RIGA TECHNICAL UNIVERSITY
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STABILITY ESTIMATION MODELS OF INTERCONNECTED POWER SYSTEM TAKING INTO CONSIDERATRION INFLUENCE OF REAL LOADS' CHARACTERISTICS

Summary of doctoral thesis

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CONFIRMATION STATEMENT

Hereby I confrim that I have worked out the present doctoral thesis, which is submitted for consideration at Riga Technical University for achieving Dr.sc.ing. degree. This doctoral thesis is not submitted to any other university for achieving scientific degree.

Eduards Antonovs..(signature)
Date..

The Doctoral thesis is written in Latvian language, it contains introduction, 5 chapters, conclusions and recomendations, list of references, and 1 appendix. Total number is 173 pages, which include 68 figures and 71 tables. The list of references includes 94 literature sources.

CONTENTS

GENERAL DESCRIPTION AND TOPICALITY OF THE WORK

Modern electrical power systems are largest interconnected power systems that consist of big amount of components and have complex configuration. Besides it continues interconnection in such power systems in large synchronous areas, therefore here arises the necessity in accurate analysis of its operation; special attention is paid to the questions of static and transient stability.

Creation of detailed model of large dimensionality power system is practically impossible and economically unprofitable even if powerful software available. In addition to conditions of competitive market relations and privatization in the area of electrical power engineering, the question of possibility of application of reduced models of power systems becomes extremely acute in perspective of unwillingness to interchange the detailed descriptions of schemes between market participants.

The problem is topical as for the possibility to receive the reduced model of power system with nonlinear loads accounting, represented in power form that is dependent from voltage and frequency (known as well as static and dynamic load characteristics).

In the majority of works, devoted to the power systems networks' reduction, there is not paid enough attentions to the loads representation character as well as research of its behavior during transient processes that leads to the significant calculation results distortion. Hereby appears the necessity of creation of methodology of power system network reduction taking into account real load characteristics.

AIM AND TASKS OF THE THESIS

The aim of this doctoral thesis is – a creation of efficient tool of research of power systems transient stability by means of receiving power systems reduced models that allow eliminating load nodes represented by its static and dynamic characteristics. To achieve the placed goals the following tasks were solved:

1. For the receipt of numeric and graphic results at the computation of static and transient stability was modeled initial model of Baltic power grid in the Eurostag software;
2. A reduced model of Baltic power grid by the means of elimination of load nodes was received, represented as:
   2.1. constant resistance (linear load);
   2.2. arbitrary nonlinear load characteristic, dependent only from voltage;

2.3. arbitrary nonlinear load characteristic, dependent from voltage and frequency;

3. An estimation of the aggregated load characteristic by the means of load nodes elimination was carried out, represented by its static and dynamic characteristics.
4. The conversion accuracy was researched by the means of comparison of received results during the computation of steady-state operation and transient process on the initial scheme and on its reduced scheme model.

METHODOLOGY OF RESEARCH AND TOOLS

1. The solution of linear and nonlinear differential equations of large dimension systems was operated.

2. Jordan’s matrix algebra

3. State of similarity theory

4. Main computational tasks of doctoral thesis were solved with Eurostag® software and also in Microsoft Office Excel® environment.

5. For the ease of analysis, software in the Microsoft VisualBasic® 6.3 programming language was created.

6. Graphical processing of results was completed with the help of CorelDRAW Graphic Suite X6® software.

SCIENTIFIC IMPORTANCE OF DOCTORAL THESIS

1. The reviewing of methods, used for modeling power systems of large dimension was carried out. A conclusion about the searching for a necessity for further improvement of the method was made.
2. The analysis of fundamental stages development of synchronously operated interconnected world’s systems was completed. A conclusion about its continuous meshing tendency and escalated question of complex power systems modeling necessity was made.
3. The method with the help of which is possible to load nodes with its real static and dynamic characteristics elimination, based on Jordan method was developed.
4. The method developed considers the real character of load characteristics on the side of low and/or medium voltage and allows receiving the value of full power of aggregated load as well as value of its load characteristics dependent from voltage and frequency received at the power system model reduction.

PRACTICAL VALUE OF DOCTORAL THESIS

1. The proposed method of power system reduced models creation may be applied for the research of static and transient stability of large scale power systems.
2. The method developed also may be applied to the unknown exponential quantities of static and dynamic characteristics of aggregated load definitions.
3. The proposed reduction method based on the Jordan's method assumes the solution of linear equations system and can be easy coded in the form of simple matrix operation.
4. With the help of proposed method it is possible to create reduced models of power systems with the high accuracy and without uncovering the confidence of initial scheme/topology.
5. The developed method of power system reduced models creation can be used by power system dispatching department for analysis and/or computation of different operations.
6. The execution of assessment criterion of power system parameters and state availability "n-1" as most frequently used can be checked at the received reduced model of power system, at that, in case of generation changes there is no necessity to do new reduction of the given model.
7. On the basis of the developed method there was created an algorithm for the elimination of power system models with the loads represented by its static and dynamic characteristics represented in exponential form.

