном времени полетных режимов. Был разработан метод описания вычислительных алгоритмов в виде символьных комбинаторных моделей (СК - моделей). Они могут быть использованы для отгимизации алгоритмов и разработки формализованных математических методов создания программного обеспечения для самолетных бортовых компьютеров, позволяющего реализовать параллельные режимы обработки результатов измерений сигналов в условиях дефицита их динамичности.