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STUDY OF NONLINEAR DYNAMICS OF SWITCHING POWER CONVERTERS

Summary of doctoral thesis

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RIGA TECHINCAL UNIVERSITY
Faculty of Electronics and Telecommunications
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STUDY OF NONLINEAR DYNAMICS OF SWITCHING POWER CONVERTERS

Summary of doctoral thesis

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CONFIRMATION

I confirm that the doctoral thesis, submitted for the degree of doctor of science in engineering at the Riga Technical University, in my own original work. The doctoral thesis has not been submitted for the degree at this or any other university.

Dmitrijs Pikulins …………………………….(Signature)

Date: ………………………

The doctoral thesis is written in Latvian, contains introduction, seven chapters, conclusions, references, 7 appendices, 117 figures, 263 pages in total. A list of references consists of 198 titles.

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List of Main Abbreviations and Terms

SPC	Switching Power Converter
EMI	Electromagnetic Interference
EMC	Electromagnetic Compatibility
CCM	Continuous Conduction Mode
DCM	Discontinuous Conduction Mode
Buck	Buck Switching Power Converter
Boost	Boost Switching Power Converter
MCBG	Method of Complete Bifurcation Groups
UPI	Unstable Periodic Infinitium
FP	Fixed Point
BG	Bifurcation Group
PD	Period-Doubling Bifurcation
SN	Saddle-Node Bifurcations
DIB	Discontinuity Induced Bifurcation

List of Main Symbols

E	Input DC voltage	D	Average duty ratio
V_{out}	Output DC voltage	d	Instantaneous duty ratio
C	Output capacitance	ϕ_{pm}	Phase margin
L	Inductance of power inductor	V_{pp}	SPC peak-to-peak output voltage ripples
R	SPC load resistance	Δi_L	Ripples of inductor current
f_{sw}	SPC switching frequency	I_{ref}	Reference current
T	SPC switching period	Pn	n-th order periodic regime
f_c	Cutoff frequency	nT	n-th order bifurcation group

GENERAL DESCRIPTION OF THE WORK

Topicality of the subject matter

The application of the new types of semiconductor devices [77, 79] and the rapid progress in the field of microelectronics define the wide opportunities in the development of switching power converters with pulse width modulation on the basis of new layout and design solutions that provide high speed operation, high efficiency and reliability, admitting small mass and dimensions. The systems of this type allow the control of energy flows, changing their parameters and implementing optimal control methods [55, 88, 91] with small energy losses.

The main operating mode of the mentioned systems is the regime of periodic oscillations, that is caused by external periodic force (in converters with pulse width modulation), or is maintained due to the properties of the system (in converters with relay based control). At the same time numerical and physical experiments reveal the existence of much more complicated dynamical regimes, including quasiperiodic and chaotic oscillations. The possibility of coexistence of various operating regimes in the parameter space has also been ascertained. Considering the mentioned properties of the system, even small influence of the noise or deviations in system parameters, that is defined by conditions of operation and the operating regimes of the load, would lead to catastrophic phenomena [75, 76], that manifest themselves as rapid change in the dynamics of the system (for example the transition from one stable periodic regime into another or catastrophic chaotization of oscillations). Mentioned phenomena not only define the increment of dynamical errors and the deterioration of qualitative parameters, but could also lead to the abrupt failures in the operation of technological systems, causing some serious crashes [81, 82].

The design process of switching power converters (SPC) is associated with the variety of specific problems and additional design tasks, that are summarized in the Fig. 1 (continuous lines represent the problems examined within the thesis). The problems of ***ensuring the stable operation of SPC*** and the ***improvement of the EMC*** are of special importance, that is why these problems are discussed in details.

The assurance of the stable operation of SPC (without subharmonic regimes and self-excited oscillations) is a complicated task [2, 8, 13, 40, 53], the solution of which requires the application of complex approaches and which is based on various trade-offs. The stability analysis method, based on the SPC frequency responses is used in the majority of practical applications because of its simplicity and efficiency [23, 31, 32, 33, 53]. Frequency responses are based on the appropriate transfer functions, which, in turn, are derived from the small signal models [57, 58]. However there are following shortcoming, inherent to this method:

- small signal models are applicable only in the presence of small state variable perturbations in the neighborhood of the steady-state operating point;
- it is not possible to predict the appearance of the variety of unwanted nonlinear phenomena (such as subharmonic oscillations and chaotic mode of operation [56]);
- there is an uncertainty in the choice of the SPC switching frequency (f_{sw}) to crossover frequency (f_c) ratio; according to different sources the value of f_{sw}/f_c is recommended to select within range : >5 [2], 4-10 [39], 3-10 [17], 10-15 [86]; it should be noted that the mentioned ranges are defined empirically and in design process specific values are selected by means of heuristic methods; it has been noted that the decrease of value of f_{sw}/f_c leads to the increase of the opportunity of appearance of undesirable dynamic mode [86]; however, the excessive increment of f_{sw}/f_c is not considered as the effective solution of the problem, as it could cause noticeable deterioration of parameters of control system (increased control time, static errors etc.); several authors note even the

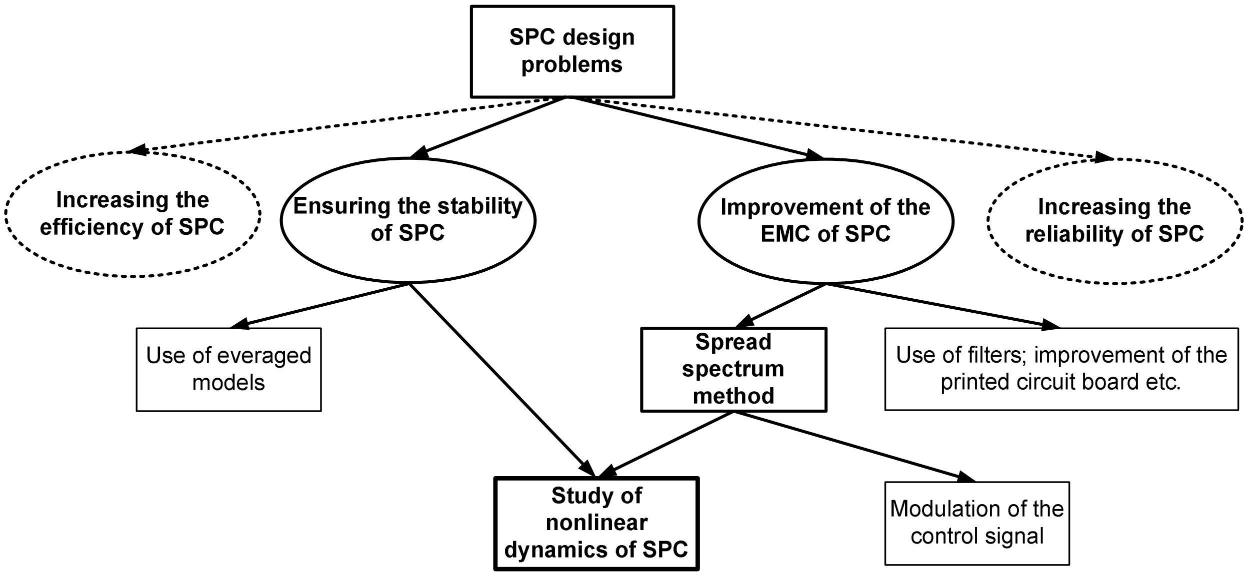


Fig. 1. Problems and possible solutions in the design of SPC

considering all stability criteria, defined for the transfer functions, it is possible to observe the appearance of unwanted dynamics within SPC [56] for small values of f_{sw}/f_c.

A necessity of the innovative design methods development is verified also by the fact that International Institute of Electrical and Electronics Engineers (IEEE) decided in 2005 to form a task force on assessing the need to include the higher order terms for small-signal analysis [66].

Thus, according to the above-mentioned facts, the development of methods for the study of complex dynamics of SPC, within which SPC are considered as essentially nonlinear systems exhibiting bifurcations and chaos, is considered to be an urgent problem.

The second considered problem is related to the high levels of electromagnetic interference (EMI) of SPC. EMI in SPC is caused mainly by rapidly switching power semiconductor devices with high current: *di/dt* and voltage *dv/dt* rate of change. With the introduction of the international electromagnetic compatibility directives (e.g. CISPR 22), there is an increasing demand for the effective methods used to solve the EMI problems of SPC [64]. The classical ways of mitigation of EMI (see Fig. 1) usually include the application of input filters, correct design of printed circuit boards, grounding and shielding, the use of soft switching techniques etc. [20,43,45,54,67]. However the mentioned techniques can substantially increase size, weight and production expenses of SPC

Over the last two decades another successful approach for EMI reduction known as spread spectrum has been developed. This method is based on modulating of the switching frequency of SPC in random, chaotic or periodic manner, as a result, the noise energy (which in the case of unmodulated control signal is concentrated in discrete harmonics) is spread over a wider frequency range, thus significantly reducing the amplitudes of EMI [32,65]. Over the last 15 years spread spectrum method has been extensively used in traditional SPC, power factor correctors, lighting equipment electronic ballasts. In spite of wide applications of spread spectrum method, this technique has several noticeable disadvantages:

- the modulation of the control signal leads to the increase of the output voltage ripples;
- in addition to high frequency ripples, the output voltage contains also low frequency ripples, caused by the process of modulation;
- the implementation of this approach requires the use of additional modulating signal generator.

To solve the first and the second problem, researchers propose the implementation of definite modulating waveforms (see e.g. [24, 25]), that allow the acceptable reduction of EMI, and have small effect on the quality of the output voltage. Several authors suggest the uncustomary solution of the third problem. It has been proved that the operation of SPC in the chaotic regime, when all waveforms become aperiodic, leads to the spreading of the EMI spectrum ***without any modifications in the structure of the control circuitry or application of external signal generators*** [15, 16]. In order to obtain the suppression of EMI it is necessary to ensure the operation of SPC in the region of ***robust chaos***, where small deviations of system parameters or external noise could not cause the occurrence of subharmonic operating regimes [4] that are not applicable to the improvement of the EMI of SPC. The determination of conditions to provide robust chaos and the estimation of the effect of system parameters on the robustness of chaotic regime is rather complicated procedure, as different types of rare attractors within the chaotic regions are observed [47], destroying the uniformity of chaos and eliminating the preferable use of the observed region. That is why in order to design the chaotic SPC it is suggested to apply modern approaches used in the study of nonlinear dynamics that allow systematical research of the division of parameter space of SPC and detection of regions of robust chaos.

In spite of essential achievements in the control and regulation theory, as well as development of the theory of nonlinear oscillations and chaos, the investigation of complex dynamics and catastrophic phenomena of systems with control based on relays or pulse-width modulators is still the subject of intensive study. This forces the implementation of great amount of experiments in order to obtain acceptable dynamic characteristics, as well as to improve the efficiency of operation and reliability.

The oscillatory properties of relay and pulse-width modulated systems have been studied in numerous works (see e.g. [44, 80, 90] etc.). Most authors pay attention to definition of conditions of existence and stability of definite periodic regimes, not taking into account the complex dynamics of the system. The objects of investigation in generalized theory of relay and nonlinear pulse systems are rather simple periodic regimes that noticeably limits the class of systems under test. The investigation of complex dynamics is provided in [18, 21, 22, 78, 85] etc., however, in general these researches are associated with the numerical investigation of concrete narrow classes of systems with pulse-width modulation.

Systems with relay control or pulse-width modulation could be described by sets of differential equations with discontinuities on definite smooth surfaces in the right-hand sides. The mentioned surfaces divide the phase space of the dynamical systems to the domains where the dynamics of the system is described by differential equations with sufficiently smooth right-hand sides. These systems are known as piecewise-smooth dynamical systems.

Three basic bifurcation scenarios are known for the transition to chaos in piecewise-smooth systems: the infinite cascade of period-doubling bifurcations (Feigenbaum scenario), the transitions through variable types of intermittency; the transition via different types of quasiperiodic scenarios [2, 11, 37, and 38].

It has been clarified in the several last decades that the mentioned scenarios do not exhaust the possible mechanisms of chaotization. There is a much broader class of bifurcation phenomena and transitions to chaotic motion (the classification of which could be found in [11, 89]), whose theoretical and numerical analysis requires the development of special approaches and new algorithms.

The phase trajectories of the mentioned systems are sewed together from various smooth domains and that is why one can in general distinguish between two types of fundamentally different types of bifurcations:

- the first type is similar to the bifurcations we know for smooth dynamical systems (i.e. local bifurcations, where definite periodic orbit loses its stability);

- the second type of bifurcations (not having analogues in smooth dynamical systems) is connected to the situations where the phase trajectory starts to intersect one of the sewing borders, resulting in the change of the number of domains, forming the trajectory of the solution; these changes in the topological structure of phase space are called C-bifurcations [89] or discontinuity induced bifurcations [11, 41, 42]; a simple type of C-bifurcation consists of the continuous transition of a solution from one type into another; however, more complicated phenomena are also possible, including period doubling, tripling, the merging and disappearance of solutions of different types etc.

The extreme complexity and diversity of nonlinear phenomena, observed in the piecewise-smooth systems, is associated with interactions of smooth and different types of discontinuity induced bifurcations. The mentioned conditions define the increasing interest in the investigation and more complete understanding of chaos and complex dynamics observed in piecewise-smooth systems [1, 3, 5, 11, and 12]. Theoretical research is induced by the wide variety if potential applications of the obtained results in different branches of science and engineering [2, 11].

It could be considered, that the main problem is derived from the fact that at the present moment there is no unified methodology for the investigation of complex dynamics and chaos within non-smooth dynamical systems that could be also used in some practical applications. The existing results mainly describe the properties of the concrete class of one or two-dimensional piecewise-smooth maps, and are not linked together by some general theoretical concepts. For the class of systems under investigation the great variety of bifurcation scenarios and causes of the catastrophic phenomena still lack the acceptable interpretation. Nevertheless, the mentioned systems are widely used in modern mechanics, electronics and power conversions circuits.

It follows from the above that the development and improvement of the techniques, used for the investigation and prediction of chaotic and catastrophic phenomena in switching power converters is urgent technical and scientific problem.

The scientific aspect of the formulated problem includes the necessity for the development of the theory of complex oscillations and investigation of bifurcation mechanisms of chaotization in piecewise-smooth dynamical.

The practical aspect of the problem includes the development of mathematical models, numerical algorithms and special software tools for the analysis of the complex dynamics and chaos in switching systems with pulse-width modulation, allowing the definite choice of structure, control schemes, parameters of compensators and type of the modulation in order to ensure the required performance in the wide range of circuit parameters and perturbations.

Numerical techniques, based on the transient processes, extensively used in the bifurcation analysis, do not allow systematic investigation of all possible important stationary regimes. Non-complete bifurcation analysis, providing tools for the construction of non-complete (also called Monte Carlo) bifurcation diagrams, are widely applied to the investigation of dynamics of various systems [9-11]. Mentioned technique for the construction of stable regimes and bifurcation diagrams does not take into account the complex topology of solution branches as well as some special stable regimes that could be found only by following unstable solution branches.

Taking into account the mentioned disadvantages of the existing methods used for the analysis of nonlinear dynamics, the innovative approach (initially applied for the analysis of subharmonic and chaotic dynamics in mechanical systems [68-74]) – Method of Complete Bifurcation Groups (MCBG) - is used within the doctoral thesis. The thesis includes the improvements of the existing and the development of new algorithms and approaches used within the MCBG for the analysis of dynamics of SPC. All developed algorithms are implemented within MATLAB environment.

Aims and tasks

The main **aims** of the thesis are:

- to study the bifurcation mechanisms of chaotization of dynamical regimes and the causes of appearance of catastrophic phenomena in widely used switching power conversion circuits, as well as to define the effect of the mentioned complex operating regimes on the most important parameters of SPC;
- to improve the existing and develop new algorithms for the analysis of subharmonic and chaotic regimes in SPC within the method of complete bifurcation groups;
- to improve the quality of the output voltage of SPC, predicting the occurrence of undesirable dynamical regimes during the design phase;
- to study the applicability of different types of chaotic operating regimes for the improvement of electromagnetic compatibility of SPC.

To achieve the set goals it was necessary to solve the following **tasks**:

- to carry out the analysis of applicability of modern analysis methods used for the investigation of complex dynamics of switching power converters;
- to develop the algorithmic base for the investigation of nonlinear dynamics of SPC, that includes:
    - the development of method, that allows the simplification of mathematical models of SPC (consisting of systems of differential equations), obtaining the corresponding Poincare maps;
    - methods and algorithms used for the analysis of the local stability of periodic regimes, including: algorithms allowing the calculation of all stable and unstable periodic regimes; methods allowing precise detection of switching moments, solving transcendental equations; methods used for the analysis of stability of periodic regimes by means of analytical and numerical calculations of monodromy matrixes;
    - the methodology and algorithms for the numerical calculation of periodic regimes, in cases when there exists the uncertainty about the number and type of stable and unstable periodic regimes, existing in certain parameter range;
    - the unified approach for the investigation of smooth and discontinuity induced bifurcations in piecewise linear systems;
    - numerical algorithms for the construction of complete bifurcation diagrams and bifurcation maps for switching systems;
    - numerical algorithms for the construction of stable and unstable manifolds of discrete maps without inversion;
    - algorithms, allowing the transition through bifurcation points and their continuation in the parameter plain, detecting bifurcation borders;
    - development of the specific programs within MATLAB/SIMULINK for the investigation of subharmonic and chaotic oscillations in SPC on the basis of MCBG;
- to establish the properties of: bifurcation maps of dynamical regimes in parameter plane; interactions of smooth and non-smooth bifurcations; transitions from periodic to chaotic modes of operation in SPC;
- to discover the features of division of parameter space to regions of periodic and chaotic regimes of operation; analyze the dependence of chaotization mechanisms through bifurcations on the chosen primary and secondary bifurcation parameters;
- to ascertain the most common methods of chaotization within switching power converters;

- to analyze the properties of bifurcation maps in parameter space, constructing bifurcation borders for smooth and non-smooth bifurcations, defining the scenarios of appearance of chaotic regimes through local and discontinuity induced bifurcations;
- to evaluate the applicability of various chaotic regimes of SPC to the mitigation of generated EMI and provide the recommendations for ensuring the robust chaotic operation;
- to provide the verification of the obtained results by means of numerical simulations, computer modeling and experiments on the basis of the developed prototype of laboratory prototype of SPC.