APROBATION OF DOCTORAL THESIS

1. "An aggregate analytical load model with voltage dependant characteristics", ECT 2011, 6 th International Conference On Electrical And Control Technologies, Kaunas, Lithuania, May 5-6, 2011.
2. "Elimination of nodes with voltage dependent load characteristics in electrical network models", EEEIC 2011, 10th International Conference on Environment and Electrical Engineering, Rome, Italy, May 8-11, 2011.
3. "Elimination of load nodes of power system with dynamic characteristics", EE 2011, 6th International Scientific Symposium on Electrical Power Engineering, ELEKTROENERGETIKA 2011, Koshice, Slovakia, September 21-23, 2011.

4. “Comparison of two Methodologies for Power System Nodal Model Reduction presenting Nonlinear Load Elimination”, EPE 2012, 13th International Scientific Conference “Electric Power Engineering 2012”, Brno, Czech, May 23-25, 2012
5. “An aggregate network model acquired by elimination of frequency dependent load nodes”, PMAPS 2012, 12tth International Conference on Probabilistic Methods Applied to Power Systems, Istanbul, Turkey, June 10-14, 2012.
6. “Transformation algorithm of complex power system by means of load nodes elimination, represented as its static and dynamic characteristics”, EPE 2012, 2012 International Conference and Exposition on Electrical and Power Engineering, Iasi, Romania, October 25-27, 2012.

PUBLICATIONS

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1. G. Georgiev, I. Zicmane, E. Antonov. Finding of the rational approach at the decision of a question of compensation in high voltage networks. Scientific proceeding of Riga Technical University, Power and Electrical Engineering, 4 series, Riga: RTU, 2009, Vol.25, pp. 65-68.
2. A.Sauhats, J.Kucaevs, V.Chuvychin, A.Utans, G.Bockareva, L.Leite, E.Antonovs. Verification of models of automaticdevices for elimination of asyncronous operation in power systems. Proceedings of the 5th International conference on Electrical and Control Technologies, Kaunas: Technologija 2010, pp. 182-186
3. G.Georgiev, I.Zicmane, S.Kovalenko, E.Antonov. An algorithm of automatic plotting of electricity supply network circuit. Proceedings of the 5th International conference on Electrical and Control Technologies, Kaunas: Technologija 2010, pp. 265-268.
4. G.Georgiev, I.Zicmane, E.Antonov, S.Kovalenko. Operational Reduction of Active Losses in High-Voltage Networks via Reactive Powers. Scientific proceeding of Riga Technical University, Power and Electrical Engineering, 4 series, Riga: RTU, 2011, Vol.28, pp.25-31.
5. A.Sauhats, V.Chuvychin, E.Antonovs, I.Zicmane, V.Strelkovs. Interconnection of Power Systems with Different Under-frequency Load Shedding Schemes. Proceedings CD of the 10th International Conference on Environment and Electrical Engineering, Rome, Italy, May 8-11, 2011, 4 p.

6. G.Georgiev, I.Zicmane, E.Antonov, S.Kovalenko. Elimination of nodes with voltage dependent load characteristics in electrical network models. Proceedings CD of the 10th International Conference on Environment and Electrical Engineering, Rome, Italy, May 8-11, 2011, 4 p.
7. G.Georgiev, I.Zicmane, E.Antonov. Elimination of load nodes of power system with dynamic characteristics. Proceedings CD of the 6th International Scientific Symposium "ELEKTROENERGETIKA-2011", Stara Lesna, Slovakia, September 21-23, 2011, pp.48-51.
8. G.Georgiev, I.Zicmane, E.Antonov. Comparison of two Methodologies for Power System Nodal Model Reduction presenting Nonlinear Load Elimination. Scientific roceedings of the 13th International Scientific Conference "Electric Power Engineering 2012", Brno, Czech, May 23-25, 2012, pp. 109-113.
9. A.Sauhats, V. Chuvychin, D. Zalostiba, E. Antonov, V. Strelkovs. Risks, Load Shedding And System Separation During Emergency Situations. Proceedings CD of the 12th International Conference on Probabilistic Methods Applied to Power Systems, Istanbul, Turkey, June 10-14, 2012, pp.911-916.
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11. G.Georgiev, I.Zicmane, E.Antonov. Transformation algorithm of complex power system by means of load nodes elimination, represented as its static and dynamic characteristics. Proceedings CD of the 2012 International Conference and Exposition on Electrical and Power Engineering, Iasi, Romania, October 25-27, 2012, 6 p.

