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# Minimax reliability of aircraft and airline

**M. Hauka, Yu. Paramonov** [1]

Minimax approach to inspection program development problem under stringent limitation of number of full-scale fatigue tests of airframe is discussed

## 1 Introduction

Development of inspection program for limitation of fatigue failure probability (FFP) of fatigue-prone aircraft (AC) and fatigue failure rate (FFR) of airline (AL) is a problem of high priority. This problem is aggravated by stringent limitation of number of full-scale fatigue tests (only one or two airframe are tested). In previous authors publications [1-2] minimax solution of the problem was offered in assumption that in exponential approximation of fatigue crack size growth function, $a(t) = a_0 \exp(Qt)$, only one parameter of fatigue crack trajectory (PFCT) is unknown. Here we consider the case of two unknown parameters.

## 2 Mathematical model of the problem

Probability of failure (PF) of fatigue-prone aircraft (AC), failure rate (FR) and gain (GL) of airline (AL) for specific inspection program can be calculated using Markov Chains (MC) and Semi-Markov process (SMP) theory if parameters of corresponding models are known. Exponential approximation of fatigue crack size growth function, $a(t) = a_0 \exp(Qt)$, where $a_0$, $Q$ are random variables , is used. Estimation of the parameter of distribution function of these variables, $\theta$, and the choice of inspection program under condition of limitation of PF or FR can be made using results of observation of some random fatigue cracks in full-scale fatigue test of the airframe. For processing of acceptance AC type test, when redesign of new AC should be made if some reliability requirements are not met, the minimax decision is used. The process of operation of AC is considered as absorbing MC with $(n+4)$ states. The states $E_1, E_2, ..., E_{n+1}$ correspond to AC operation in time intervals $[t_0, t_1), [t_1, t_2), ..., [t_n, t_{SL})$, where $n$ is an inspection number, $t_{SL}$is specified life (SL), i. e. AC retirement time. States $E_{n+2}$, $E_{n+3}$, and $E_{n+4}$ are absorbing states: AC is descarded from service when the SL is reached or fatigue failure (FF) or fatigue crack detection (CD) take place. In corresponding matrix for operation process of AL the states $E_{n+2}$, $E_{n+3}$ and $E_{n+4}$ are not absorbing but correspond to return of MC to state $E_1$(AL operation returns to first interval). In matrix of transition probabilities of AC ,$P_{AC}$ , there are three units in three last lines in diagonal, but for corresponding lines in matrix for AL, $P_{AL}$, the units are in first column, corresponding to state $E_1$. Using $P_{AC}$ we can get the probability of FF of AC and cumulative distribution function, mean and variance of AC life. Using $P_{AL}$ we can get the stationary probabilities of AL operation $\{\pi_1, ..., \pi_{n+1}, \pi_{n+2}, ..., \pi_{n+4}\}$. Here $\pi_{n+3}$defines the part of MC steps, when FF takes place and MC appears in state $E_{n+3}$. The FR, $\lambda_F$ , and the gain of this process, $g$, are calculated using the theory of SMP with

[1]Riga Technical University, Aviation Institute. E-mail: maris.hauka@gmail.com, yuri.paramonov@gmail.com





reword, taking into accout the reword of succesful operation in one time unit, the cost of acquisition of new AC after SL, FF or CD take place,... If the gain is measured in time unit then $L_{n+3} = g/\pi_3$ is a mean time between FF; the intensity of fatigue failure $\lambda_F = 1/L_{n+3}$.

If we need only to provide AC reliability, R, then the vector $t = [t_1, t_2, ..., t_n, t_{SL}]$ should be defined by a p-set function $t(\hat{\theta})$ of estimate of parameter $\theta$ [1]. This means that $p_f(\theta, t) = \Sigma_{i=1}^{n+1} \mathbf{P}(T_{i-1} \leq T_d < T_c < T_i)$ should not exceed some small value $p = 1 - R$ for all unknown $\theta$. Here random vector $[T_1, T_2, ..., T_n, t_{SL}] = t(\hat{\theta})$, $T_d, T_c$ are times when fatigue crack become detectable ( with probability 1) and critical (fatigue failure) correspondingly. But in general case we need to limit FR of AL and to have a good gain. Consider the simplified case when all intervals are equal. Then the procedure of choice of $n$ is the following. If $\theta$ is known we calculate the gain as function of $n$,$g(n)$, and choose the number $n_g$ corresponding to the maximum of gain : $n_g = \arg\max g(n)$. Then we calculate expected value of intensity of fatigue failure, $\lambda_F(n)$, and choose $n_\lambda$ in such a way that for any $n \geq n_\lambda$ the function $\lambda_F(n)$ will be equal or less than some value $\lambda_{FD}$ (the designed intensity of fatigue failure): $n_\lambda = \min\{n : \lambda_F(n) \leq \lambda_{FD}, \text{ for all } n \geq n_\lambda\}$. And finally we should choose $n = \max(n_g, n_\lambda)$. *But in fact we do not know $\theta$ and we can only get some estimate of this parameter using test results, $\hat{\theta}$.* So real intensity will be a function of random variable, $\lambda_F(\hat{\theta}, \lambda_{FD})$. We can limit the mean value of this function if we take into account that really full-scale fatigue test is an approval test. Redesign of airframe will be made (service of aircraft of tested design version of aircraft will not be allowed) if some requirement is not met (for example, if it appears, that estimate of mean time to failure is too small). If full-scale fatigue test is approved and there is no necessity to make airframe redesign let us write $\hat{\theta} \in \Theta_0$, where $\Theta_0$ is some part of parameter space. Corresponding expected value of fatigue failure intensity function $\mathbf{E}\{\lambda_F(\hat{\theta}, \lambda_{FD}, \Theta_0)\}$ , where $\lambda_F(\hat{\theta}, \lambda_{FD}, \Theta_0) = \lambda_F(\hat{\theta}, \lambda_{FD})$ if $\hat{\theta} \in \Theta_0$ and $\lambda_F(\hat{\theta}, \lambda_{FD}, \Theta_0) = 0$ if $\hat{\theta} \notin \Theta_0$, is a function of $\theta$, which has maximum because for "bad" $\theta$ we make redesign of airframe but for "very good" we do not need any inspection.

It is shown that for this type of approval test using minimax approach by the choice of designed $\lambda_{FD}$ and $\Theta_0$ we can meet the reliability requirement defined by specific aviation regulation. Numerical examples are given.

## References

[1] *Paramonov Yu., Kuznetsov A., Kleinhofs M.*, Reliabilty of fatigue-prone airframes and composite materials. RTU, Riga, 2011, 122 pages.

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**Abstracts**

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