Research Results and Scientific Novelty

- For the first time the innovative approach – MCBG – was applied for the investigation of dynamics of SPC, allowing: the analysis of structure and properties of maps of dynamical regimes in parameter space; the construction of complete bifurcation diagrams and defining the most typical properties of interplay of smooth and non-smooth bifurcations; detection of multistability regions (and existence of rare attractors) in the wide region within the parameter space, that allows the appearance of catastrophic dynamics in the presence of small parameter variations or external noise.
- The following mechanisms of chaotization and appearance of complex dynamics in SPC have been established:
    - chaotic or subharmonic dynamics appear if for the fixed parameter values coexist several locally stable periodic or chaotic regimes (including rare attractors) and the amplitude of the noise in the system is capable of exceeding the radius of basins of attractions of these regimes;
    - chaotic oscillations could arise if there is only one periodic regime and the influence of external noise causes flipping of the phase point between various positions within the same trajectory if the basin of attraction the this regime has small radius;
    - discontinuity induced bifurcations may cause: the appearance of sudden abrupt period doublings; the formation of the set of periodic regimes with different characteristics; smooth transition from one periodic regime into another; the development of unstable periodic infinitium region of one periodic group in a single point, defining the abrupt transition to chaotic regime of operation;
    - studying the dynamics of SPC, it is possible to observe small stable periodic regions – rare attractors, that could be found within the chaotic modes of operation or as coexisting attractors; some special conditions have been defined for the boost converters, ensuring that all rare attractors disappear or become unstable;
    - ***for the first time*** it has been shown that in SPC the appearance of regions of unstable periodic infinitium could be caused by non-smooth bifurcations, when the wide parameter range of period doubling cascade and the consequent formation of new unstable periodic regimes shrinks to one point in the parameter plane;
    - ***new kinds of tip type rare attractor and protuberances were found***, formed by the interactions of smooth and discontinuity induced bifurcations;
    - ***for the first time*** the appearance of specific period-doubling saddle-node bifurcations and their effect on the formation of robust chaotic region and the appearance of ***new types of bifurcation groups*** has been explored.
- The following chaotization mechanisms have been revealed:
    - infinite period doubling cascade ending with aperiodic operating regime;
    - sudden appearance of locally stable periodic regimes with different dynamic characteristics, within each of the mentioned regimes the finite or infinite

period doubling cascade with transition to chaotic mode of operation is observed;
    - rapid transition from stable periodic regime to chaos;
    - finite sequence of smooth period doubling and various types of non-smooth bifurcations ending with chaotization of oscillations.
- It has been proved that the nature of many widely observed but insufficiently explored phenomena (such as complex interactions of smooth and non-smooth bifurcations in hybrid systems) could be explained, using the branches of unstable periodic regimes, constructed within the MCBG, as well as studying the dynamics of corresponding multipliers as circuit parameters are varied.
- The necessary conditions for ensuring robust chaotic operation (that could be used in order to improve the EMC of SPC by means of spread spectrum technique) were defined for the boost converter under current mode control; the influence of the compensating ramp signal on the robustness of obtained chaotic operation has been explored.
- The following algorithms used in the study of nonlinear dynamics in systems with pulse-width modulation have been improved and developed:
    - method allowing the obtaining of mathematical models of SPC from the systems of differential equations with discontinuities by means of construction of corresponding Poincare maps, which has been demonstrated for the voltage mode controlled *buck* converter;
    - numerical method and algorithm for the calculation of periodic regimes, that was adopted for the investigation of dynamical properties of switching power converters, allowing the detection (with predefined precision) of all stable and unstable periodic regimes for fixed parameter values, as well as the division of the obtained regimes to corresponding bifurcations groups;
    - algorithm and methodology for the calculation of periodic regimes, based on the application of the method of Newton-Kantorovich as well as solution of systems of transcendental equations in correspondence to switching moments; both approaches allow detection of stable and unstable periodic regimes in systems, that are described by iterative maps or sets of differential equations with discontinuous right-hand side;
    - algorithm used for the analysis of local stability of periodic regimes, which is based on the linearization of the obtained stroboscopic map in the neighborhood of the fixed point and the consequent analytical or numerical calculation of monodromy matrix, allowing the investigation of properties of bifurcations and stability borders as well as the proper identification of smooth and non-smooth bifurcations;
    - numerical algorithms, that could be used in order to analyze smooth and discontinuity induced bifurcations, construct complete bifurcation diagrams and bifurcation maps, calculate stable and unstable manifolds, allowing the investigation of complex dynamics, bifurcation scenarios and mechanisms of chaotization in systems with pulse-width modulation.

Defendable theses

The following theses are being proposed and defended:

1. the proper design of stable and reliable SPC is not possible without the global analysis of their nonlinear dynamics on the basis of modern techniques utilized for the investigation of chaos and bifurcations;
2. the global analysis of the qualitative nonlinear dynamics of SPC on the basis of new algorithms developed within the Method of Complete Bifurcation Groups (MCBG) allows the improvement of the voltage conversion quality and the increment of

reliability of SPC in the design stage, predicting and avoiding the occurrence of undesirable dynamic effects in the operation of SPC;
3. the development of new algorithms for MCBG allows providing the analysis of fundamentally different types of bifurcations – smooth and discontinuity induced ones – and estimation of their effect on the dynamics of SPC, investigating the development of constructed unstable branches of periodic regimes, playing ***an essential*** role in the process of development of bifurcation scenarios;
4. MCBG and the improvement of it's algorithms, utilizing the analytical calculations of monodromy matrixes, allows essential improvement of the efficiency of construction of complete bifurcation diagrams, detecting the regions of robust chaos (that could be used in order to improve the electromagnetic compatibility of SPC) as well as the estimation of the influence of different parameters of converters under study on the robustness of chaos.

Methodology of investigation

The development of methods and algorithms used in the design of SPC, providing the possibilities to avoid the occurrence of undesirable dynamical regimes is not possible without the analysis of nonlinear dynamics of the converters. Techniques utilized in publications devoted to the investigation of dynamics of SPC [2, 11, 60-63] does not allow the acquisition of the complete understanding of the dynamics of converters, as generally these approaches are based on the so called ***brute-force*** or Monte Carlo methods, the application of which does not allow the systematic calculation, classification and investigation of interactions of all possible periodic regimes.

Taking into account the disadvantages and shortcomings of the mentioned approaches, the innovative method of the investigation of nonlinear dynamics of complex systems– MCBG (developed in 1993-2012 in the Institute of Mechanics of Riga Technical University in the research group "Nonlinear dynamics, chaos, catastrophes and control" under the guidance of prof. M.V. Zarzhevsky) – was used and improved within the thesis. The theory of MCBG and the appropriate set of algorithms and programs allow the detection of new periodic and chaotic regimes, new bifurcation groups and provide the tools for the investigation of their interactions, within typical and widely applied nonlinear models.

The most important results of the investigation were obtained by means of numerical techniques based on the MCBG. All the models of SPC used within the doctoral thesis were transformed to the discrete-time maps. The validity of the obtained results was verified by means of MATLAB and PSpice simulations, laboratory experiments as well as comparison of the obtained data with some results published by other authors.

Objects of investigation

The objects of research are the main topologies of switching power converter, operating in typical regimes:

- buck converter under voltage mode control, operating in discontinuous current mode;
- boost converter under voltage mode control, operating in discontinuous current mode;
- boost converter under current mode control, operating ins continuous current mode (with/without compensating ramp);
- buck converter under voltage mode control (with proportional compensator), operating in continuous current mode.

Practical value of the work

- It is shown within the thesis that the application of MCBG allows the implementation of research of nonlinear dynamics and stability of SPC, thus significantly facilitating the design process of the mentioned devices. The numerical methods developed for the analysis of nonlinear dynamics of SPC allows establishing the coherence between the structure of the system, parameters of the converter and oscillatory properties, detect potentially dangerous operating regimes, predict the occurrence of catastrophic phenomena.

- The developed set of mathematical models, numerical algorithms and programs together provides the necessary tools for the complete bifurcation analysis of SPC in parameter and phase spaces, deriving practical recommendations for the appropriate choice of the structure, parameter set and compensation networks of converters, in order to fulfill necessary quality requirements.
- The introduced unified approach for the description of bifurcation scenarios of chaotization and appearance of catastrophic phenomena in SPC allows providing the analysis of specific features of the constructed complete bifurcation diagrams and bifurcation maps.
- The developed methods for the analysis of stability could be used in order to ensure the required operating regime of SPC (i.e. not allowing subharmonic operation in cases it could significantly affect the quality of obtained voltage, or intentionally ensuring robust chaotic operating regime in order to improve some characteristics of switching converter).
- The developed programs could be integrated into SPC automatic design software to provide the investigation of nonlinear dynamics of switching converters at definite design stages.

Approbation and publicity

The main results of the doctoral thesis have been presented at the following scientific conferences:

- „The 12th International Conference of ELECTRONICS", Kaunas, Lithuania, May 18-20, 2008;
- „The 13th International Conference of ELECTRONICS", Kaunas, Lithuania, May 12-14, 2009;
- „The 14th International Conference of ELECTRONICS", Kaunas, Lithuania, May 18-20, 2010;
- „The 15th International Conference of ELECTRONICS", Kaunas, Lithuania, May 17-19, 2011;
- „The 16th International Conference of ELECTRONICS", Palanga, Lithuania, June18-20, 2012;
- „The 50th RTU International Conference", Riga, Latvia, Oct. 14-16, 2009;
- „The 3rd Chaotic Modeling and Simulation International Conference", Hanja, Crete, Greece, June 1-5, 2010;
- „The 9th International Symposium on Electronics and Telecommunications", Timisoara, Romania, Nov. 11-12, 2010;
- International Scientific Conference „The Role and Opportunities of Youth in the Development in Engineering Science", Daugavpils, Latvia, Apr. 28, 2011;
- International Scientific Conference „The Role and Opportunities of Youth in the Development in Engineering Science", Daugavpils, Latvia, Apr. 26, 2012;
- „2nd International Symposium AR'11", Jurmala, Latvia, May 16-20, 2011;
- „7th International Conference-Workshop Compatibility and Power Electronics CPE" Tallin, Estonia, June 3, 2011;
- International Scientific Conference „Technology. Environment. Resources.", Rezekne, Latvia, June 20-22, 2011;

- „52nd RTU International Scientific Conference (Section – Power and Electrical Engineering Conference)”, Riga, Latvia, Oct. 13-14, 2011;
- 52nd RTU International Scientific Conference (Section – Electronics, Telecommunications and eSociety), Riga, Latvia, Oct. 13-14, 2011;
- „4th International Interdisciplinary Chaos Symposium on Chaos and Complex Systems”, Antalya, Turkey, Apr. 29 – May 2, 2012.

The research results have been published in 16 publications:

1. J.Jankovskis, D.Stepins, **D.Pikulins**, S.Tjukovs. Examination of Different Spread Spectrum Techniques for EMI Suppression in dc/dc Converters // Electronics and Electrical Engineering – Kaunas: Technologija, 2008. - No.6 (86). - pp. 60 – 64.
2. **Pikulins D**., Tjukovs S. Investigation of EMI reduction and output voltage ripple minimization using interleaved buck converters // Scientific Proceedings of RTU. Series 7. Telecommunication and Electronics, 2008, Vol.8, pp. 27.-30.
3. **Pikulins D.** Tools for Investigation of Dynamics of DC-DC Converters within Matlab/Simulink // CHAOS THEORY Modeling, Simulation and Applications Selected Papers from the 3rd Chaotic Modeling and Simulation International Conference (CHAOS2010)), World Scientific Publishing, 2011. – pp. 317-325.
4. Jankovskis J., Stepins D., **Pikulins D.** Improving Effectiveness of the Use of Frequency Modulation in Power Converters // Proceedings of the 12th Biennial Baltic Electronics Conference (BEC2010), Estonia, Tallinn, Oct. 4-6 , 2010. – pp. 327-330.
5. Jankovskis J., **Pikulins D**., Stepins D. Effects of Increasing Switching Frequency in Frequency Modulated Power Converters // Proceedings of the ”2010 9th International Symposium on ELECTRONICS AND TELECOMMUNICATIONS”, Romania, Timisoara, Nov. 11-12, 2010. – pp. 115-118.
6. **Pikulins D**. Some Applications of Numerical Path-following in the Analysis of Dynamics of Switching Converters // Student Forum Proceedings of the 7th International Conference-Workshop Compatibility and Power Electronics CPE 2011, Estonia, Tallinn, June 3, 2011. – pp. 11-14.
7. Zakrzhevsky M., Schukins I., Frolov V., Klokovs A., Jevstignejevs V., Smirnova R., **Pikulins D**. RARE ATTRACTORS IN DISCRETE NONLINEAR DYNAMICAL SYSTEMS // Proceedings of the 2nd International Symposium RA’11, Latvia, Jurmala, May 16-20, 2011. – pp. 21-25.
8. **Pikulins D.** SMOOTH AND NONSMOOTH NONLINEAR PHENOMENA IN DC-DC CONVERTERS // Proceedings of the 2nd International Symposium RA’11, Latvia, Jurmala, May 16-20, 2011. – pp. 26-30.
9. **Pikulins D.** Nonlinear Dynamics of Buck Converter // Proceedings of the 8th International Scientific and Practical Conference “Environment. Technology. Resources”, Latvia, Rezekne, June 20-22, 2011. – pp. 156-162.
10. **Pikulins D.** The Investigation of Complex Behavior in Buck Converters by Means of Matlab and Simulink // Scientific Journal of RTU. 7. ser., Telecommunications and Electronics. - 9. vol. (2009), pp. 24-33.
11. Jankovskis J., Stepins D., **Pikulins D**. Lowering of EMI in boost type PFC by the use of spread spectrum. Electronics and Electrical Engineering – Kaunas: Technologija, 2009. - No.6 (94). – pp. 15-18.
12. Jankovskis J., **Pikulins D**., Stepins D. Efficiency of PFC Operating in Spread Spectrum Mode for EMI Reduction // Electronics and Electrical Engineering. – Kaunas: Technologija, -No.7 (2010), pp. 13-16.
13. **Pikulin D**. Effects of Non-smooth Phenomena on the Dynamics of DC-DC Converters // Scientific Proceedings of RTU. 4. ser., Electronics and Electrical Engineering. – vol. 29. (2011), pp. 119-122.

14. **Pikulins D**. The Complete Bifurcation Analysis of Boost DC-DC Converter // Scientific Proceedings of RTU. 7. ser., Telecommunications and Electronics. – vol.11. (2011), pp. 22-26.
15. Zakrzhevsky M., Schukin I., Yevstigneev V., Klokov A., **Pikulin.D**. Complete Bifurcation Analysis of Discrete Nonlinear Dynamical Systems (book in print).
16. **Pikulins D**. Subharmonic Oscillations and Chaos in DC-DC Switching Converters // Electronics and Electrical Engineering. – Kaunas: Technologija, -No.X (2012), pp. X-X (accepted).

Two papers (4 and 5) are available in the IEEEXplore Digital Library, one paper (3) is available in World Scientific Books digital library; articles (2, 10-14, 16) are available in EBSCO, ProQuest, VINITI data bases.

Structure of the Thesis

The work consists of an introduction, seven chapters, conclusions and appendixes.

In the introduction the topicality of the carried research is substantiated, the main goals and methodologies of investigation are defined; the scientific novelty and several practical applications are pointed out; defendable theses are formulated.

In the first chapter the main topologies of SPC as well as typical control schemes and their functioning principles are considered.

The second and the third chapters are dedicated to the review of chaos and bifurcation theory, emphasizing the tools of chaotic identification and characterization that will be applied to the investigation and interpretation of complex dynamics of SPC.

The ideological structure of chapters 4-7, depicting author's contribution, is schematically shown in the Fig. 2, indicating also the papers of the author of the thesis corresponding to solutions of appropriate problems.

In the fourth chapter the concept of the MCBG is presented, including some specific definitions and notions as well as the existing and innovative algorithms developed for the investigation of nonlinear dynamics of SPC.

The fifth chapter is dedicated to the investigation of nonlinear dynamics of buck and boost converters operating in discontinuous current mode.

In chapters six and seven the bifurcation sequences and routes to chaos in the boost converter under current mode control and the buck converter under voltage mode control, both operating in the continuous current mode, is considered. Applying techniques introduced in chapters three and four, the border collision phenomena and its effect on the dynamics of SPC is studied in detail.

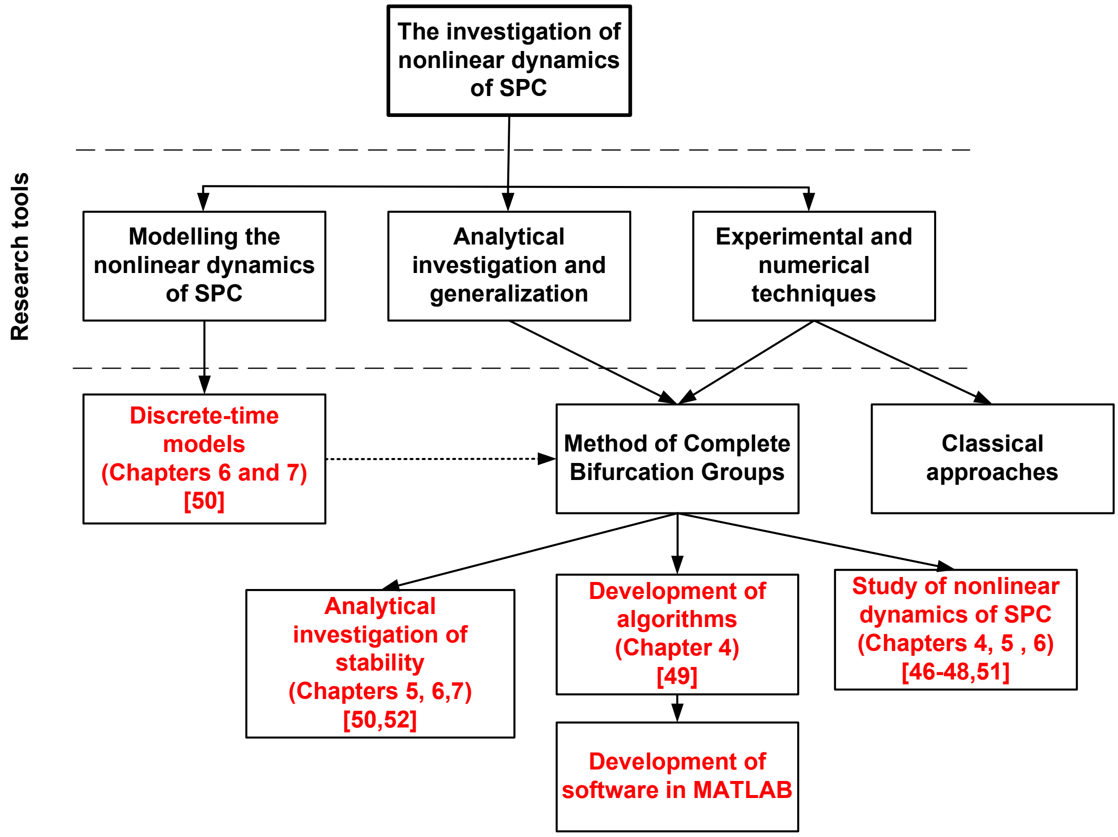


Fig. 2. Ideological structure of doctoral thesis, depicting (with color) author's contribution

SUMMARY OF THE THESIS

1. THE ANALYSIS OF OPERATION OF SWITCHING POWER CONVERTERS (SPC)

In the first chapter of the doctoral thesis the principles of operation of the basic SPC topologies and their control schemes are considered; the conditions for the operation of converters in DCM or CCM are defined; the applicability of methods used for the evaluation of stability of SPC, on the basis of averaged modeling approaches, to detection of various types of nonlinear phenomena is verified; the main idea of the spread spectrum technique is outlined and the possibilities of application of the chaotic modes of operation for the reduction of EMI levels in SPC is specified.

Power electronics and its branches, associated with the conversion of electrical energy, is fast growing scientific and technological area, providing results that could be applied to the development of other branches of electronics. The principles of energy conversion, component and technological base are persistently updated and every 3-4 years the generation changes are observed in this area. Power supplies of radioelectronic equipment form the wide range of transistor devices, the operation of which should fulfill some specific requirements.