STRUCTURE AND CONTENTS OF DOCTORAL THESIS

The Doctoral thesis is written in Latvian language, it contains introduction, 5 chapters, conclusions and recomendations, list of references, and 1 appendix. Total number is 173 pages, which include 68 figures and 71 tables. The list of references includes 94 literature sources.

1. MAIN ASPECTS OF DEVELOPMENT, REDUCTION AND MODELING OF POWER SYSTEMS.

In this chapter a reviewing of methods applied for modeling of large power system is performed. Questions of necessity and possibility of such modeling and its part at the behavior of power system analysis are overviewed. Special attention is given to questions of development of synchronously interconnected power system.

The construction and development of cross-boarder transmission lines was started in the beginning of XX century, what lead to appearance of the first interconnections and their synchronous operation. Today all Europe is subdivided in 4 large synchronously interconnected power systems (UCTE, IPS/UPS, NORDEL, TESIS), some of them are interconnected through direct current cables [1].

At the planning of development, designing and operations control of large power systems it is necessary to solve large spectrum of engineering and techno-economic tasks that have analytical and computational character. For their practical implementation must take a number of assumptions, which may lead to significant distortions of the results received [2,3].

However the difficulty and the essentially limited possibilities of real power systems' results receipt indicates about the necessity of creation of experimental system. Obviously as such may be used only the model system [3]. The notion of a model always requires the introduction of the concept of similarity, that is reciprocal single-valued conformance between objects [3].

The method of electrodynamic modeling of power systems began to develop in the 20th of XX century due to increase of power and meshing of power system schemes. First attempts of powerful model's creation were not successful, therefore were created and operated electrodynamic models of lower-power. Moreover, one of the first attempts of studying and reproduction of process running in the power systems were based on the real experiments [3]. Only a bit later of the middle of XX century were created first computational tables (models) of direct and alternating current [2,3].

The following expansion of interconnected power systems scale, the amount of power stations, substations, network elements and etc. arising, as well as interconnection deepening of their interrelations and wide implementation of automation equipment, serves as a unique stimulus to start use digital computing techniques in the tasks of power engineering [2].

Today exists a set of engineering softwares for analysis and computation of power systems, that allow to simulate processes in the time frame from one second till few days and research schemes from 1 till 10000 nodes, but due to the large dimensionality of modern power systems still exists a number of problems of insertion of its detailed model in the software, that is – appears the necessity to create equivalent models which images steady state and dynamic characteristics of initial full model.

In this chapter are examined methods of power systems models reduction (Gauss–Rutishauser, method of discharged matrix, nodes combination by Dimo method, nodes combination by Zukov method), which applies at the computation of line circuits and doesn't allow to consider all parameters of power systems, which have important part in the transient processes research. Thus none of the methods mentioned above does not allow to exclude of "nonlinear" nodes, that is nodes specified with the actual static or dynamic load characteristics.

2. JORDAN'S ELIMINATION / TRANSFORMATION METHOD AND ITS APPLICATION TO THE POWER ENGINEERING TASKS

In this chapter the application of Jordan's elimination method for the power systems models reduction by means of load nodes elimination, which loads are represented as constant resistance is offered.

Linear method of Jordan's elimination examined in this chapter is modification of Gauss method and is used for the transformation of linear equation system with the aim to change the unknown with arbitrarily tacked free term that is by means of elimination of correspondent variable out of vector of unknown. Only one condition - at each step of Jordan's elimination it is necessary to choose *pivotal element* which is not equal to zero and corresponding unknown independent variable U_i and known dependent variable J_i, on which will be complete the operation of its places changing in the table [4-6].

The feature of the given method is that it is not oriented on the required system solution, but more often is used only for the system transformation with the aim to receive more useful description of examined model. Moreover while using this method for the power system, it is possible to work not obligatory with a full network but also with its separate part, which contains elimination nodes and adjacent to it. As well as reduction can be complete separately by the voltage groups, as transformer nodes doesn't get under elimination then it is possible not to consider transformer brunches.

Let's examine, as an example, the part of power grid scheme of alternating current which consists of three nodes with 3 driving currents, fig. 2.1.

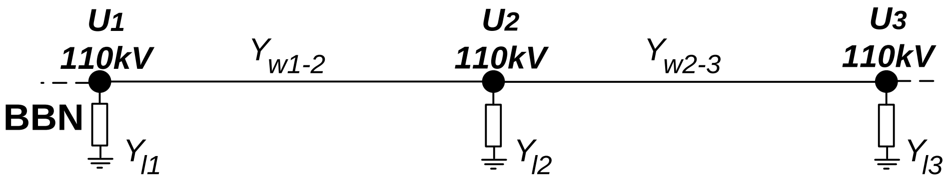


Fig. 2.1. Initial network scheme

All loads given as constant resistances that is the system have linear dependence:

$$P = P_0 \cdot \left(\frac{U}{U_0}\right)^{\alpha} \; , \; Q = Q_0 \cdot \left(\frac{U}{U_0}\right)^{\beta} , \tag{2.1}$$

where - exponential quantities and $\alpha = 2$, $\beta = 2$

The system of linear nodes equations by I Kirchoff's law in the matrix form with complex variables will have next notation:

$$\boldsymbol{Y \cdot U = 0,} \tag{2.2}$$

where the values of loads constant admittances are added to the diagonal elements of nodal admittances matrix Y [7,8].