The overview of main topologies of switching power supplies

DC-DC converters, studied within the doctoral thesis, convert the input DC voltage to the appropriate regulated output voltage. SPC have to ensure constant output voltage, varying load resistance and the value of input voltage. In contradistinction to linear converters, the values of the output voltage could be smaller (e.g. in buck converter) or greater (e.g. in boost and buck-boost converters) than the input voltage.

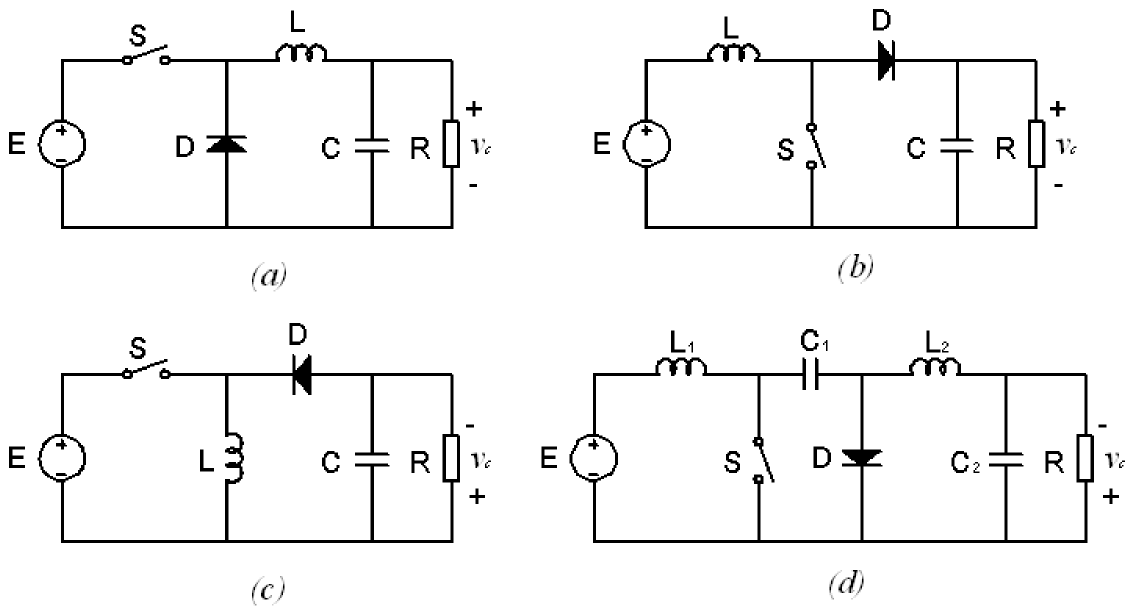


Fig. 1.1. (a)-(c) Main SPC topologies: (a) *buck* converter; (b) *boost* converter; (c) *buck-boost* converter; (d) their combination in the Cuk converter

The majority of SPC are implemented on the basis of the main simple converter topologies shown in the Fig. 1.1. Typically, the switching element and the diode are turned *ON* and *OFF* in a cyclic and complementary manner. The switch is directly controlled by the pulse-width modulated signals which is derived from a feedback circuit, but the diode turns on and off depending upon its terminal conditions. When the switch is closed, (*ON* position) the diode is reverse biased and hence open, the inductor current ramps up. When the switch is turned *OFF*, the diode is forward biased and behaves as a short circuit, the inductor current ramps down. The described process repeats cyclically. The system could be plainly described by a set of state equations, each responsible for one particular switch state. For the operation described above it is possible to define two state equations:

$$\text{Switch is } ON\text{, diode is off: } \dot{x} = A_1 x + B_1 E\,, \tag{1.1}$$

$$\text{Switch is } OFF\text{, diode is on: } \dot{x} = A_2 x + B_2 E\,, \tag{1.2}$$

where x - state vector usually consisting of all capacitor voltages and inductor currents;

$\dot{x}$ - dx/dt;

$A_{1,2}$, $B_{1,2}$ - system matrixes;

E - input voltage.

Furthermore, as the conduction of the diode is determined by its own terminal condition, there is a possibility that the diode can turn itself off even when the switch is *OFF*. In the power electronics literature, this operation has been termed ***discontinuous conduction mode (DCM)***, as opposed to ***continuous conduction mode (CCM)*** where the switch and the diode operate strictly in complementary fashion. The new state equation appears for the situation where both switch and diode are off:

$$\text{Switch } OFF\text{, diode off: } \dot{x} = A_3 x + B_3 E\,. \tag{1.3}$$

In practice the choice between DCM and CCM is often an engineering decision. CCM is more suited for high power application, whereas DCM is limited to low power applications On the other hand, DCM gives a more straightforward control design and generally yields faster transient responses.

The study of stability of SPC based on averaged models

In order to ensure the stable operation of SPC (i.e. without subharmonic and self-excited oscillations) the design process includes the analysis based on frequency transfer functions of the converters. Transfer characteristics are constructed, using the corresponding transfer functions, and the subsequent verification of compliance with definite stability criteria is provided [29].

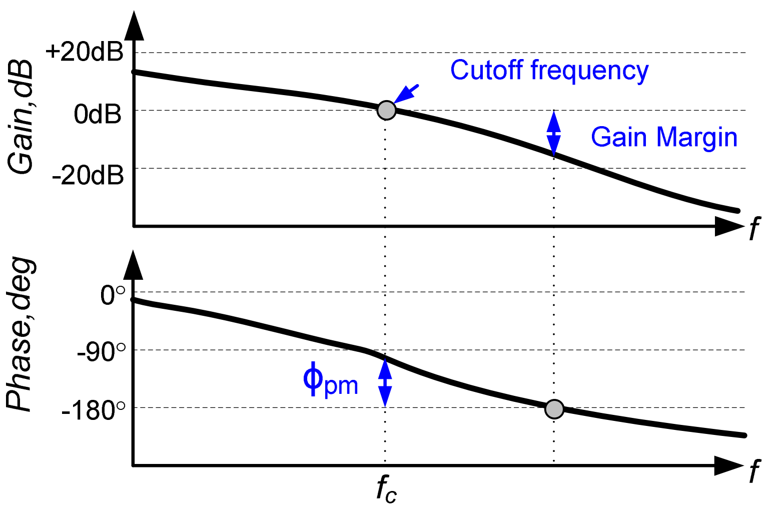


Fig. 1.2. Definitions of cutoff frequency, phase and gain margins

The following criteria ensuring the stable operating of SPC are formulated [7]:

1. at the crossover frequency f_c the ***phase delay*** introduced by the feedback loop should be smaller than 180^0; at f_c the ***phase margin*** ϕ_{pm} is defined, which could be calculated as the difference between the phase angle of the transfer function and -180^0 (see Fig. 1.2); in order to ensure the stable operation of the converter theoretically it is sufficient to have $\phi_{pm} \approx 1^0$, however generally the feedback loop is designed to provide $\phi_{pm} \approx 20^0$-60^0 , as small values of ϕ_{pm} could lead to high ***overshoot*** voltages;
2. ***gain margin*** is defined as the difference between 0 dB and the magnitude of the transfer function at frequency, where the phase angle may (but not always) cross -180^0; if the phase angle reaches the value of -180^0, the gain margin should be at least 10dB in order not to allow the appearance of undesirable oscillations as circuit parameters are varied.

Thereby it is supposed, that if the control-to-output transfer function of the SPC complies with the mentioned stability requirements, is not possible to observe subharmonic and self-excited oscillations within the converter.

The application of spread spectrum method to the attenuation of EMI noise

One of the main disadvantages of SPC is the high level of electromagnetic interference (EMI). EMI in SPC is caused mainly by rapidly switching power semiconductor devices with high rate of change of currents and voltages. The generated conducted EMI could interfere with other electronic devices, including the control circuitry of SPC itself.

One of the most successful approaches for EMI reduction, allowing the attenuation of noise levels within the source of generation, is a spread spectrum technique (SST), which is widely used in all kinds of electronic devices (e.g. microprocessors, D-class amplifiers, telecommunication systems etc. [26-28,43,54]). The method is based on the modulation of switching frequency in random, periodic or aperiodic manner, as a result the energy, concentrated in discrete harmonics, is spread over a wider frequency range (see Fig. 1.3).

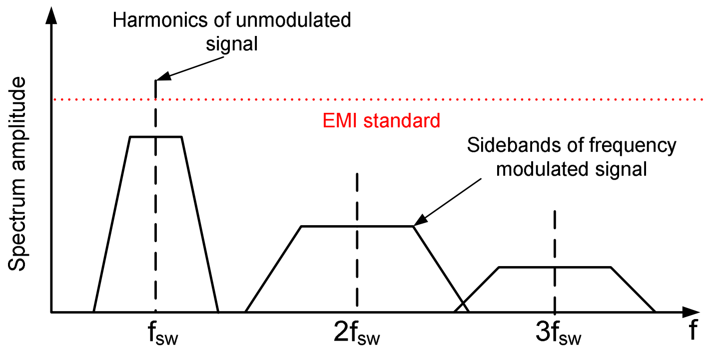


Fig. 1.3. Spectra of unmodulated and frequency modulated switching signals

More detailed analysis of the EMI reduction techniques, based on the application of random and chaotic signals could be observed in the following publications [24, 30, 65]. Comparing analytically and experimentally obtained results with those of periodic modulation, authors show that the application of chaotic modulation causes better EMI reduction, having negligible effect on the magnitude of the output voltage ripples.

All widely used SST implementation schemes suffer from one common drawback – the necessity of introduction into control circuitry an additional modulating signals source (that could be separate signal generator or integrated source). In order to avoid the use of additional circuitry, it has been proposed [16] to use the operation of SPC in chaotic mode, ensuring that voltages and currents become aperiodic and their spectra are inherently spread.

Some recommendations for ensuring the robust chaotic operation of SPC (that could be efficiently applied for the reduction of EMI levels), are given within the doctoral thesis.

2. CHAOS AND BIFURCATIONS IN NONLINEAR DYNAMICAL SYSTEMS

In the second chapter some most important aspects of the theory of chaos are summarized, including: the concept of dynamical system; classification of fixed points in the case of two-dimensional systems; classification of smooth and non-smooth bifurcations and the description of their main properties; description of various types of crisis and their influence on the global dynamics of nonlinear systems; main properties and the preconditions of occurrence of deterministic chaos.

The properties of nonlinear systems, described within the chapter, are utilized in the thesis in the process of investigation of subharmonic and aperiodic regimes and explanation of main chaotization scenarios.

The survey on nonlinear dynamical systems

The switching power converters investigated within this doctoral thesis can be modeled by means of continuous-time differential or discrete-time difference equations. In general, any system that could be put in one of the mentioned forms is called a ***dynamical system (DS)*** in the sense that its behavior varies as a function of time.

In the typical *RLC* circuits, for example, the capacitor voltages and inductor currents form a set of independent state variables. The basic constitutive laws of all elements (i.e., $v_R = i_R R$ for resistors, $L(di_L/dt)=v_L$ for inductors, $C(dv_C/dt)=i_C$ for capacitors etc.), together with the relevant independent Kirchhoff's law equations, define the connecting function. The study of dynamics of the system is essentially an investigation of how the state variables change in time. Mathematically this is done by relating the rate of change of these state variables to their current values via differential equations:

$$\frac{dx(t)}{dt} = f(x(t), \mu, t)\,, \tag{2.1}$$

where x - vector consisting of the state variables,
f - the connecting function,
μ - vector of system parameters.

Studying the dynamical behavior of a given system, one has to compute the trajectory starting from a given initial condition. However, it is generally not necessary to compute all possible trajectories in order to study a given system. In may be noted, that the left-hand side of (2.1) gives the rate of change of the state variables. The equation (2.2) thus defines a vector at every point of the state space. The properties of a system can be studied by studying this ***vector field***.

The analysis of the dynamics of nonlinear systems could be carried out on the basis of discrete-time models that are defined in the following way:

$$x_{n+1} = f(x_n)\ . \tag{2.2}$$

At the beginning of the investigation one first finds the fixed points of the definite period *T* - $x_{n+T} = x_n = x_T^*$. Than it is possible to locally linearize the discrete system in the neighborhood of fixed point, obtaining the Jacobian matrix. The eigenvalues (multipliers) of the Jacobian matrix indicate the stability of the fixed point – in the case of discrete-time system the FP is stable if modules of all eigenvalues have magnitude of less than unity.

Smooth and non-smooth bifurcations

The dynamical system may have several equilibrium points: for the definite set of initial conditions and parameters the system may converge to one of them. The approached equilibrium point is called an ***attractor***. If parameters of the system change the existing equilibrium solution may become unstable and system will converge to another solution. This phenomenon is called ***bifurcation***. In general the bifurcation could be considered as an abrupt change in system's qualitative dynamics as one of the parameters is smoothly changed.

In the dynamical systems with switching (including SPC) it is possible to distinguish two inherently different types of bifurcations:

1. the first category includes all classical bifurcations that could be observed within smooth systems; these include the local bifurcations (period-doubling, saddle-node, pitchfork and Hoph bifurcations), where the periodic regime loses its stability as one of its multipliers (or a pair of multipliers) smoothly crosses the unity circle (see Fig. 2.1), and global bifurcations (e.g. homoclinic), where a connection is established from an unstable solution and back to the same solution along its stable directions;

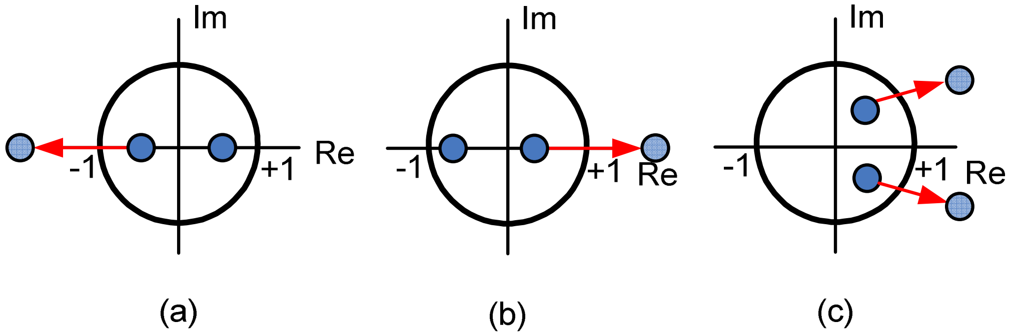


Fig. 2.1. Multipliers path at smooth bifurcation point for two-dimensional system: (a) period-doubling (b) saddle-node or pitchfork; (c) Hopf bifurcation

2. the second type, referred as discontinuity induced bifurcations (DIB), is connected with situations where the trajectory of the system intersects one of the sewing surfaces – i.e. the surfaces that divide the phase space into domains of qualitatively different dynamics; within each such domain the system is smooth, but the equation of motion change abruptly at the border of domains; this type of bifurcations, which typically involves abrupt jumps in multipliers of the orbit (see Fig. 2.2), cannot occur in smooth dynamical systems.

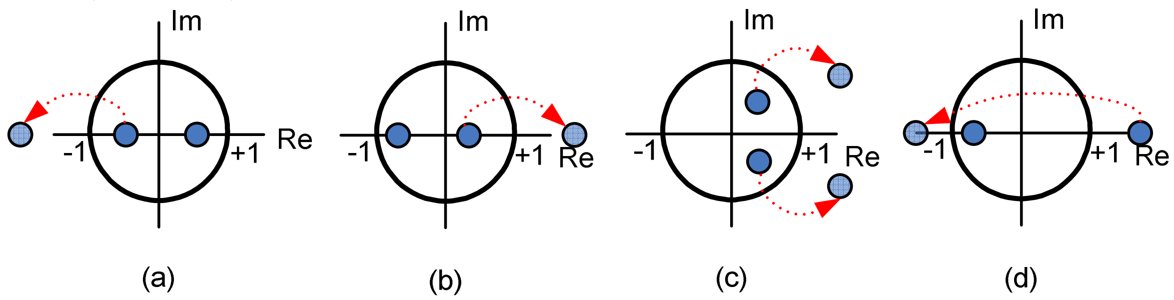


Fig. 2.2. Multipliers path at non-smooth bifurcation point for two-dimensional system (a) DI period-doubling; (b) DI saddle-node or pitchfork; (c) DI Hopf; (d) period-doubling saddle-node bifurcation

A simple type of DIB consists in the continuous transformation of a solution from one type into another. However, more complicated phenomena are also possible, including new types of direct transition to chaos, merging and disappearance of solutions of different types.

Deterministic chaos

Chaos is a particular behavior of nonlinear systems, which is characterized by an aperiodic and apparently random trajectory. In addition, the trajectory is unpredictable in the long term, meaning that knowing the trajectory at one time moment gives no information about where exactly the trajectory will be in the far future [35].

The most typical features of chaos are:

- **nonlinearity:** chaotic behavior cannot occur in a linear system; nonlinearity is necessary but not sufficient condition for the occurrence of chaos;
- **determinism:** chaotic motion must follow one or more deterministic equations that do not contain any random factors; the system states of past, present and future are controlled by deterministic, rather than probabilistic, underlying rules;
- **sensitive dependence on initial conditions:** a small change in the initial state of the system can lead to extremely different behavior in its final state;
- **aperiodicity:** chaotic orbits are aperiodic, but not all aperiodic orbits are chaotic (e.g. quasiperiodic orbits are aperiodic, but not chaotic.

Invariant collectors and crisis

Outside the small neighborhood of a fixed point the description of linearized system behavior, presented in the previous sections, is no longer valid. For example, if the FP is a saddle, the eigenvectors in the small neighborhood have the property that if the initial condition is placed on the eigenvector, subsequent iterates remain on the eigenvector. Outside the small neighborhood the lines with this property no longer remain straight lines. One therefore identify curved lines passing through the FP with the property that any initial condition placed on the line forever remains on it under iteration of the discrete-time model. Such curves are called ***invariant manifolds*** (see Fig. 2.3).

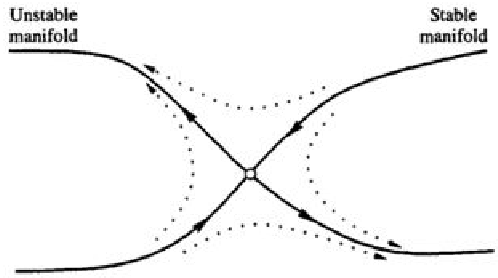


Fig. 2.3. The unstable manifold of a saddle FP attracts points in the state space while the stable manifold repels [2]

The term ***"crisis"*** was first introduced by Grebogi, Ott and Yorke [19] in order to describe the abrupt qualitative changes in the chaotic dynamics of dissipative dynamical systems as a control parameter is varied. Crisis occurs when the chaotic attractor collides with an unstable periodic solution (or its manifold). There are three types of crisis, according to the nature of discontinuity induced in the chaotic attractor [2]:

- in the first type, the chaotic attractor is suddenly destroyed as the control parameter a passes through its critical crisis value a_c; this is called ***boundary*** or an ***exterior crisis*** (see Fig. 2.4,(a));
- in the second type, the size of the chaotic attractor suddenly increases a is varied through a_c – this is called an ***interior crisis*** (see Fig. 2.4,(b)); during this crisis the chaotic attractor collides with an unstable FP of equilibrium solution that is in the interior of the basin of attraction;
- in the third type, two or more chaotic attractors of a system merge to form one chaotic attractor as a is varied through a_c – this is called an attractor merging crisis (see Fig 2.4,(c)); the new chaotic attractor can be larger in size than the union of the chaotic attractors before crisis.

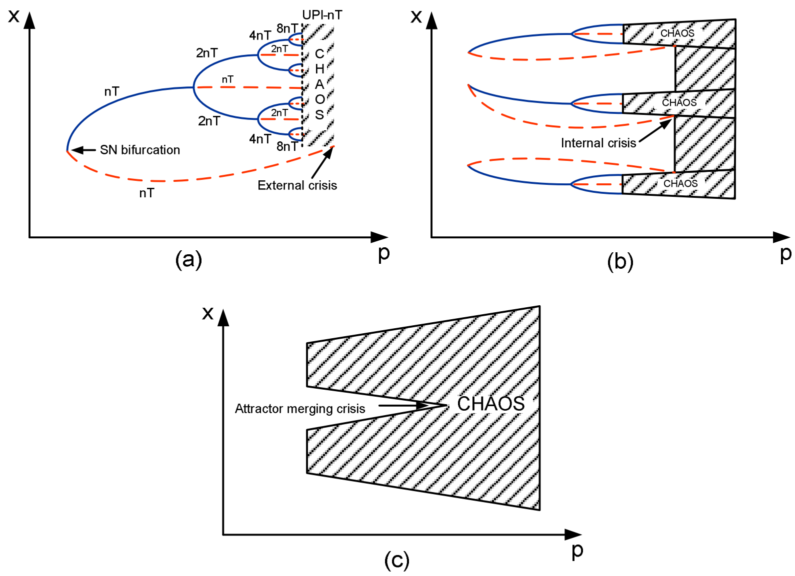


Fig. 2.4. Different types of crisis: (a) external crisis; (b) internal crisis; (c) attractor merging crisis

Stable branches of bifurcation diagram are shown as dark continuous lines, unstable – as light dashed lines, shaded areas – domains of chaotic operation

3. METHODS USED FOR THE GLOBAL STUDY OF NONLINEAR DYNAMICS OF SWITCHING POWER CONVERTERS

The third chapter presents a brief overview of tools suitable for the investigation of nonlinear phenomena in SPC. Special attention is devoted to the principles of definition of discrete-time models using stroboscopic mapping, as this kind of models will be utilized during the investigation of chaos and bifurcations in SPC by means of MCBG in the following chapters.

The global analysis of the dynamical system could be provided under fixed of varying system parameters. The global analysis of the system under fixed parameters includes the investigation of all possible stable and unstable regimes and the construction of basins of their attraction. The global analysis under varying parameters includes the investigation of possible stationary and nonstationary (transient) regimes and their bifurcations, as well as determination of changes in the structure of basins of attraction as one or several system parameters are varied.

The investigation of dynamics of the system with fixed parameters; discrete-time models

Studying the dynamics of the system with fixed parameter set, several properties of its evolution may be of interest: waveforms of selected variables, steady-state trajectories in the state space (attractors), frequency spectra of selected variables etc.

In their original forms the obtained data may not provide much insight in to the behavior of the system (e.g. it is not always possible to distinguish the chaotic operation from quasiperiodic operation from the obtained waveforms); however, if the mentioned properties are presented in certain formats, identification of a particular behavior can be more easily accomplished. Here we focus on three specific formats:

- sampled data or stroboscopic maps;
- phase portraits or two-dimensional projections;
- Poincaré sections.

One of the most important steps in the process of investigation of bifurcations and chaos in definite dynamical system is the appropriate choice of the model. The selection of the suitable description of the system is important for obtaining precise results that could be later compared to the experimentally obtained data. The dynamics of SPC could be described by system of differential equations together with appropriate definition of switching conditions. There models describe the dynamics of the system in continuous-time domain and could be used in order to obtain analytical and numerical description of the physical system under test. However, if the main goal of the researcher is to understand the nature of nonlinear phenomena, exhibited by continuous-time system, the discrete-time models are widely used.

The extensively used discrete-time maps, depending on the sampling moment could be divided in the following groups [10]:

- stroboscopic maps;
- switching maps:
  - S-switching maps;
  - A-switching maps.

All types of maps are depicted in the Fig. 3.1 together with control signals of SPC under voltage mode and under current mode control.

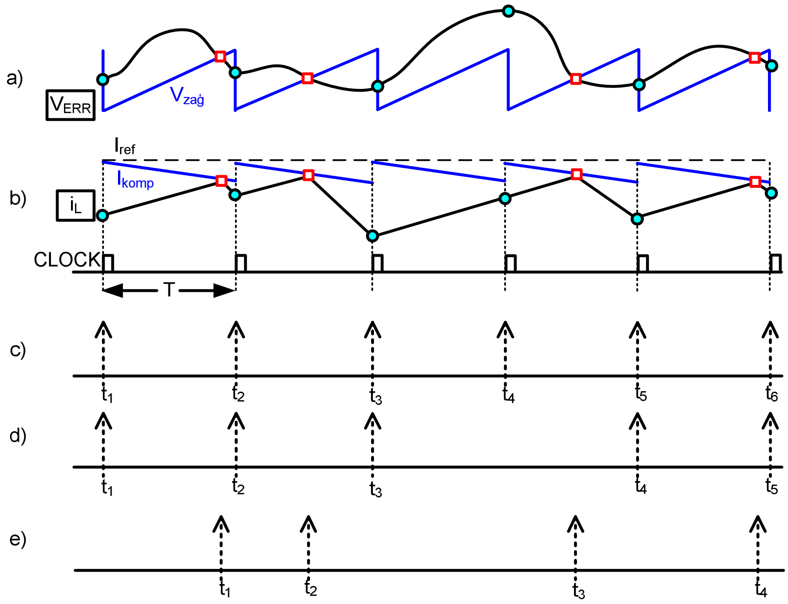


Fig. 3.1. (a) Typical ramp and error waveforms of SPC under voltage mode control; (b) typical inductor current and reference current waveforms for the SPC under current mode control with compensating ramp; (c) samples of stroboscopic map (marked as colored circles in the (a) and (b)); (d) samples of S-switching map; (e) samples of A-switching maps map (marked as colored squires in the (a) and (b))

It should be noted, that all sampling methods shown in the Fig. 3.1 are almost equivalent for the analysis of the periodic behavior of the circuit. Things may be different if the dynamics of the system becomes aperiodic. Under this situation the sampling times for the stroboscopic map are still periodic, but those corresponding to *A-switching* and *S-switching* maps will be determined by the satisfaction of the switching conditions whenever they occur. Studying the nonlinear dynamics of SPC within the doctoral thesis the stroboscopic maps are used for the following reasons:

- SPC including the pulse-width modulator are systems with external periodic source and the derivation of the stroboscopic map is more obvious and simple procedure;
- the applied MCBG is based on the use of models, defined be means of stroboscopic technique.

System dynamics under varying parameters

Nonlinear systems can behave in many different ways depending upon the values of parameters. The transition from one type of the behavior to another can happen abruptly when some circuit parameters are smoothly varied. As it has been already mentioned this sudden change in the behavior of the system is called bifurcation.

The most commonly used tool for capturing bifurcation phenomena is the ***bifurcation diagram***, which is essentially a summary chart of the different types of behavior exhibited by a system when some parameters are varied. The simplest case corresponds to the variation of only one parameter and the bifurcation diagram consist of a $p - x$ plot, where sampled data x are plotted against the chosen parameter p.

In practice in order to study the dynamics of the system under varying parameters the ***Monte Carlo bifurcation diagrams*** are used, which are based on the simplest transient processes and scanning of the phase space. The obtained diagrams give little insight in the dynamics of the system as parameters are varied and suffer from several noticeable disadvantages mentioned in the introduction of thesis.

4. METHOD OF COMPLETE BIFURCATION GROUPS AND ITS MODIFICATIONS FOR THE GLOBAL STUDY OF NONLINEAR DYNAMICS OF SPC

The fourth chapter includes the description of the main definitions and specific concepts of the MCBG, as well as main algorithms, used for the investigation of dynamics of nonlinear systems, and their improvements, accommodating the use of the mentioned approaches to the study of complex nonlinear dynamics of SPC, including the algorithms for the construction of bifurcation maps and invariant manifolds developed by the author of the thesis.

As it has been already mentioned in the introduction, the doctoral thesis is dedicated to the investigation of possible applications of MCBG to the global analysis of nonlinear dynamics of SPC. The choice of the direction of investigation is determined by the fact, that many important and even dangerous operating regimes could remain unnoticed in the traditional analytical or numerical investigation of dynamics of SPC.

The MCBG and the associated set of approaches are not fully developed, as the fundamental concepts of the method have been defined during several last years. Therefore this chapter includes: the description of the main concepts of MCBG, the improvements of the appropriate algorithms, the general recommendations for the application of the method to the analysis of nonlinear dynamics of SPC.

The fundamental concepts of the MCBG

The MCBG and the appropriate set of algorithms and programs allow the detection of new periodic and chaotic regimes, new bifurcation groups and provide the tools for the investigation of their interactions, within typical and widely applied nonlinear models. The MCBG has been developed for periodic systems that could be described by systems of differential or difference equations. The term ***periodic system*** within the MCBG is defined as dynamical system for which at least one periodic regime could be found in the defined range of parameters.

MCBG includes the following fundamental concepts [74]:

1. ***periodic skeleton*** of the dynamical system for a point p_0 of its parameter space and ***passports of periodic and chaotic orbits***;
2. ***complete bifurcation group nT*** with all stable and unstable branches of periodic points, connected in bifurcation points ;
3. ***UPI subgroup*** is a part of complete bifurcation group nT with unstable periodic infinitium (UPI) and subsequent chaotic mode of operation; the UPI subgroup consists

of only unstable orbits of bifurcation group under consideration and the birth of UPI in smooth systems is caused by the period-doubling cascade [71,72]; however, in non-smooth systems UPI may occur after the DIB, when the full period-doubling cascade shrinks to one point in the parameter space;

4. ***rare attractors (RA)*** are structurally stable attractors existing in restricted (narrow) domain of parameter space; rare attractors may be periodic of chaotic and form an important part of topological structure of all nonlinear dynamical systems.

Four types of rare attractors are defined within the MCBG:

1. tip type RA (see Fig.4.1);
2. egg-like RA (see Fig.4.2,(a));
3. kink or hysteresis RA (see Fig.4.2,(b));
4. isola isle RA (see e.g. [74]).

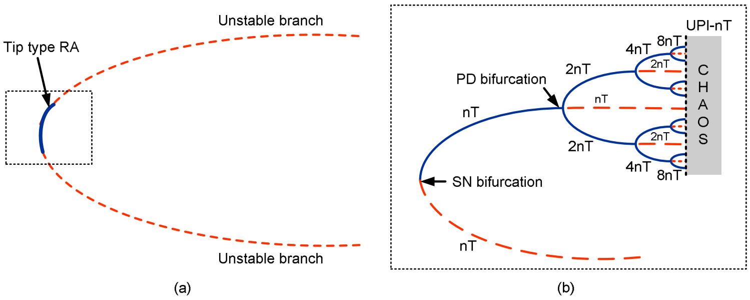


Fig. 4.1. Tip type rare attractor: (a) long unstable branch with small stable domain – tip type RA; (b) the characteristic structure of tip type RA: the attractor is formed by the saddle-node bifurcation from one side and by period-doubling cascade and subsequent chaos from another
Stable branches of bifurcation diagram are shown as dark continuous lines, unstable – as light dashed lines.

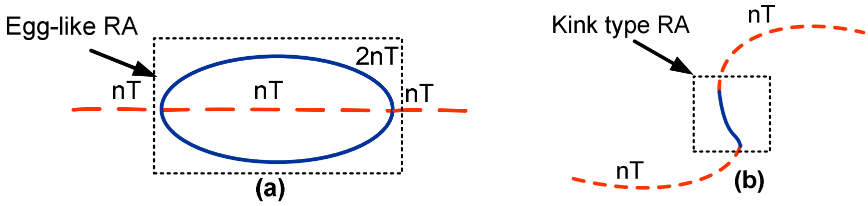


Fig. 4.2. (a) The example of egg-like RA: from both sides the attractor is formed by subcritical or supercritical period doubling bifurcation; (b) kink type RA – at the turning point of long unstable branch a small stable domain is observed
Stable branches of bifurcation diagram are shown as dark continuous lines, unstable – as light dashed lines.

5. **_complex protuberances_** grown up from internal bifurcation points (where nT solutions change their stability) at corresponding parameter values p_1 and p_2 (see Fig. 4.3); protuberances may extend far from points p_1 and p_2, and their topological structures may be very complex even in the case of simple systems; the theory based on the investigation of protuberances allows prediction and explanation of some unexpected phenomena observed in a wide domains of parameter space and includes UPI and RA;

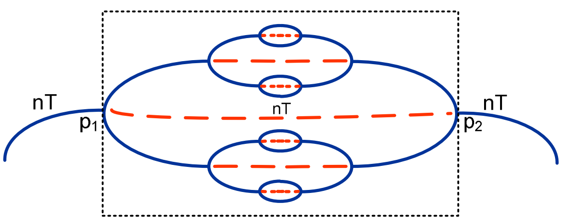


Fig. 4.3. Protuberance with complicated structure
Stable branches of bifurcation diagram are shown as dark continuous lines, unstable – as light dashed lines. Protuberance appears between two bifurcation points p_1 and p_2, where nT regime changes its stability.

6. ***typical bifurcation topological groups*** with all stable and unstable orbits and UPI, and their interaction in the same parameter space;
7. topological structure of ***chaotic attractors and chaotic transients***, the mechanisms of their birth and disappearance; the dynamical system may have several UPIs in the same parameter region, forming the structure of chaotic attractor.

Algorithms used within MCBG and their improvements

- ***The application of the Newton-Kantorovich method (NKM) to calculation of periodic regimes of dynamical systems***

The Point Mapping Approach (PMA) [87] is widely used in the process of investigation of dynamical systems. The detection of fixed points usually is carried out using the method of simple iteration, as well as methods of secondary mappings [87]. The method of simple iteration exhibits low convergence speed and does not allow the detection of definite types of saddle FP, that play an important role in the process of investigation of phase space. Thereby for the completeness of the analysis more complicated methods should be applied to the mapping operator – e.g. Newton method, the generalization of which was introduced by L.Kantorovich [83, 84].

- ***The analysis of stability of periodic regimes***

The investigation of the stability of periodic regimes [2, 11] is based on the theory of Floquet, manifesting that the periodic solution of the system is stable only if all its multipliers in the complex plane are located inside the unity circle, i.e. $|\lambda_i| \leq 1$ for all i. As the multipliers are calculated as eigenvalues of the corresponding monodromy matrix (which is calculated for the periodic regime under investigation), the main objective, defining the stability, is reduced to the construction of monodromy matrix G.

Let's assume that the system under investigation is defined using the following discrete-time operator F:

$$x^* = F_T(x^*) \ , \tag{4.1}$$

where $x=(x_1,x_2,\dots,x_n)^T$ - the vector of system's phase coordinates;
$F_T(x)=(F(x_1),F(x_2),\dots,F(x_n))^T$ - the discrete-time operator.

For the analysis of stability of the regime under investigation (i.e. the calculation of all multipliers λ), the following characteristic equation is estimated:

$$|G - I\lambda| = 0 \, . \tag{4.2}$$

In practice the analytical estimation of the monodromy matrix G is rather complicated task, that is why the simple numerical approximation is widely utilized:

$$\begin{vmatrix} \frac{F_1(x_1+\Delta, x_2,\dots,x_n)-F_1(x)}{\Delta}-\lambda & \frac{F_1(x_1, x_2+\Delta,\dots,x_n)-F_1(x)}{\Delta} & \dots & \frac{F_1(x_1, x_2,\dots,x_n+\Delta)-F_1(x)}{\Delta} \\ \frac{F_2(x_1+\Delta, x_2,\dots,x_n)-F_1(x)}{\Delta} & \frac{F_2(x_1, x_2+\Delta,\dots,x_n)-F_2(x)}{\Delta}-\lambda & \dots & \frac{F_2(x_1, x_2,\dots,x_n+\Delta)-F_2(x)}{\Delta} \\ \dots & \dots & \dots & \dots \\ \frac{F_n(x_1+\Delta, x_2,\dots,x_n)-F_1(x)}{\Delta} & \frac{F_n(x_1, x_2+\Delta,\dots,x_n)-F_n(x)}{\Delta} & \dots & \frac{F_n(x_1, x_2,\dots,x_n+\Delta)-F_n(x)}{\Delta}-\lambda \end{vmatrix} = 0 \, , \tag{4.3}$$

where Δ - small perturbation used for the numerical calculation of the derivative.

It should be noted, that the parameter of discretization should be small enough in order to reduce the effect of nonlinearity of operator F on the calculation of matrix G_k.

The described approach used for the analysis of stability of periodic regimes is universal and could be applied to analytical equations, as well as used within numerical routines. Applying the NKM, the majority of calculation time is devoted to the numerical estimation of monodromy matrix that noticeably slows down the process of investigation.

The improvement of the method for the detection of FP and estimation of their stability, based on the analytical construction of monodromy matrices and the calculation of corresponding multipliers, is proposed in the doctoral thesis. The obtained analytical equations are utilized for the verification of the numerically obtained results and implemented in the software, developed by the author of the thesis, noticeably accelerating the performance of the FP location and their stability estimation algorithms.

- ***The construction of the complete bifurcation diagrams and the investigation of rare attractors***

One of the most important steps in the process of investigation of nonlinear dynamical systems is the construction of bifurcation diagrams. The obtained information allows the estimation of qualitative dynamics of the system as one of the systems parameters (simple or combined) is varied. The bifurcation diagrams provide the information about all bifurcation points, the regions of parameter space with chaotic dynamics, the location of rare attractors, defining sudden changes in the dynamics of the system.

The following diagrams are constructed within the doctoral thesis:

- the complete bifurcation diagrams, using the numerical continuation technique – allowing the construction of branches for stable and unstable periodic regimes as well as location of new BG and RA;
- Monte Carlo bifurcation diagrams based on the transient processes – the mentioned approach is utilized for the construction of aperiodic regions of bifurcation diagrams.

Constructing the complete bifurcation diagrams, the main difficulties arise as the bifurcation branch changes its direction to the opposite one (e.g. after saddle-node bifurcation). In this case the widely accepted method called the "***arc-length continuation***" is applied [34].

The arc-length continuation has several drawbacks:

- the process of continuation of bifurcation branches uses complex parameter linked to the chosen bifurcation parameter, which should be evaluated at every point of the diagram, requiring additional calculation resources and noticeably increasing calculation time;
- the arc-length continuation rarely allows passing the points of non-smooth bifurcations and further construction of bifurcation diagrams [11].

Taking into account the mentioned reasons, the innovative approach has been proposed in the doctoral thesis, allowing fast and effective construction of bifurcation branches, slowing down the calculation only within the small neighborhood of bifurcation points and allowing the continuation, passing smooth as well as non-smooth bifurcation points, observed in the dynamics of switching converters. The new approach is based on the concept of periodic skeleton within the MCBG.