Using matrix notation we will receive;

Table 2.1

	U1	U2	U3
J1	Y_{11}	$-Y_{12}$	0
J2	$-Y_{21}$	Y_{22}	$-Y_{23}$
J3	0	$-Y_{32}$	Y_{33}

g loads
$Y_{sl1} + 1/2\omega c_1$
$Y_{sl2} + 1/2(\omega c_1 + \omega c_2)$
$Y_{sl3} + 1/2\omega c_2$

where $Y_l = G_l + jB_l$, $Y_w = G_w + jB_w$

It is necessary to note that for the mostly visualization in the present work the vector of independent variables [U] will be written in the upper row of nodal admittances matrix [Y].

In the table 2.1 to the loads susceptance adding ½ of lines capacitive susceptance [9,10].

Lat's assume that it is necessary to eliminate 2nd node, fig. 2.1. One step of Jordan's elimination with pivotal second row and second column consist of whole set of rules of transformation the systems coefficients:

1. pivotal element changes to the inverse value:

$$Y'_{22} = 1/Y_{22}, \tag{2.3}$$

2. non-transformed elements of pivotal column (that is elements which don’t belong to the pivotal raw or column) multiplies on the inverse value of pivotal element:

$$Y'_{n2} = Y_{n2} \cdot 1/Y_{22}, \tag{2.4}$$

3. non-transformed elements of pivotal raw multiply on the inverse value of pivotal element but taken with inverse sign:

$$Y'_{1m} = -Y_{1m} \cdot 1/Y_{22}, \tag{2.5}$$

4. non-transformed elements of matrix are transformed into correspondence with equation:

$$Y'_{33} = Y_{33} - Y_{32} \cdot Y_{23} \cdot 1/Y_{22}, \tag{2.6}$$

where with index $'$ are indicated elements received after Jordan’s transformation that is numerical values of its elements are different from initial [4-6].

Thereby after all above-mentioned transformations the second (pivotal) column and second (pivotal) raw are allowed to be eliminated:

Table 2.2

	U1	***U3***
J1	Y'_{11}	$-Y'_{13}$
J3	$-Y'_{31}$	Y'_{33}

g loads
Y'_{sl1}
Y'_{sl3}

It should be remembered that active and reactive component of loads are represented by the equivalent admittances, which values are determined by means of subtraction of nondiagonal elements sum from diagonal element, taken with inverse sign [8]:

$$Y'_{sl1} = Y'_{11} + \left(-Y'_{13}\right). \tag{2.7}$$

A new value of lines admittances is defined from correspondent nondiagonal element of transformed matrix.

New transformed scheme, fig. 2.2 corresponds to the reduced system.

Fig. 2.2. Circuit layout received after Jordan’s elimination

Received scheme model is accurate equivalent of initial scheme, that is in the received model of reduced scheme towards the nodes 1 and 3 is saved the operation of initial scheme [11-16] and I Kirchoff's law is applied. At this the matrix diagonal elements still should be equal to the sum of nondiagonal elements of raw but taken with inverse sign and to the admittance of new equivalent load. By-turn, the sum of new loads power received after the elimination must be approximately equal to the sum of loads power before elimination. At that, the active component of load may rise a little due to the adding to it the lines capacitive susceptance with inverse sign [11,13,15]. Voltages and their angles at the all remained nodes remain the same as well as the balancing power hasn't changed. Thereby received equations may be used for the following elimination [11,13,15].

Examined method supports high accuracy of received results at the computation in the area of steady state and transient processes of power systems. The load of eliminated node is automatically spreads between the adjacent nodes (located topologically close by it), by this way not disturbing the operation of power system other part. Based on this it is possible to functionally change the divided part of initial scheme with one aggregated load.

Using proposed method it is possible to eliminate load nodes using matrix operations that allow saving time and with one step to eliminate any necessary set of nodes of power system as well as it is very significant at the computer implementation.

The elimination allows receiving the convenient visualization of power system scheme topology thereby to ease the analysis of cause-effect relations that appear on the equivalent lines between generators stations.

3. POWER SYSTEM NETWORK REDUCTION BY MEANS OF LOAD NODES ELIMINATION REPRESENTED OF ITS CONSTANT ADMITTANCE

In this chapter an algorithm based on the Jordan's elimination method is introduced and considered its application to the part of power system scheme reduction where all loads are represented as constant admittances.

Based on the proposed technique of power systems models reduction was developed an algorithm, fig. 3.1, which allows eliminating load nodes from power system's scheme, which are given as constant admittance/impedance and at this keeping only those nodes that were specified in the special list (block 4). Nodes elimination is performed by using Jordan elimination method in a complex form. This is a step by step method - elimination of the one node at each step. To ease the process, all activity is performed in relative units.

Algorithm's 2nd block serves for scheme parameters input, such as nodes numbers and its voltages, lines and their parameters (R, X, Bc); power of loads (P, Q) and its static or dynamic load characteristics (exponential quantity $\alpha, \beta, \gamma, \delta$).

In the third block it is necessary to specify basis conditions – base power S_{base} and base voltage U_{base}. Whereupon, a reduction of scheme parameters to the selected basis conditions is fetched in this block.

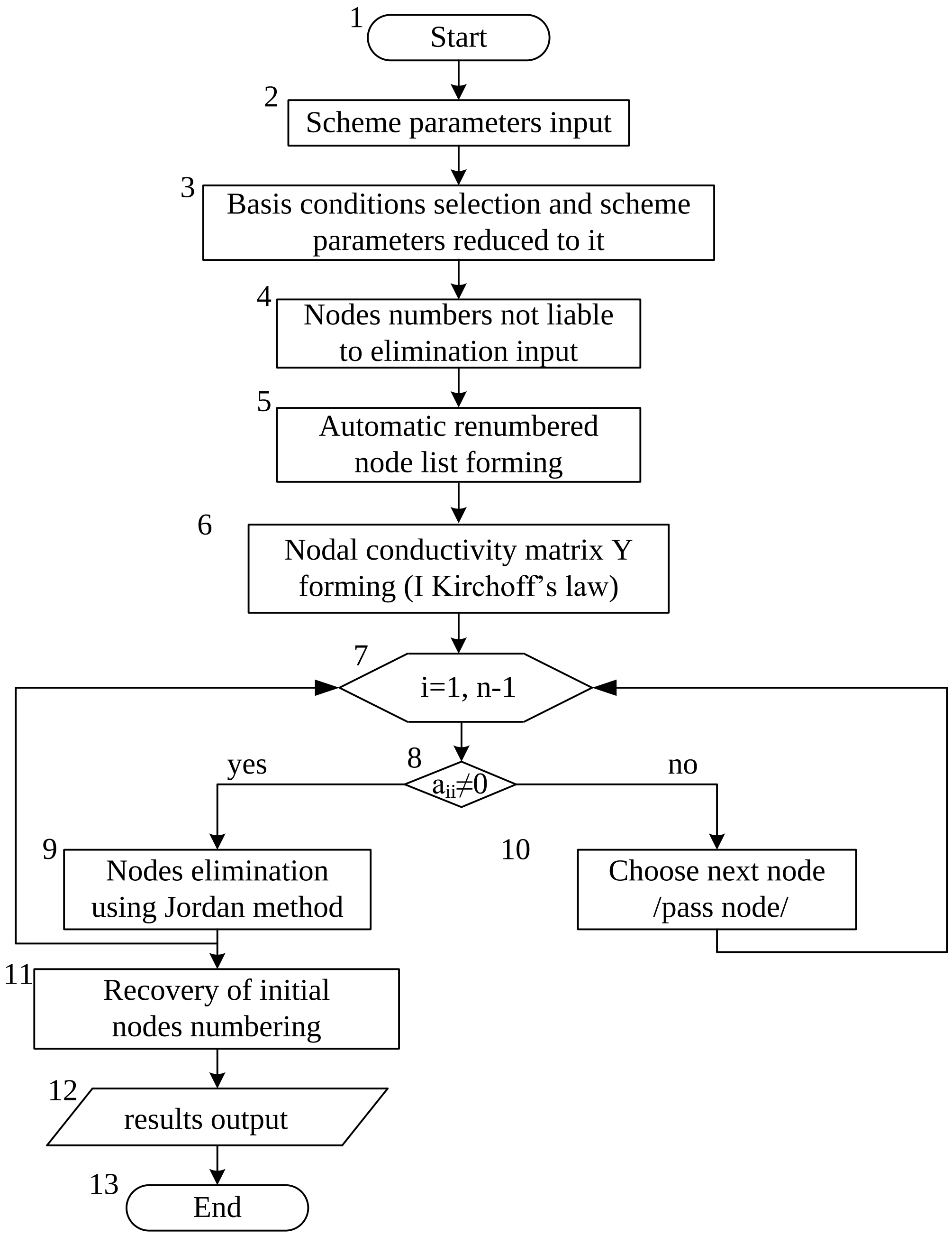


Fig. 3.1. Algorithm block scheme

The 4th block is used for the nodes numbers input which are exempt to elimination, that is - nodes, which must remain after the scheme transformation.