Further the basic idea of the algorithm proposed for passing the critical points and implemented within the software SMPS CHAOS, developed by the author, is explained. Let's assume that the construction of bifurcation branch of nT regime in the direction of exceeding the bifurcation parameter begins with the stable periodic regime (see Fig. 4.7). Arriving at point 1 the algorithm states that the further decrease of the parameter leads to the disappearance of the periodic regime. The described situation could be observed, in example, in the case of saddle-node bifurcation, when the stable periodic regime loses its stability and merges with the unstable periodic regime. In order to construct the bifurcation diagram the transition to the point 3 is required. The mentioned evolution is possible if the transition from point 1 to point 2 is carried out (i.e. the "step forward" is performed) and for parameter value

p_2 (see Fig. 4.7) the periodic skeleton for the system is constructed, including only periodic nT regimes. The data included in the periodic skeleton is used for the selection of the corresponding point for further continuation. It should be noted that approaching the bifurcation point or the fold of bifurcation branch, the corresponding step size control algorithm noticeably decreases the value of calculation step, in order to detect the position of the turning point with high precision.

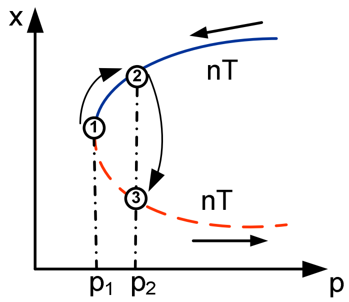


Fig. 4.7. The algorithm of passing critical points
Stable branches of bifurcation diagram are shown as dark continuous lines, unstable – as light dashed lines.

In contradistinction to widely used algorithms, based on the concept of arc-length continuation, the method provided by the author allows without any modifications the application to passing smooth and non-smooth bifurcation points as well as turning points of bifurcation branches. The mentioned algorithms require additional calculation resources only within the close neighborhood of turning points (when the construction of periodic skeleton is provided), noticeably decreasing the total calculation time.

The construction of bifurcation maps in the parameter space

Providing the analysis of the dynamics of nonlinear systems it is frequently required to study the qualitative dynamical changes as two or more system parameters are varied (e.g. during the operation of SPC the load resistance and input voltage could be varied). Therefore, one of the most important tasks in the bifurcation analysis of dynamical systems if the construction of ***bifurcation maps***, dividing the two-parameter plane to regions with qualitatively equivalent dynamics (periodic regimes and chaotic dynamics).

One of the most widely used methods for the construction of bifurcation maps is the cell-to-cell mapping approach [59], which is based on the division of the parameter space to equal cells and simple iterations from each of them, defining the periodicity of the regime under consideration. The mentioned approach has several noticeable disadvantages:

- large computation times;
- if several coexisting attractors are found in the system, for the detection of each of them it is necessary to use the set of initial conditions, that drastically increases the calculation time;
- often it is not possible to detect precisely the location of bifurcation border for two different regimes.

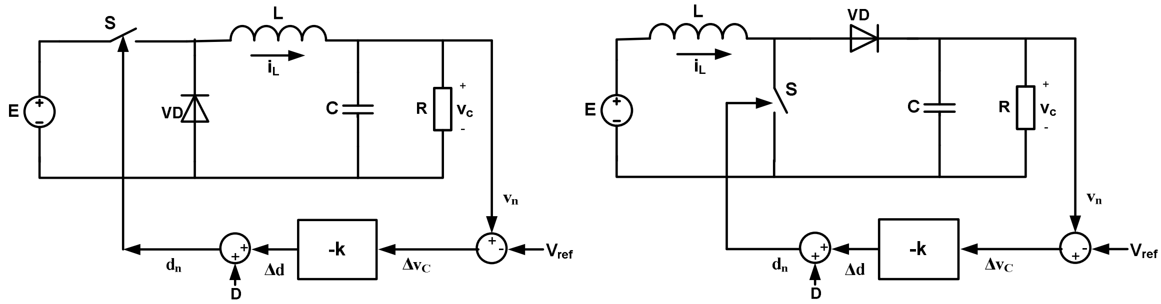
Taking into account the mentioned drawbacks the implementation of the complete bifurcation analysis within MCBG is based on the more efficient approach – ***movement along bifurcation borders***. Within the framework of this approach the coordinates of definite bifurcation points are defined and the subsequent continuation of these points in two-parameter plane is provided. SMPS CHAOS software utilizes the modification of the mentioned approach, allowing the decrease of computation time and continuation of the non-smooth bifurcation points. The detailed description of the algorithm developed by the author could be found in the Appendix 1 of the doctoral thesis.

5. THE STUDY OF NONLINEAR DYNAMICS OF BUCK AND BOOST CONVERTERS OPERATING IN DISCONTINUOUS CURRENT MODE (DCM)

The fifth chapter is devoted to the investigation of nonlinear dynamics of buck and boost converters, operating in DCM. Firstly the discrete-time models, describing the operation of the mentioned converters are defined. Then the discrete-time models are used providing analytical and numerical investigation of the dynamics of SPC on the basis of MCBG: the bifurcation maps are constructed in different parameter planes, complete bifurcation diagrams are obtained, the most significant features of nonlinear dynamics of buck and boost converters are studied. Analytically and numerically obtained results are verified (for the buck converter) by means of PSpice modeling and laboratory experiments. In the last part of the chapter the stability analysis of the converters is provided on the basis of bode plots, obtained from the averaged models. The applicability of this widely used approach to prediction of different types of bifurcation in SPC, operating in DCM in verified.

Discrete-time models of buck and boost converters

The switching process of SPC is controlled by periodic signals, therefore the operation of the circuit could be described by means of stroboscopic map, in which the values of state variables x at time moment $t=nT$ are expressed in terms of the values of x at $t=(n-1)T$. This discrete-time model facilitates the analysis of nonlinear dynamics of SPC and numerical calculations.



(a) (b)

Fig. 5.1. (a) *Buck* and (b) *boost* converters under voltage mode control

C.K.Tse in his publications [61, 62] provides the discrete-time models for the description of dynamics of *buck* and *boost* converters with proportional feedback loop, operating in DCM (see Fig. 5.1). The mentioned author has also experimentally proved in [62] that in spite of several simplifications, including series expansion and neglecting higher order terms, the proposed models accurately describe the dynamics of the converters under study, allowing the prediction of various types of nonlinear phenomena. The models provided by Tse are the first order iterative mappings:

$$\textit{buck} \quad v_{C,n+1} = \alpha v_{C,n} + \frac{\beta(H(D-k(v_{C,n}-V_{ref})))^2 E(E-v_{C,n})}{v_{C,n}}; \tag{5.1}$$

$$\textit{boost} \quad v_{C,n+1} = \alpha v_{C,n} + \frac{\beta(H(D-k(v_{C,n}-V_{ref})))^2 E^2}{v_{C,n}-E}, \tag{5.2}$$

where

$$\textit{buck}\ D = \sqrt{\frac{(1-\alpha)V_C^2}{\beta E(E-V_C)}}\,;\ \textit{boost}\ D = \sqrt{\frac{(1-\alpha)(V_C-E)V_C}{\beta E^2}}\,;\ \alpha = 1-\frac{T}{RC}+\frac{T^2}{2C^2R^2}$$

$$\beta = \frac{T^2}{2LC}$$

where D - steady state (average) duty cycle;

V_C - steady state (average) capacitor voltage (V_C=V_{ref});

k - small signal feedback gain;

V_{ref} - reference voltage;

H(.) - function accounting for the limited range of the duty cycle [0...1].

The models provided by Tse are the first order difference equations as in the DCM the inductor current i_L is equal zero at every switching instant nT and does not act as the state variable and the dynamics of the systems is governed by equation $v_{n+1}(v_n)$ with the capacitor voltage as the only state variable.

The obtained models are utilized further in this chapter, studying the nonlinear dynamics of *buck* and *boost* converters by means of analytical and numerical techniques.

The analytical investigation of dynamics of buck and boost SPC operating in DCM

Providing the analytical investigation of the discrete-time model of the *buck* converter:

- it has been ascertained, that during the operation of the converter in DCM various types of smooth and non-smooth bifurcations could be observed, caused by the saturation of the duty cycle of control signal in pulse-width modulator (see intervals 2, 4 in the Fig. 5.2), and the appearance of which could be predicted utilizing the borders

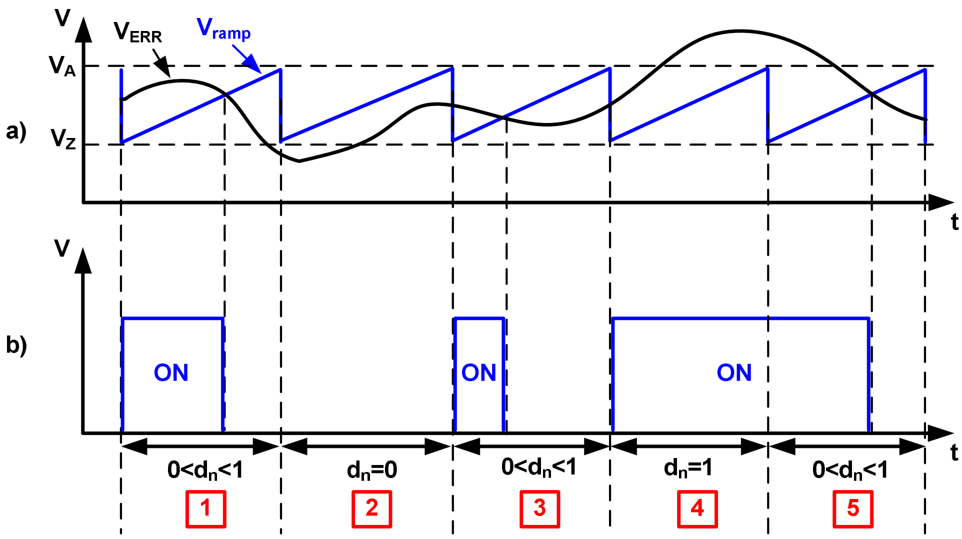


Fig. 5.2. The signals of the PWM: a) error and sawtooth voltages (b) PWM control signal

defined in the following equations:

$$\text{if } d_n=0, \text{ then } v_{C,n\_BORD\_0} = \frac{D}{k} + V_{ref}; \tag{5.3}$$

$$\text{if } d_n=1, \text{ then } v_{C,n\_BORD\_1} = \frac{D-1}{k} + V_{ref}; \tag{5.4}$$

- it has been detected, that the introduction of the proportional feedback loop and increment of the small signal feedback gain in the *buck* converter, firstly causes the appearance of period doubling cascade, leading to the development of chaotic dynamics, and only after that the non-smooth bifurcations are observed;
- the bifurcation sequence mentioned in the previous paragraph allowed providing the analytical investigation of period doubling cascades:

- it has been shown that the stability of P1 and higher periodic regimes could be estimated, using the following analytical equation:

$$\lambda = \left.\frac{\partial(v_{C,n+1})}{\partial(v_{C,n})}\right| = \alpha + \frac{\beta E[2k(-kv_{C,n}^3 + (D+V_{ref}k)v_{C,n}^2 - EDV_{ref}) - Ek^2(V_{ref}^2 - v_{C,n}^2) - ED^2]}{v_{C,n}^2} \tag{5.5}$$

- the converter operates in a stable P1 regime until the module of the characteristic multiplier λ is smaller than 1, i.e.:

$$|\lambda| = \left|\alpha - \frac{\beta ED[2kV_{ref}(E - V_{ref}) + DE]}{V_{ref}^2}\right| < 1, \tag{5.6}$$

  that allows defining the critical value of the small signal feedback gain k_{krit}, beyond which the period-doubling bifurcations (subharmonic oscillations and chaos) are observed in the system [61]:

$$k_{krit} = \frac{(1+\alpha)V_{ref}^2 - \beta E^2 D^2}{2\beta EDV_{ref}(E - V_{ref})}; \tag{5.7}$$

- the diagrams, allowing the detection of k_{krit}, varying the input voltage and the load resistance of the *buck* converter, have been obtained (see Fig. 5.3); it has been explored, that the acceptable range of the values of k, ensuring the operation of the converter in stable P1 regime, has to be detected at the maximal value of input voltage and for the minimal value of the load resistance, defined by design requirements.

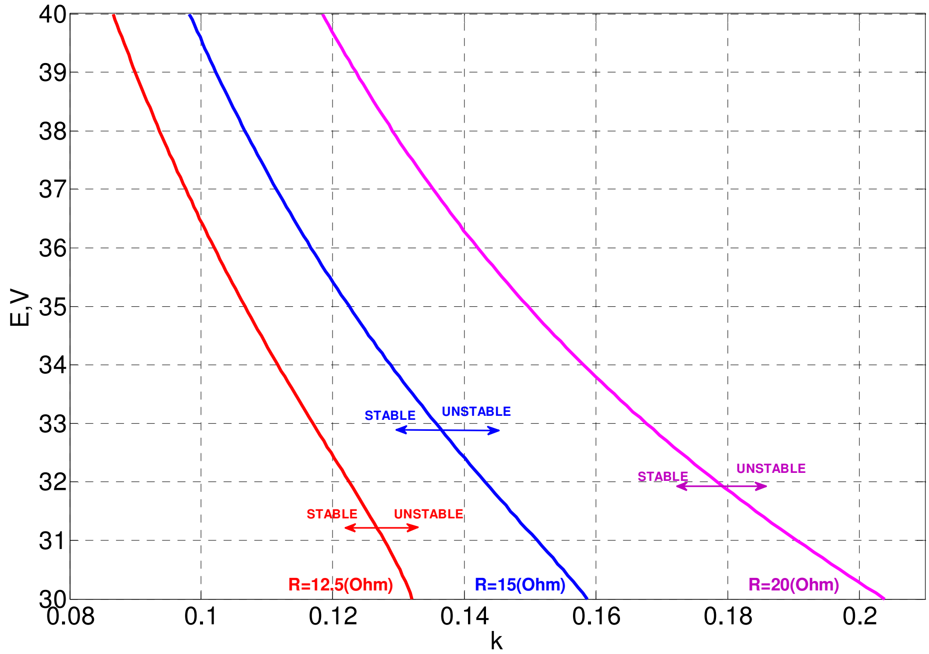


Fig. 5.3. The dependence of the critical value of the small signal feedback gain on the output voltage for different values of the load resistance

To the right of every line the unstable (subharmonic) region is located, to the left – stable P1 region.

It should be noted that all conclusions could be assigned to the dynamics of *boost* converter, the analytical investigation of which is provided in the Appendix 3 of the doctoral thesis.

At the beginning of investigation the primary and secondary bifurcation parameters (input voltage *E* and small signal feedback gain *k*), defining the qualitative changes in the dynamics of SPC, as well as dimensions and location of regions with different types of periodic (or chaotic) regimes in the parameter plane, are selected.

Providing the study of nonlinear dynamics of *buck* and *boost* converters, by means of MCBG, PSpice modeling and laboratory experiments, the following conclusions were obtained, associated with:

A. stability and chaotization of SPC:

- the bifurcation maps obtained during the investigation (see e.g. Fig. 5.4) allows obtaining the information about the division of the parameter plane into regions with different periodic and chaotic operating regimes;

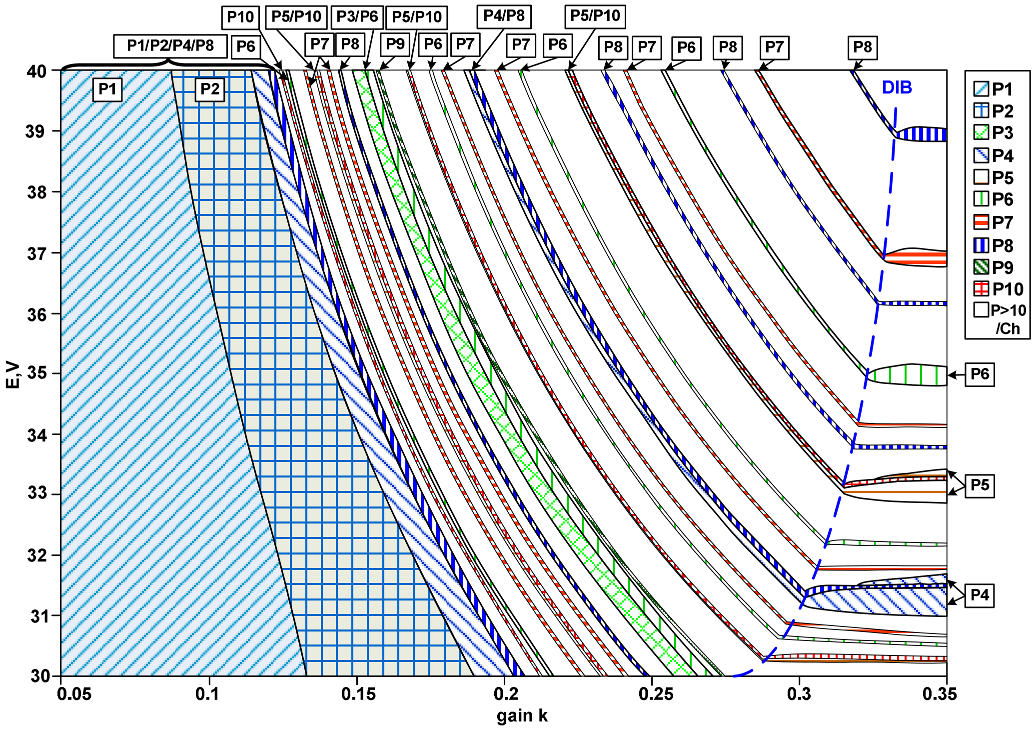


Fig. 5.4. The bifurcation map of the *buck* converter, operating in DCM, in the *E-k* parameter plane

  - ***the constructed maps could be used in the process of design of SPC, selecting the appropriate operating region as far from bifurcation boundaries as possible;***

- the constructed complete bifurcation diagrams allowed the investigation of typical bifurcation groups, chaotization scenarios of *buck* and *boost* converters, the possibilities of appearance of rare attractors, protuberances and submerged isles:
  - it has been revealed, that changing the value of the small signal feedback gain causes the appearance of smooth period doubling cascades (see Fig. 5.5) with the further chaotization at accumulation point (in which the UPI of the appropriate BG appears and the location of which could be detected by means of Feigenbaum Universality Theory);
    - ***the constructed BG, numerically and experimentally obtained results show that the increase of the period within the period doubling cascade, causes significant increase of the output voltage ripples;***

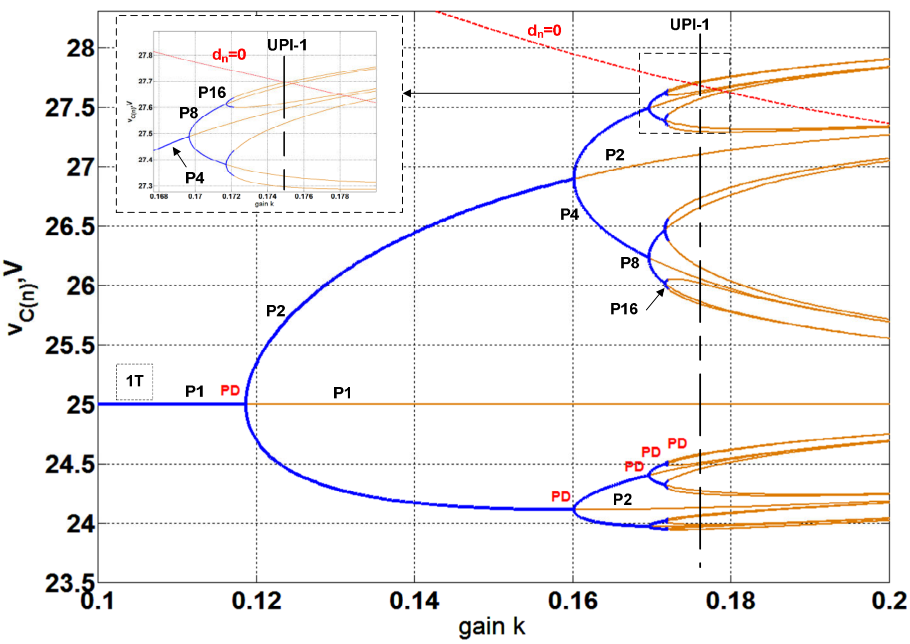


Fig. 5.5. The complete bifurcation diagram of the *buck* converter for *E*=33 V depicting *1T* BG
Stable (dark lines) and unstable (light lines) periodic regimes up to P8 are depicted in the diagram. The following designations are used: PD – period-doubling bifurcation; UPI – unstable periodic infinitium. The dark dashed line represents the border, defining the saturation of the instantaneous value of duty ratio.