The 5th block automatically creates renumbered nodes list: defines the nodes number in the scheme and classifies its numbers in increasing order. Whereat new ordered numbers by increase (1,2,3,...,n), that support coincidence of nodes number and number of the counter of next used cycles are assigned. Initial numbers of nodes return at the block number 11.

The 6th block forms the nodal conductivity matrix Y in accordance to the I Kirchoff's law $YU = 0$. Lets remind that non diagonal elements of matrix with coordinate i and j corresponds to lines admittance with numbers i and j, but with inverse sign [17,18]. Matrix completes with zero for nonexistent lines. Each diagonal element is equal to the sum of elements in its row, but in reverse sign. To the each diagonal element, also is added admittance on the ground of proper load of this node [7,8].

Cycle block 7 contains the step of cycle and condition of end of the cycle.

The 8th block chooses each next node for elimination and checks condition of inequality to zero of pivotal element that is the only one condition at each step of Jordan's elimination [4-6]. If this condition doesn't execute, the program goes to the block nr. 10. For the topologically interconnected power system the probability that diagonal element of nodal conductivity matrix will be equal to 0 is very low.

On each step of the elimination, in the block nr. 9 availability check of nodes number liable to elimination in the block number 4, in the nodes list not liable to elimination is completed. If this node is missing in the list, its elimination by the Jordan's method occurs, that is over all elements of nodal conductivity matrix in correspondence with Jordan's elimination rules. If the number of node liable to elimination is in the block number 4, than this node will be leave and program will transfer to the next node. Each cycle eliminates only 1 node and then the program returns to the start of cycle in order that goes to the next excludable node. After the check of all scheme's nodes with the following elimination of its part, in the block forms new nodal conductivity matrix which corresponds to the obtained reduced scheme composed of only the nodes specified in the block 4.

Block 10 will skip the earlier chosen node due to the non-compliance of condition in the block number 8 and will return the program to the start of the cycle (block number 7), until the condition in the block 8 is executed.

In the block number 11 the initial nodes numbering is returned to the start.

Block 12 displays the results, exactly active and reactive impedances for the neogenic lines, the values of active and reactive power for the remaining load nodes [11,12].

For the case study was used the scheme of “БРЭЛЛ”, the content of power system IPS/UPS, with the next assumptions:

- Part of initial scheme IPS/UPS (Russia, Belorussia, Lithuania, Estonia) initially are represented as equivalent.
- The scheme of Latvia’s power system that is a component of the scheme is considered without taking into account the topology of distribution network (20 kV, 0,4 kV), but taking into account the loads of step-down transformers HV/MV. At this all nodes and lines at the voltage of 110 kV and 330 kV are represented fully.

For the ease of analysis of computation the considered scheme is conventionally divided in 6 areas, table 3.1.

Let’s reduce the initial scheme of Latvia’s power system at the voltage of 110kV which consists of 247 nodes, table 3.1, using an algorithm described above, keeping some important distributing points.

Thereby the amount of load nodes for the Latvia’s power system decreased more than 3 times, that is from 247 to 79 nodes.

Table 3.1.

Scheme’s element	Region (area)	Initial scheme	Reduced model of the scheme
Node		291	126
	Belorussia (8)	4	4
	Estonia (9)	5	5
	Finland (Estlink) (10)	1	1
	Latvia (11)	247	79
	Lithuania (12)	17	17
	Russia (13)	17	17
Line		345	129
Transformer		60	60
Load		186	67
Generator		31	31

While comparing the results of load flow computation based on the initial and obtained reduced model of scheme, the relative error for the received values of voltage didn’t exceed 0,009% and for the voltage angles - 0,05%. From the practical point of view such errors are allowed to be considered as insignificant.

Total results of load flow computation of initial and obtained reduced model of scheme are given in the tables 3.2 and 3.3.

Table 3.2.

Results of load flow computation of initial scheme

Area number	Active power (MWt)			Reactive power (MWAR)		
	Load	Losses	Export	Load	Losses	Export
8	4400,00	11,38	-501,38	1400,00	-437,32	103,42
9	1020,00	25,06	254,94	183,00	-510,90	166,78
10	223,41	0,05	-223,47	0,00	6,14	-6,14
11	857,59	28,99	266,84	317,90	-459,77	-95,01
12	1265,00	9,45	-699,45	250,00	-710,63	149,80
13	158305,00	59,22	905,92	34785,00	-3606,19	-321,74
Total	166071,00	134,15	3,41	36935,91	-5718,67	-2,90
	MWAR generated by network and capacitors -3178,77 MWt losses 7,82					

Table 3.3.

Results of load flow computation of reduced scheme model

Area number	Active power (MWt)			Reactive power (MWAR)		
	Load	Losses	Export	Load	Losses	Export
8	4400,00	11,38	-501,38	1400,00	-437,32	103,45
9	1020,00	25,06	254,94	183,00	-510,91	166,80
10	223,41	0,05	-223,46	0,00	6,14	-6,14
11	859,36	27,10	266,94	133,82	-275,34	-95,14
12	1265,00	9,45	-699,45	247,34	-707,97	149,89
13	158305,00	59,22	905,84	34785,00	-3606,19	-321,64
Total	166072,78	132,26	3,43	36749,16	-5531,60	-2,77
	MWar generated by network and capacitors -3178,75 MWt losses 7,82					

Relative errors for the total loads by areas are given in the table 3.4.

Table 3.4.

Relative error of total load comparing the results of initial scheme and its reduced model

Area number	Relative error of active load ε(P),%	Relative error of reactive load ε(Q),%
8	0	0
9	0	0
10	0	0
11	0,206392	57,905
12	0	1,064
13	0	0
Total	0,001072	0,505606

Different kinds of short-circuits were studied, for example – graphic results of comparison at the solid fault at the node 904 (Pļaviņu HES) are represented at the fig. 3.2.

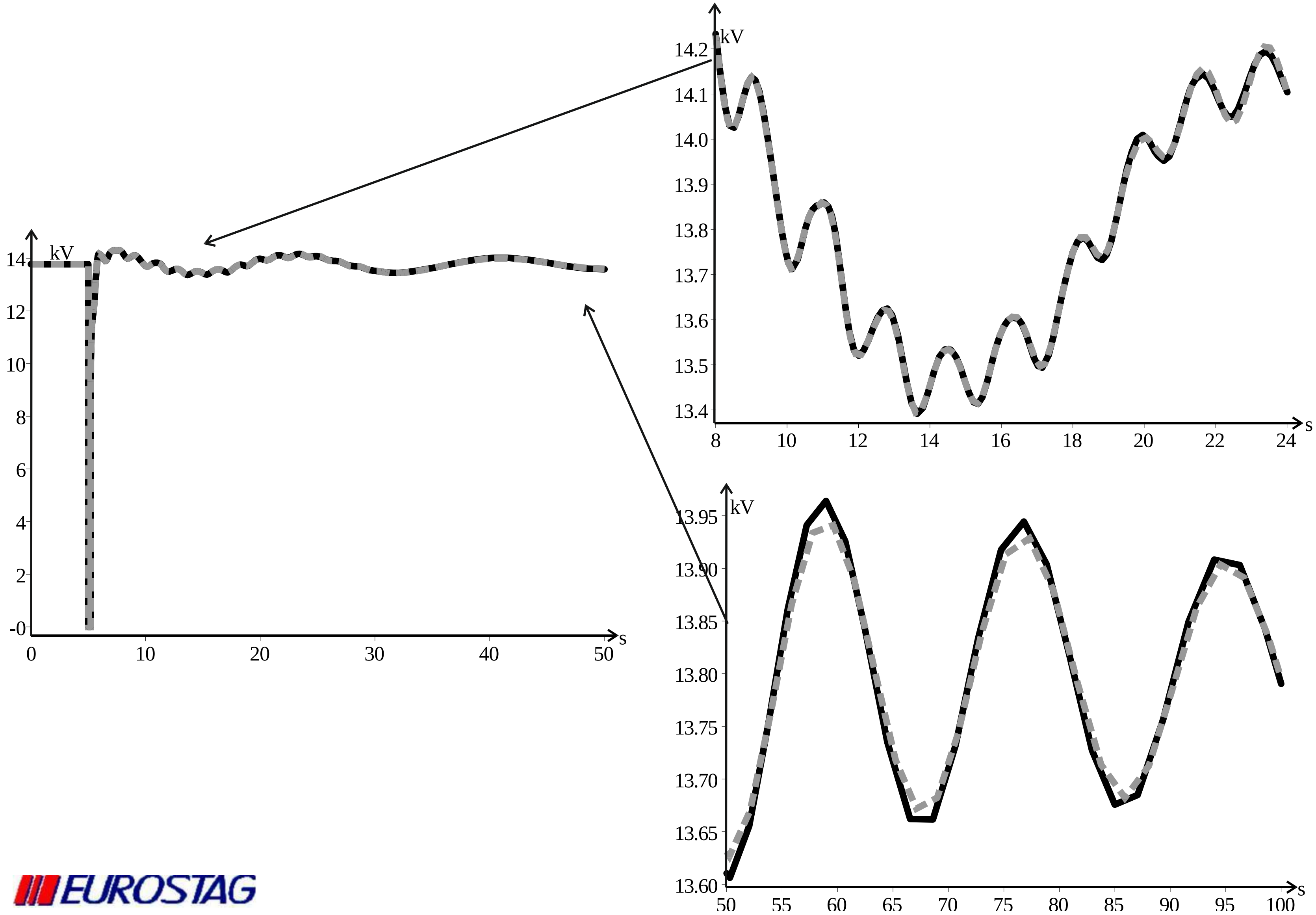


Fig. 3.2. changes of voltage at the node 904: full line-initial scheme, doted line-reduced model of scheme.