- the effects of the non-smooth phenomena (such as the saturation of the duty cycle) on the dynamics of converters have been studied, estimating that the collision with the border, defined in the equation (5.3), does not cause the qualitative changes in the structure of BG (determining the operation of SPC);
  - ***the mentioned observation shows that the appearance of the skipped cycles in the operation of SPC (i.e. when the error signal at the input of comparator becomes smaller that the lowest value of the sawtooth signal and the switching element remains OFF for the whole period – see Fig. 5.2) does not cause any significant qualitative changes in the dynamics of the converter;***
- it has been shown that the collision with the border, defined in the equation (5.4) (when the switching element remains *ON* for the whole period), causes the appearance of non-smooth period-doubling and saddle-node bifurcations, resulting in the abrupt changes in the dynamics of converters;
  - ***if the error signal at the input of comparator exceeds the highest value of the sawtooth signal and the switching element remains ON for the whole period, the buck and boost concreters exhibit sudden transition to subharmonic operating regimes with the subsequent chaotization;***
- it has been shown, that the development of global chaotic attractors of the converter is defined by the interaction of chaotic attractors appearing in the individual BG (as a result of crisis event – see Fig. 5.6), or by contact of these attractors with the unstable branches of the smallest periodic regime of BG under investigation (i.e. by expansion of attractor within the unstable manifold of the saddle point);
  - ***as the dimensions of the unstable manifold could be greater that the chaotic attractors of individual BG, the appearance of internal crisis may cause significant and abrupt increase of the output voltage ripples;***

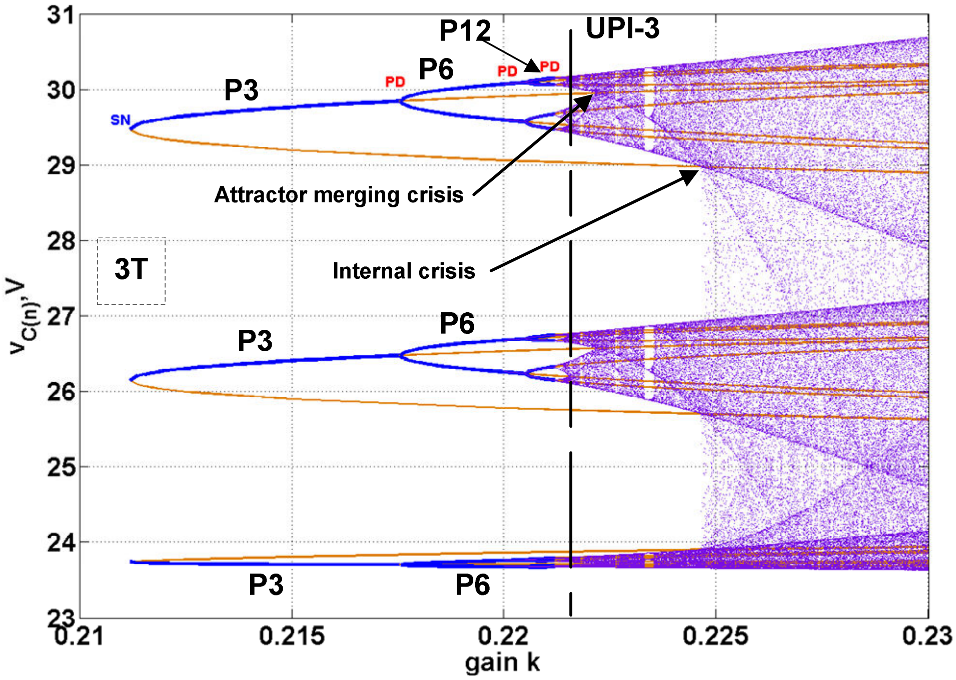


Fig. 5.6. The complete bifurcation diagram of the *buck* converter for *E*=33 V depicting *3T* BG merged with the Monte Carlo bifurcation diagram

Stable (dark lines) and unstable (light lines) periodic regimes are depicted in the diagram. The following designations are used: PD- period-doubling bifurcation; SN-saddle-node bifurcation; UPI- unstable periodic infinitium.

- the formation of complex protuberances and submerged isles within the individual BG has been studied (e.g. see Fig. 5.7);

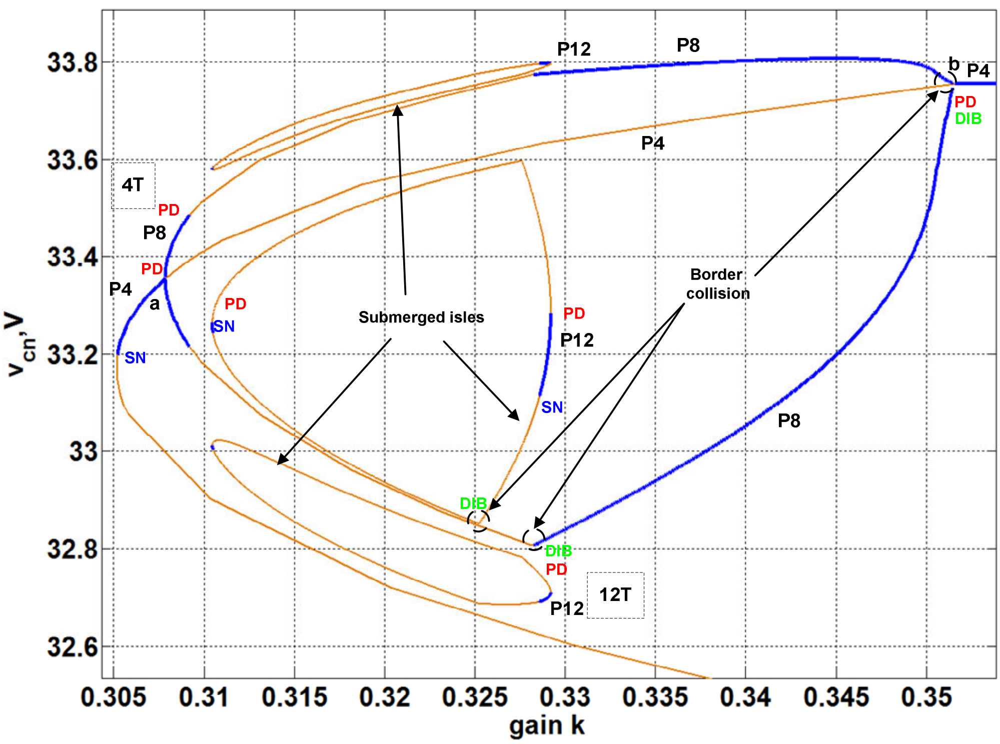


Fig. 5.7. (a) The complete bifurcation diagram of the *boost* converter for *E*=11.4 V depicting *4T* BG

Stable (dark lines) and unstable (light lines) periodic regimes up to P8 are depicted in the diagram. The following designations are used: PD- period-doubling bifurcation; SN-saddle-node bifurcation; DIB- discontinuity induced bifurcation.

- ***small deviations of the input voltage and the influence of external noise causes the transition to subharmonic and chaotic modes of operation, as well as skipping to the rare attractors, coexisting within the framework of submerged isles, leading to extremely complex dynamics of the converter;***
- the analysis of the averaged models of *buck* and *boost* converters operating in DCM, allows ascertaining that these models are not capable of predicting the appearance of subharmonic and chaotic modes of operation;

B. the improvement of EMC of SPC:

- it is shown that within the chaotic region a great variety of BG (caused by the appearance of smooth and non-smooth saddle-node bifurcations) exist, within which the tip type rare attractors are observed, defining non-robustness of chaotic operation;
  - ***the appearance of periodic windows and other types of rare attractors does not allow the use of chaotic modes of operation of SPC in order to improve the EMC, as even small noises (always present in the real converter) cause unpredictable transitions from chaotic mode of operation to subharmonic regions;***
- the verification of results allows ascertaining that the investigation on the basis of MCBG provides precise data, characterizing the dynamics of SPC.

6. THE INVESTIGATION OF NONLINEAR DYNAMICS OF CURRENT MODE CONTROLLED BOOST CONVERTER OPERATING IN CONTINUOUS CURRENT MODE

Chapter six is devoted to the study of nonlinear dynamics of the current mode controlled boost converter by means of discrete-time modeling, analytical investigation techniques, numerical calculations on the basis of the algorithms developed by the author of the thesis as well as modeling in SIMULINK software. The main attention is paid to the dynamics of the inner current loop that can become unstable under certain conditions, causing the variety of nonlinear phenomena.

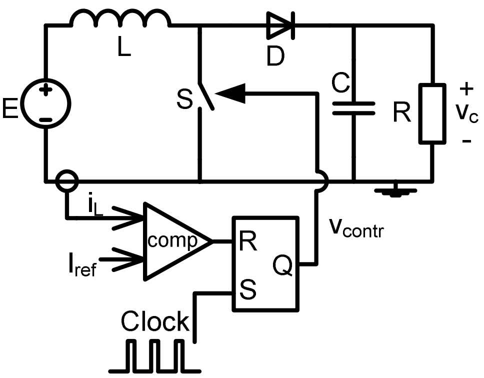
The implementation of the discrete-time model of the boost converter

In almost all practically used *boost* SPC the control of the output voltage is implemented, using two feedback loops – the inner current loop and the external voltage loop. Note that as the main focus is on the dynamics of the inner current loop, it suffices to consider the system without the voltage feedback loop (see Fig. 6.1, (a)). This assumption is acceptable as the operation of the outer feedback loop is usually much slower and its purpose is to adjust the values of reference current in accordance to changing load resistance. Therefore, the exclusion of the dynamics of the voltage feedback loop does not affect the high frequency dynamics of internal current loop.

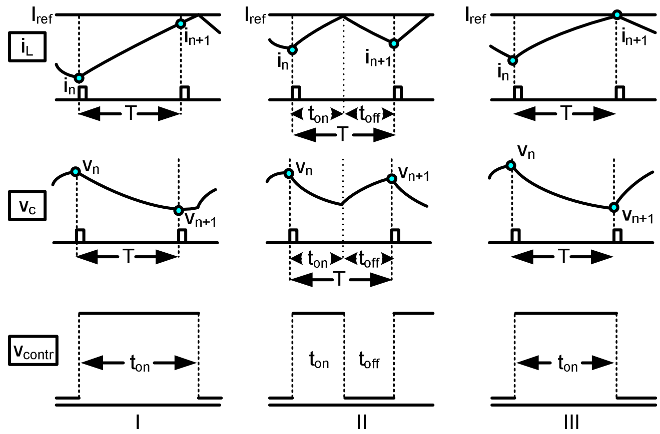
Nevertheless the operation of the *boost* converter could be described by systems of differential equations, providing the analytical and numerical analysis requires the use of discrete-time models. It is possible to obtain the discrete-time model of the current mode controlled *boost* converter in the closed form without any simplifications [6, 14, 52] (that were used, defining the corresponding models of the *buck* and *boost* converters operating in DCM).

Taking into account the observation that there exist three possible switching combinations between two consequent clock pulses (see Fig. 6.1, (b)), the corresponding discrete-time model describing the operation of the dynamics of SPC is defined for every situation, eventually merging the obtained difference equations and defining the following switching condition:

$$I_{border} = I_{ref} - \frac{ET}{L}. \tag{6.1}$$



(a)



(b)

Fig. 6.1. (a) The current mode controlled *boost* converter; (b) the waveforms of the inductor current, capacitor voltage and the control signal

If the inductor current at the beginning of the observed interval is $i_n<I_{border},$ then the next sample should be obtained using the following equations:

$$\begin{cases} v_{(n+1)} = v_n e^{-T/RC} \\ i_{(n+1)} = i_n + ET/L \end{cases}, \tag{6.2}$$

however, if $i_n \geq I_{border}$, equations (6.3) and (6.4) should be used:

$$v_{n+1} = e^{-mt_{off}}\left[K_1 \cos(\mu t_{off}) + K_2 \sin(\mu t_{off})\right] + E\,; \tag{6.3}$$

$$i_{n+1} = e^{-mt_{off}}\left[C\left[-m(K_1 \cos(\mu t_{off}) + K_2 \sin(\mu t_{off})) + \mu(-K_1 \sin(\mu t_{off}) + K_2 \cos(\mu t_{off}))\right] + \frac{\left[K_1 \cos(\mu t_{off}) + K_2 \sin(\mu t_{off})\right]}{R}\right] + \frac{E}{R}, \tag{6.4}$$

where $K_1 = v_n e^{-2mt_{on}} - E$; $K_2 = \left(I_{ref}/C - m(v_n e^{-2mt_{on}} + E)\right)/\mu$;

$t_{on} = (I_{ref} - i_n)L/E$; $t_{off} = T - t_{on}$; $\mu = \sqrt{p^2 - m^2}$; $m = 1/2RC$ and $p = \sqrt{1/LC}$.

Utilizing the obtained model it is possible to study the nonlinear dynamics of the *boost* converter by means of MCBG, avoiding the solution of systems of differential equations and additional construction of Poincare maps (the obtained model in point of fact is the stroboscopic map).

The study of dynamics of current mode controlled boost converter using the MCBG and SIMULINK modeling

Providing the analysis of nonlinear dynamics of *boost* converter, on the basis of MCBG and SIMULINK modeling, the following conclusions in the chapter six were obtained, associated with:

A. stability and chaotization of SPC:

- if the condition $T/RC >> 1$ is satisfied, the manifestations of smooth bifurcations are observed, when the loss of stability of periodic regimes and the appearance of new regimes is defined by period-doubling and saddle-node bifurcations (see Fig. 6.2);

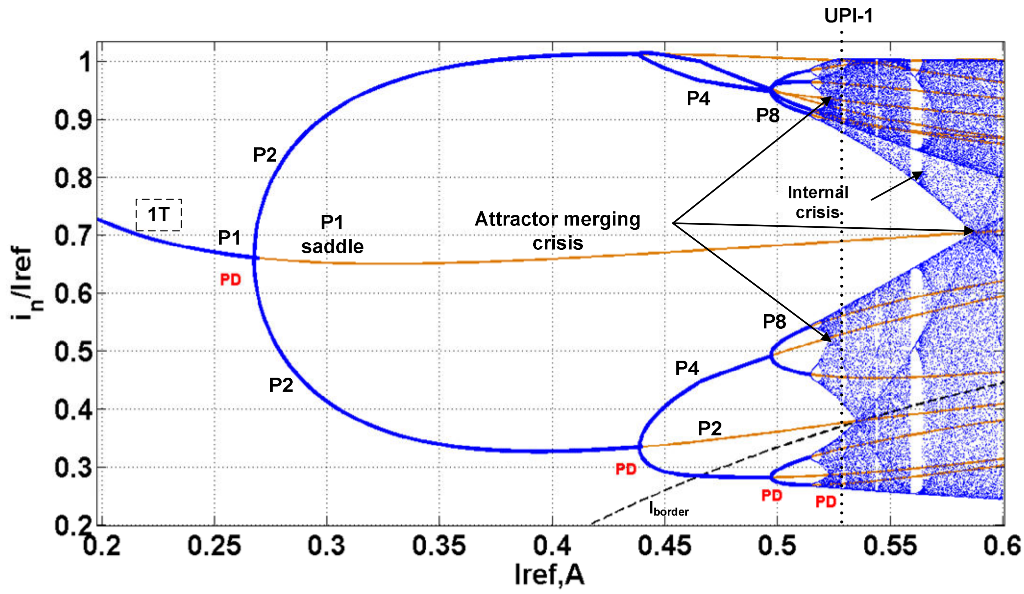


Fig. 6.2. The complete bifurcation diagram of the *boost* converter for $T/RC >> 1$, depicting *1T* BG merged with the Monte Carlo bifurcation diagram

Stable (dark lines) and unstable (light lines) periodic regimes up to P8 are depicted in the diagram. The following designations are used: PD- period-doubling bifurcation; SN-saddle-node bifurcation; UPI- unstable periodic infinitium. The border defined in (6.1) is depicted as the dark dashed line.

- the formation of global chaotic attractor is defined by attractor merging, internal and external crisis, causing the increase of voltage and current ripples of SPC;
- the construction of branches of bifurcation diagrams, corresponding to unstable periodic regimes, is extremely important in identifying and explaining the causes of different nonlinear phenomena in SPC:
  - the investigation of dynamics of unstable periodic regimes, varying circuit parameters, allows the prediction of appearance of different types of crisis (see Fig. 6.2);
  - the structure of unstable manifold of saddle type fixed point defines the parameters and location of new subharmonic and chaotic attractors in state space (as well as corresponding values un inductor current and capacitor voltage);
  - the branches of complete bifurcation diagrams, corresponding to unstable periodic regimes, allows the study of fundamentally different types of bifurcations (smooth and DIB) within one diagram, providing also the precise classification of new regimes;
- ***the new kind of tip type rare attractor***, defined by the period tripling and appearance of the whole set of unstable periodic regimes at the point of DIB, could be observed in the *boost* converters;
- if the condition $T/RC << 1$ is satisfied for the *boost* converter under investigation, the dynamics of the system is defined by effects of different types of DIB, causing the

appearance of non-smooth period-doubling and saddle-node bifurcation, leading to abrupt qualitative changes in the dynamics of SPC (see Fig. 6.3);

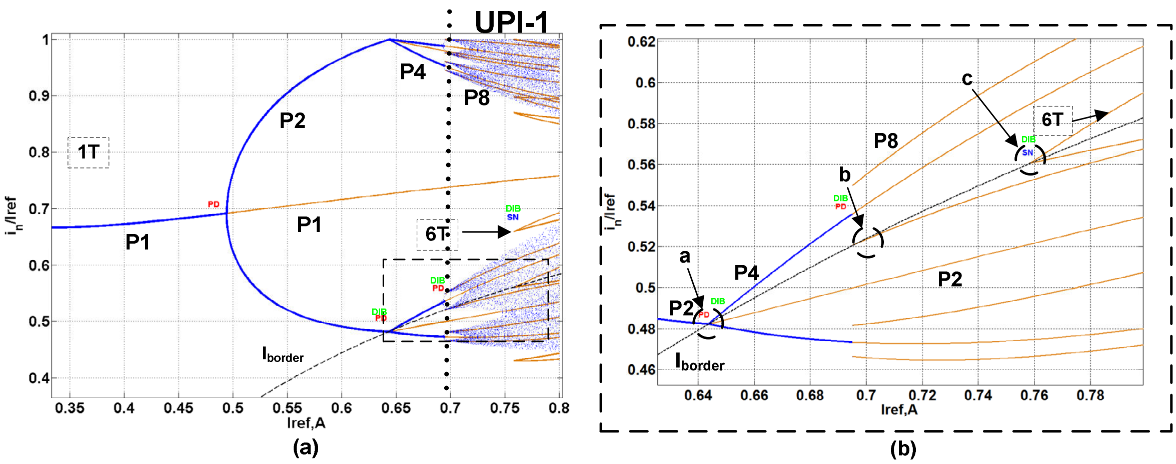


Fig. 6.3. (a) The complete bifurcation diagram of *boost* converter for $T/RC \ll 1$ depicting *1T* and *6T* BG; (b) the fragment of the diagram, showing the appearance of non-smooth period-doubling and saddle-node bifurcations

Stable (dark lines) and unstable (light lines) periodic regimes up to P8 are depicted in the diagram. The following designations are used: PD- period-doubling bifurcation; SN-saddle-node bifurcation; DIB- discontinuity induced bifurcation. The border defined in (6.1) is depicted as the dark dashed line.

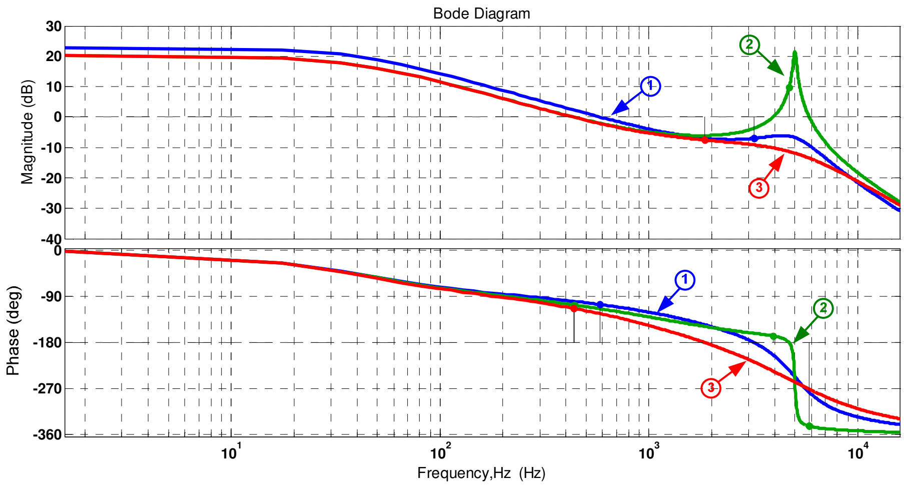


Fig. 6.4. The bode plots obtained from the Ridley model

The diagram 1 corresponds to the stable period-1 regime (phase margin is 76^0 and the gain margin is 7dB); diagram 2 corresponds to the chaotic regime of operation of SPC (the magnitude plot crosses 0dB at f=5850 Hz ***without*** any phase margin, there is also no gain margin at f=4700 Hz, that indicates the possibility of occurrence of undesirable subharmonic oscillations and overall instability of the system), the diagram 3 corresponds to the stable P1 regime, utilizing compensating ramp (phase margin is 67^0 and gain margin is 7.5dB).

- the averaged model of the *boost* SPC, proposed by Ridley [55], allows predicting the appearance of not only subharmonic, but also chaotic modes of operation in the current mode controlled converter, as well as estimating the effect of compensating ramp on the stability of P1 regime (see Fig. 6.4);

B. the improvement of EMC of SPC:

- in order to improve the EMC of SPC it is possible to use the regions of robust chaos, exhibited by converter without compensating ramp if $T/RC \ll 1$;
- the occurrence of robust chaos is defined by:
  - ***for the first time observed*** creation of the regions of unstable periodic infinitiums after the DIB when the whole period doubling cascade develops in the one point of the parameter space (see Fig. 6.5);

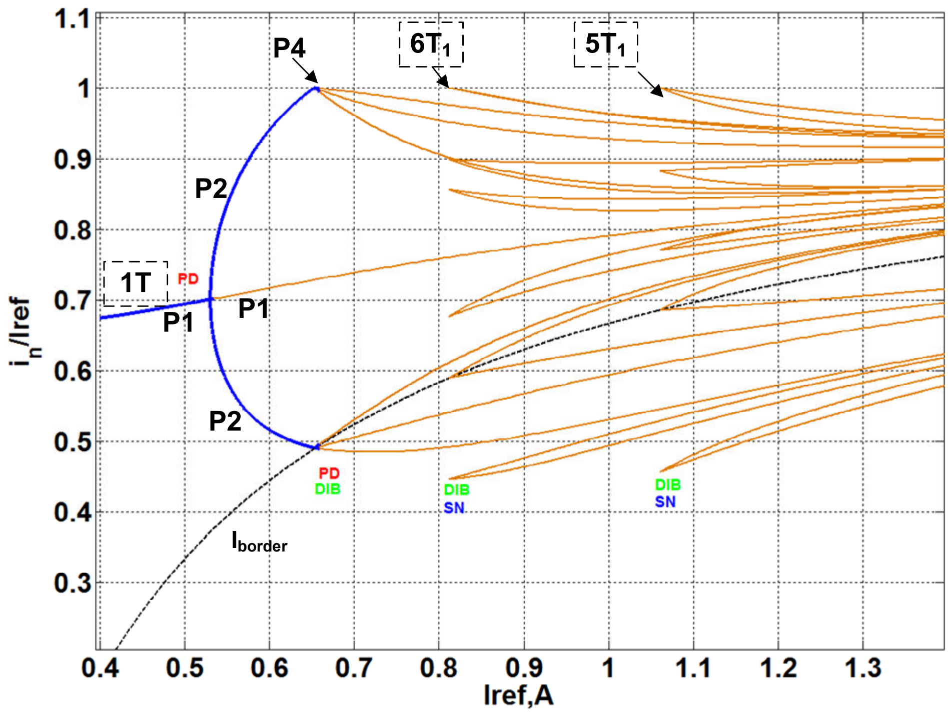


Fig. 6.5. The complete bifurcation diagram of the *boost* converter for $T/RC \ll 1$, depicting *1T*, *5T* and *6T* BG

Stable (dark lines) and unstable (light lines) periodic regimes up to P6 are depicted in the diagram. The following designations are used: PD- period-doubling bifurcation; SN-saddle-node bifurcation; DIB- discontinuity induced bifurcation.
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- atypical period-doubling saddle-node bifurcation, resulting in the developing of only unstable periodic regimes, excluding the appearance of periodic windows (see $6T_1$ and $5T_1$ BG in Fig. 6.5);

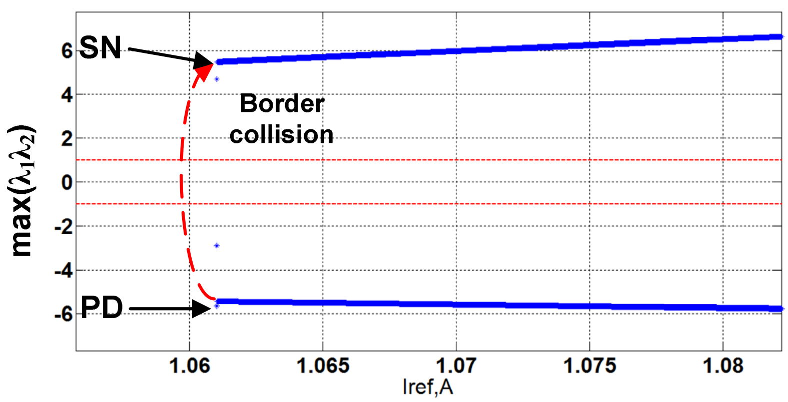

Fig. 6.17. The dependence of multipliers of the P5 BG on I_{ref}

Dashed horizontal lines represent critical values of multipliers (-1 and +1), at which the bifurcations occur. The following designations are used: PD – period-doubling bifurcation; SN – saddle-node bifurcation.

- the introduction of compensating ramp signal significantly increases the region of existence of stable P1 operation, as well as changes the whole division of the parameter space into the regions of stability of subharmonic and chaotic regimes, excluding the possibility of existence of the region with robust chaotic operation; therefore is not permitted to use the compensating ramp in the feedback of the converter if the improvement of the EMC of the converter by means of spread spectrum technique is under consideration.

7. THE INVESTIGATION OF NONLINEAR DYNAMICS OF THE VOLTAGE MODE CONTROLLED BUCK CONVERTER OPERATING IN CONTINUOUS CURRENT MODE

The seventh chapter is devoted to the study of nonlinear dynamics of voltage mode controlled buck converter on the basis of stroboscopic mapping, analytical and numerical approaches, as well as laboratory experiments. In contradistinction to the SPC described in the previous chapters, the dynamics of the buck converter in CCM with the voltage feedback loop could not be described by closed form discrete-time model. Therefore for the construction of Poincare section, author proposes specific technique, based on the reduction of the problem to the solution of transcendental equations, calculating the switching instants.

The implementation of the discrete-time model of the buck converter

The dynamics of the *buck* converter operating in continuous current mode (with voltage feedback applied) is under consideration in the following chapter. The development of the discrete-time model along with the detailed analysis of the dynamics of the converters is provided.

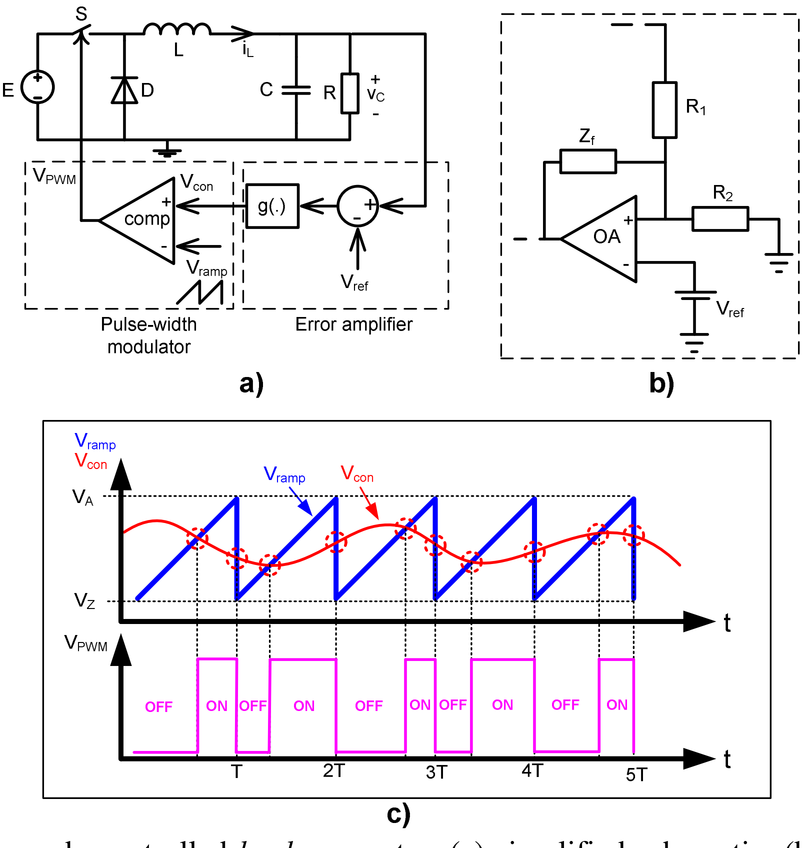

Fig. 7.1. Voltage mode controlled *buck* converter: (a) simplified schematic; (b) the structure of error amplifier (c) the acquisition of the output signal of pulse-width modulator

In the circuit with the voltage feedback loop, the value of the reference voltage is subtracted from the output voltage and amplified, in order to obtain the control voltage:

$$V_{con}(t) = A(v_C - V_{ref}), \tag{7.1}$$

that is compared to the sawtooth signal, defined by the following equation:

$$V_{ramp}(t) = V_Z + (V_A - V_Z)\left(\frac{t}{T}\bmod 1\right), \tag{7.2}$$

where A - the proportional feedback gain;
V_Z - the lowest value of the sawtooth voltage;
V_A - the highest value of the sawtooth voltage;
mod1 - the remainder after division.

The pulse-width modulated signal is obtained in the output of the comparator and further applied to the switching element. The switch is *ON*, if $V_{con}(t) \le V_{ramp}$ and *OFF*, if $V_{con}(t) > V_{ramp}$ (see Fig. 7.1,(c)).

The operation of the system could be described by the following systems of differential equations:

$$OFF \text{ interval} \begin{cases} \dfrac{dv_C}{dt} = -\dfrac{v_C}{CR} + \dfrac{i_L}{C} \\ \dfrac{di_L}{dt} = -\dfrac{v_C}{L} \end{cases} \rightarrow \ddot{v}_C + \dot{v}_C \frac{1}{RC} + v_C \frac{1}{LC} = 0\,; \tag{7.3}$$

$$ON \text{ interval} \begin{cases} \dfrac{dv_C}{dt} = -\dfrac{v_C}{CR} + \dfrac{i_L}{C} \\ \dfrac{di_L}{dt} = -\dfrac{v_C}{L} + \dfrac{E}{L} \end{cases} \rightarrow \ddot{v}_C + \dot{v}_C \frac{1}{RC} + v_C \frac{1}{LC} = \frac{E}{LC}\,. \tag{7.4}$$

Solving the equations (7.3) and (7.4), the values of $\dot{v}_C$ and v_C are obtained, the value of the inductor current could be calculated expressing it from the first equation of the systems of differential equations (7.3) and (7.4):

$$i_L = \dot{v}_C C + v_C / R\,. \tag{7.5}$$

For the sake of simplicity the following designations are introduced:

$$m = 1/2RC\,;\; p = \sqrt{1/LC}\,;\; \mu = \sqrt{p^2 - m^2}\,. \tag{7.6}$$

Therefore the solutions of equations (7.3) and (7.4) are the following:

$$OFF \text{ interval} \begin{cases} v_C(t) = e^{-mt}(K_1 \cos(\mu t) + K_2 \sin(\mu t)) \\ \dot{v}_C(t) = e^{-mt}\left[\lambda(K_1 \cos(\mu t) + K_2 \sin(\mu t)) + \mu(-K_1 \sin(\mu t) + K_2 \cos(\mu t))\right] \end{cases}; \tag{7.7}$$

$$ON \text{ interval} \begin{cases} v_C(t) = e^{-mt}(K_3 \cos(\mu t) + K_4 \sin(\mu t)) + E \\ \dot{v}_C(t) = e^{-mt}\left[-m(K_3 \cos(\mu t) + K_4 \sin(\mu t)) + \mu(-K_3 \sin(\mu t) + K_4 \cos(\mu t))\right] \end{cases}. \tag{7.8}$$

It could be seen that the equations (7.7) and (7.8) include four constants (K_1-K_4) that could be obtained, substituting the corresponding initial values.

It should be noted, that using the equations (7.7) and (7.8), the algorithm, allowing the modeling of the dynamics of *buck* converter operating in CCM with definite constant step could be developed. The critical point in the operation of the algorithm is the detection of switching moments and the recalculation of the corresponding constants for the next configuration of the circuit. Even choosing very small calculation step, there still remains the possibility, that the algorithm will skip the switching point (see Fig. 7.1), the detection of which is crucial for the calculation of constants for equations (7.7) and (7.8), defining the further development of the system and leading to possible accumulation of computation errors. For the solution of the mentioned problem it is possible to decrease the calculation step, a number of times increasing the calculation time and requiring greater amount of data to be stored. For the most efficient utilization of computation resources and reduction of calculation time, the specific precise method of calculation of switching instances is proposed.

In the beginning of modeling the admissible calculation step Δt is chosen, verifying at every moment if the sign of definite switching function $\delta(v_C, t)$ has changed (defining the transition to the next configuration). The switching function for the voltage mode controlled *buck* converter with proportional compensator could be defined using the equation relating the sawtooth signal (7.2) and the amplified error signal (7.1):

$$\delta(v_C,t) = V_{ramp} - V_{con} = V_Z + (V_A - V_Z)\left(\frac{t}{T}\bmod 1\right) - A(v_C(t) - V_{ref}). \qquad (7.9)$$

Using the introduced function it is possible to redefine the application conditions of equations (7.7) and (7.8). Therefore, if $\delta(v_C,t)<0$ the dynamics of the systems is governed by (7.7), however, if $\delta(v_C,t)\geq 0$, equation (7.8) should be used.

If during the operation of the algorithm at the time moment t_x the change of the sign of $\delta(v_C,t)$ is detected (i.e. the transition to the new configuration appears), then for the precise detection of the switching moment the following transcendental equation should be solved:

$$\delta(v_C,t) = V_{ramp} - V_{con} = V_Z + (V_A - V_Z)\left(\frac{t}{T}\bmod 1\right) - A(v_C(t) - V_{ref}) = 0 \qquad (7.10)$$

in the time interval $[t_x - \Delta t;\ t_x]$. After obtaining t_x – the solution of equation (7.10) it is necessary to recalculate the last point $v_C(t_x)$, $\dot{v}_C(t_x)$ obtained for the previous configuration, utilizing results for the calculation of constants K_1, K_2 and K_3, K_4. The described approach allows the detection of switching moments with the required accuracy that is defined by the algorithm of solution of transcendental equation.

Applying the models of the *buck* converter defined within this chapter, as well as taking into account the specific properties of switching instances, the corresponding discrete-time model is defined in the form of stroboscopic map, obtaining the samples of inductor current and the capacitor voltage at the end of every period of the sawtooth signal.

The analytical approach of stability estimation of periodic regimes

SPC are designed to operate with the constant switching frequency equal to that of the external control clock. The stability of the existing periodic mode may, however, be lost due to the variation of system parameters such as the input voltage *(E)* or the load resistance *(R)*, resulting in subharmonic oscillations. In order to avoid the occurrence of subharmonic and chaotic oscillations in practical converters, it is normal practice to specify the range of parameters (such as *E* and *R*) within which the converter will operate reliably in the steady state. Therefore, it is necessary to estimate the high-frequency instability margins in parameter space, ensuring periodic operation of SPC without the onset of subharmonic or aperiodic oscillations.

To address the mentioned problem, several specific methods of analysis of nonlinear dynamics on the basis of construction of Poincare section (described in the 3rd chapter) are proposed. The obtained discrete-time models allow reducing the problem of stability of the trajectory in the state space to that of the stability of the fixed point. For the estimation of stability of the calculated fixed points the Floquet theory [36] (originally developed to study the stability of periodic orbits) and its extensions, developed by Aizerman, Gantmakher and Filippov to analyze different types of impacting motions and stick-slip oscillations in mechanical switching systems are utilized. It is shown that the developed theoretical structure allows fast and reliable stability estimation of periodic regimes of SPC which belong to the same class of switching dynamical systems as the mechanical systems for which the theory was originally developed.

The outline of the Fillipov method

Studying the nonlinear dynamics of SPC one is interested in the stability of a periodic orbit that starts at a specific state at the beginning of switching cycle and returns to the same state after *n* switching periods. The stability of such a periodic orbit could be studied in terms of the evolution of perturbation. If the initial condition is perturbed and the solution converges back to the orbit, then the operating regime is stable. The stability margin can be assessed from the rate of convergence.

Suppose a given system has an initial condition $x(t_0)$ at time t_0 and we perturb it to $\bar{x}(t_0)$ such that the size of perturbation is $\Delta x(t_0) = x(t_0) - \bar{x}(t_0)$. If the original trajectory and the perturbed trajectory evolve in time, the perturbation at the end of the period can be related to the initial perturbation by the following equation:

$$\Delta x(t) = \Phi \Delta x(t_0), \tag{7.11}$$

where Φ is the state transition matrix, which is a function of the initial state, the initial and the final time. In linear time-invariant systems, the state transition matrix is given by the matrix exponential:

$$\Phi = e^{A(t-t_0)}, \tag{7.12}$$

where A is the state matrix that appears in the state equation $\dot{x} = Ax + Bu$.

For any SPC, the state evolves through subsystems that are linear and time-invariant, therefore, for the evolution of perturbation through each subsystem, the state transition matrix can be obtained by means of equation (7.12) (if the initial time, the final time and the initial conditions are known).

Suppose the state of the systems evolves from the instant t_A to the instant t_B and the state transition matrix for the observed period of time is Φ_1. Then the state evolves from the instant t_B to t_C, and the state transition matrix in that interval is. If the evolution from t_A to t_C is smooth (differentiable at every point), then the state transition matrix from t_A to t_C is simply the product of the two mentioned matrices $\Phi_2\Phi_1$. However, if a switching occurs at point B – the evolution at this point becomes non-smooth as the governing equations before and after the switching event are different. Aizerman and Gantmakher, as well as Filippov [36] showed that in such a situation one has to additionally consider the evolution of the perturbation across the switching event and construct the state transition matrix that relates the perturbation just after the switching event to that just before:

$$\Delta x(t_{B+}) = S\Delta x(t_{B-}), \tag{7.13}$$

where matrix S is called ***"jump matrix"*** [36] or ***„updating matrix"***.

Studying the stability of periodic orbits, it is necessary to calculate the transition matrix over a whole switching cycle. This matrix is called ***the monodromy matrix***. Rejoining the previous example and assuming that the switching in the system occurs at the time moment t_B, the monodromy matrix is expressed as:

$$\Phi_{mon} = \Phi_2 \times S_{12} \times \Phi_1. \tag{7.14}$$

If the modules of all eigenvalues (Floquet multipliers) of the monodromy matrix are less than unity (i.e. lie inside the unit circle), perturbations will die down and the periodic regimes will be stable.

Applying the methodology described in [36], the expression for the calculation of jump matrix for the voltage mode controlled *buck* converter operating in CCM could be obtained in the following form:

$$S = \begin{bmatrix} \dfrac{1}{E/L} & 0 \\ \dfrac{\dfrac{Ri_L(t) - v_C(t)}{RC} - \dfrac{V_A - V_Z}{AT}}{} & 1 \end{bmatrix}. \tag{7.15}$$

The obtained jump matrix together with the corresponding state transition matrixes, defined for the *ON* and *OFF* intervals using the equation (7.12), allows the construction of monodromy matrix for the *buck* converter in order to estimate the stability of certain periodic regime.

The described methodology was used in order to estimate the stability of periodic regimes of the *buck* SPC analytically, and integrated into the software developed by the author, allowing the decrease of calculation time and increase of calculation accuracy.

The investigation of the dynamics of voltage mode controlled buck SPC operating in CCM by means of MCBG and laboratory experiments

The analysis of nonlinear dynamics of the voltage mode controlled *buck* converter, provided in chapter 7 of the thesis, on the basis of analytical approaches, MCBG and experimental measurements, allows the definition of the following conclusions, associated with:

A. stability and chaotization of SPC:

- as pointed out within the chapter, it is not possible to obtain the iterative mapping in the closed form for the SPC under investigation, however the appropriate methodology for the construction of Poincare map, based on the precise differential equations and numerical solutions of transcendental equations, is shown;
- the application of Fillipov method to the analysis of the stability of periodic regimes in SPC allows the noticeable improvement of the efficiency of constructing the complete one parameter bifurcation diagrams, providing the possibility of obtaining the analytical monodromy matrices for the estimation of stability of certain regimes;
- the main nonlinear phenomena in voltage mode controlled *buck* converter, operating in CCM, could be characterized by the appearance of the cascade of smooth period doubling bifurcations with the subsequent chaotization of oscillations in accordance with the classical Feigenbaum scenario (see Fig. 7.2);

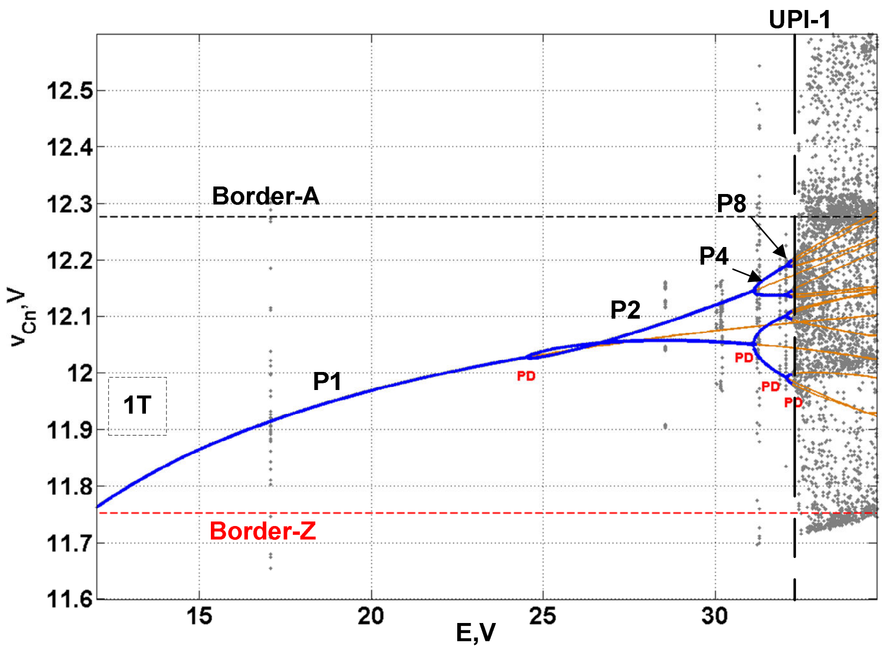

Fig. 7.2. The complete bifurcation diagram for the *buck* converter depicting *1T* BG combined with the Monte Carlo bifurcation diagram

Stable (dark lines) and unstable (light lines) periodic regimes up to P8 are depicted in the diagram. The following designations are used: PD- period-doubling bifurcation; UPI- unstable periodic infinitium. The diagram shows the classical period doubling route to chaos after the point of accumulation of unstable periodic orbits.

- the appearance of rare attractors in the dynamics of *buck* SPC could cause the essential increase of the output voltage and inductor current ripples; e.g., the dimensions of the phase portraits depicted in the Fig. 7.3 allows demonstrating that as the system exhibits the transition (caused by external noise or parameter fluctuations) from the P1 regime to the P3 rare attractor, the noticeable increase in the inductor current and capacitor voltage ripples is observed, as:
    - for the P1 regime $V_{p\text{-}p} \approx$ 40 mV and $\Delta i_L \approx$30 mA;
    - for the P3 regime $V_{p\text{-}p} \approx$ 300 mV and $\Delta i_L \approx$90 mA.

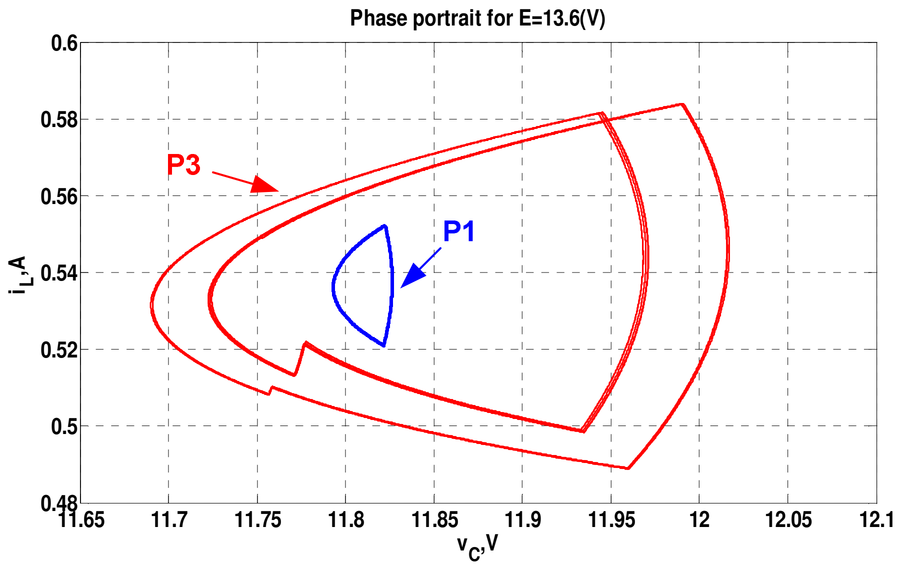

Fig. 7.3. Phase portraits depicting the coexistence of P1 regime and P3 rare attractor

- specific period-doubling saddle-node bifurcations appear in the *buck* converter as a result of border collision phenomena observed in the dynamics of the system, causing the appearance of the ***bifurcation groups of new type***, containing ***only unstable*** periodic regimes (see 5T BG in the Fig. 7.4);

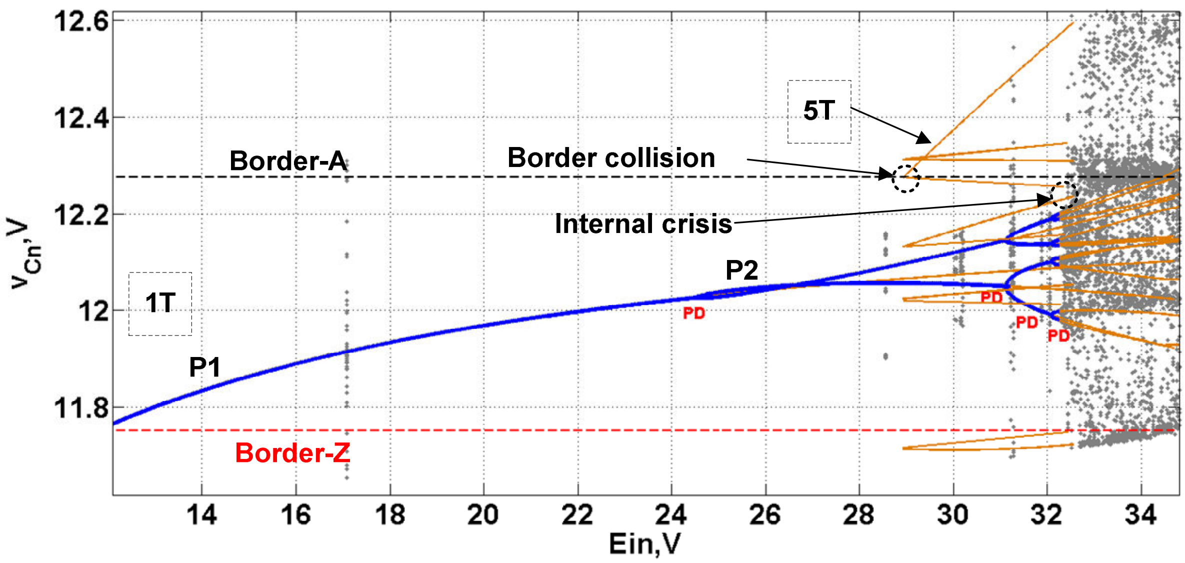

Fig. 7.4. The complete bifurcation diagram for the *buck* converter depicting *1T* and *5T* BG
Stable (dark lines) and unstable (light lines) periodic regimes are depicted in the diagram. The following designations are used: PD- period-doubling bifurcation.

- the new bifurcation groups could not be observed experimentally, however they have a crucial role in the process ob global chaotization of dynamics of the converter: if the chaotic attractor of 1T BG makes contact with the unstable manifold of the saddle point of new BG the sudden expansion of the chaotic attractor is observed (the increase of $V_{p\text{-}p}$ and Δi_L in this case is defined by the dimensions of the unstable manifold– see Fig. 7.4 and 7.5).

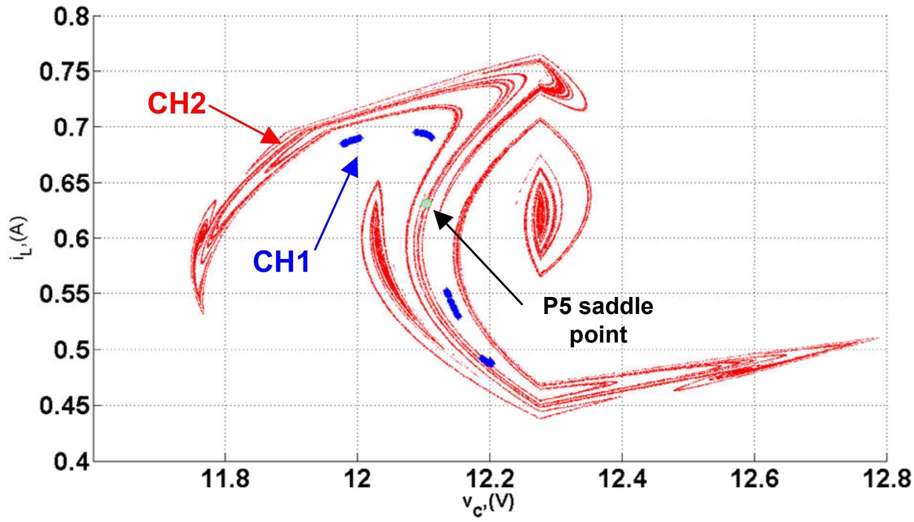

Fig. 7.5 The chaotic attractor CH1 at E=32.5 V and the unstable manifold of the P5 saddle type fixed point at E=33 V

B. the improvement of EMC of SPC:

- the coexistence of rare periodic and chaotic attractors is observed in the wide parameter space of voltage mode controlled buck converter, (e.g. see *6T* BG in the Fig. 7.6), that indicates that the robust chaotic regime could not be observed within the system, not allowing the use of the obtained chaotic operation for the improvement of EMC of switching power converter by means of spread spectrum approach.

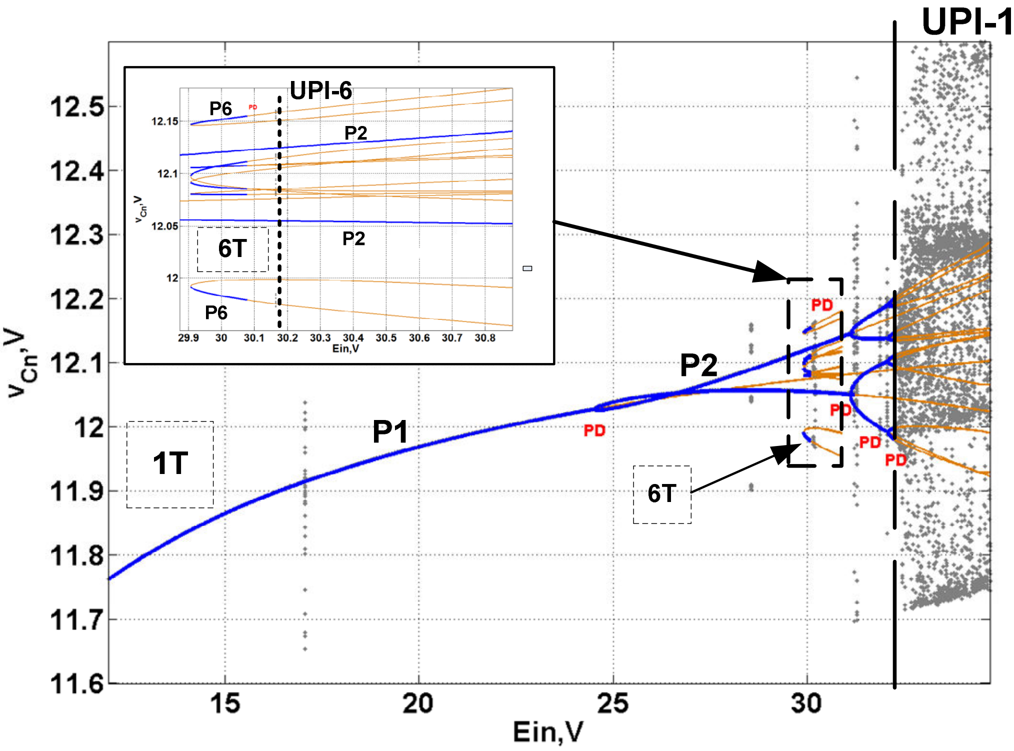

Fig. 7.6 The complete bifurcation diagram for the *buck* converter depicting *1T* and *6T* BG
Stable (dark lines) and unstable (light lines) periodic regimes are depicted in the diagram. The following designations are used: PD- period-doubling bifurcation; UPI- unstable periodic infinitium. The coexistence of rare attractor and P2 regime is shown.

CONCLUSIONS

In the process of design of switching power converters along with the other problems, ensuring the stability and improving the EMC of SPC is considered. Within the doctoral thesis it is suggested to use the modern approaches of the analysis of nonlinear dynamics in order to cope with the mentioned problems.

The results obtained in the doctoral thesis prove that the design of stable switching power converters is possible effectively applying the Method of Complete Bifurcation Groups, that allows the prediction and avoiding the occurrence of undesirable regimes in the operation of converters. It is shown that the application of widely used averaged modeling approach is limited – this method does not allow the prediction of subharmonic and chaotic modes of operation (see chapter 5.5) or distinct between periodic and chaotic regimes (see chapter 6.4). The mentioned disadvantages of the models promoted the utilization of innovative approach – MCBG, the main algorithms of which, applicable to the investigation of nonlinear dynamics of SPC, where developed and implemented by the author of the doctoral thesis in MATLAB environment.

One of the most significant features of the MCBG, in comparison to other methods used for the investigation of nonlinear dynamics, is the construction of the branches of bifurcation diagrams corresponding to unstable periodic regimes. Nevertheless the mentioned regimes could not be observed experimentally (as the smallest deviations in circuit parameters or the external noise cause the disappearance of these regimes), it has been shown within the thesis that they have a crucial role in the development of dynamics of SPC, as:

- the branches of bifurcation diagrams, corresponding to unstable periodic regimes allows the detection of rare attractors and verification of the appearance of different types of crisis (causing significant increase of voltage and current ripples in SPC), as well as the study of interactions of two fundamentally different types of bifurcations (smooth and DIB) within one bifurcation diagram and precisely classify the new operating regimes;
- the structure of the unstable manifolds of saddle points, constructed within the MCBG, defines the parameters and location of subharmonic and chaotic attractors (consequently, the corresponding values of inductor current and capacitor voltages).

For the improvement of the electromagnetic compatibility of SPC applying the spread spectrum technique (see chapter 1.3) it is proposed in the thesis to utilize the chaotic modes of operation of power converters. The possibilities of ensuring robust chaotic operation in different types of SPC, conditions for the existence of corresponding chaotic regions, as well as the influence of the parameters on their implementation is studied.

The programs implementing the algorithms developed within the thesis could be integrated into the commonly used SPC design software, allowing prediction of nonlinear dynamics of SPC during the design phase, ensuring the required regime of operation (i.e. avoiding subharmonic and chaotic modes of operation, as it could cause unreliable operation of the system, or intentionally ensuring chaotic operation in order to improve some characteristics of the converter).

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