Studying the transient processes at different kinds of short-circuits (at the generation node or in the line) the behavior of initial and reduced models of the scheme practically coincide. Obtained maximal relative error for the voltage is in the range from 0,1758% till 0,9597%, what allows to use the reduced model for transient processes study.

Negligible disarrangement of curves of voltage changes may be caused by differences in the integrations errors which are caused by difference of the schemes' topology dimensionality.

Though the high accuracy of obtained results of reduced model scheme at the load flow computation and transient processes study, the application of Jordan's elimination method have significant defect. Its application is limited by idealized case when all loads are represented by constant admittances but not with their real load characteristics that is such approach is too far from the behavior of loads in the real power system.

4. THE POWER SYSTEM NETWORK REDUCTION CONSIDERING THE STATIC AND DYNAMIC LOADS CHARACTERISTICS.

In this chapter - the power systems loads models and influence of loads modeling on the character of transient processes are considered. The original modification of methodology based on the Jordan's method by nodes elimination with its real load characteristics is proposed.

The accuracy of power system's loads representation at its modeling have significant influence on the results of transient processes computation and continue to stay sufficiently difficult and topical task [19-22].

The load combines a bundle of separate consumers with different kind of characters; in turn the different dynamic of loads is caused with dependence from operation time and kind of disturbances which influence the power system.

Two kinds of load characteristics are recognized – *static* and *dynamic*:

Static load characteristics represent the dependence of active P and reactive Q power at the low changeable parameters of operation voltages or frequencies, where each of its values corresponds to new steady state:

$$P = \varphi(f, U), \quad Q = \psi(f, U) \tag{4.1}$$

Dynamic characteristics represent the dependence of active P and reactive Q power at the quick changes of voltages and frequencies in the power systems, that are caused more often by dynamic electromechanically transient processes at which the real behavior of loads is impossible to neglect [7,23,24].

4.1. Elimination of voltage dependent loads represented by its arbitrary load characteristics in power form

At the following analysis the mathematical derivation of all formulas will be completed only on the example of active power P and exponential quantity α, γ, but it should be noted that all formulas given below, include obtained conclusions by analogy they are fair also for the definition of reactive power Q and exponential quantity β, δ [11-16].

There are a number of variants of loads representation at the power systems modeling. For example, power system load can be represented as dependent only from voltage in power form (4.1), which has a wide range of exponential quantities α and β [25-27] in the general case is a parabola.

$$P = P_0 \cdot \left(\frac{U}{U_0} \right)^{\alpha}, \quad Q = Q_0 \cdot \left(\frac{U}{U_0} \right)^{\beta}, \tag{4.1}$$

In the real power system there is a wide range of indicators describing the exponential quantities α and β , equation 4.1, gives the nonlinear characteristic to the circuit. In this case let's switch the function (4.1) with another, close to it by the form, specifying it as a polynomial of second degree, but without free terms [28]:

$$P = a \cdot U^2 + b \cdot U, \tag{4.2}$$

The following estimation of coefficients a and b with receiving the best approach turns into the task of approximation.

For the approximate representation of the given function, let's use the least square method (LSM), in accordance to which the minimized functional will have following view [29]:

$$\Phi = \int_{U_1}^{U_2} \left[a \cdot U^2 + b \cdot U - P_0 \cdot \left(\frac{U}{U_0} \right)^{\alpha} \right]^2 dU \to \min, \tag{4.3}$$

where Φ - sum squared error of approximation; $U_1 \div U_2$ - is the range of expected voltage change from which value depends the accuracy of approximation, that is as less is the range, as more exact will be the approximation (maximal possible range at $U_1 = 0$).

Using the received coefficients a and b from (4.3.) placing them in the (4.2), we will receive the power expression for the arbitrary value of voltage at the accepted exponential quantity α .

Let's compare the loads characteristics received by the approximation of LSM with loads characteristics given in accordance with (4.1.). Graphical comparison at the different quantities α is represented on the fig. 4.1, [11,12,14-16].

As a result of approximation, the initial load characteristic in the polynomial form will get the following view:

$$P = P_0 \cdot \left(\frac{U}{U_0}\right)^{\alpha} \approx a \cdot U^2 + b \cdot U\,, \tag{4.4}$$

$$I_{act} = \left(\frac{P}{U}\right) \approx \left(a \cdot U^2 + b \cdot U\right)/U = a \cdot U + b\,, \tag{4.5}$$

where $I_{act.}$ - the active component of load current, that is the value which have the linear dependence from voltage.

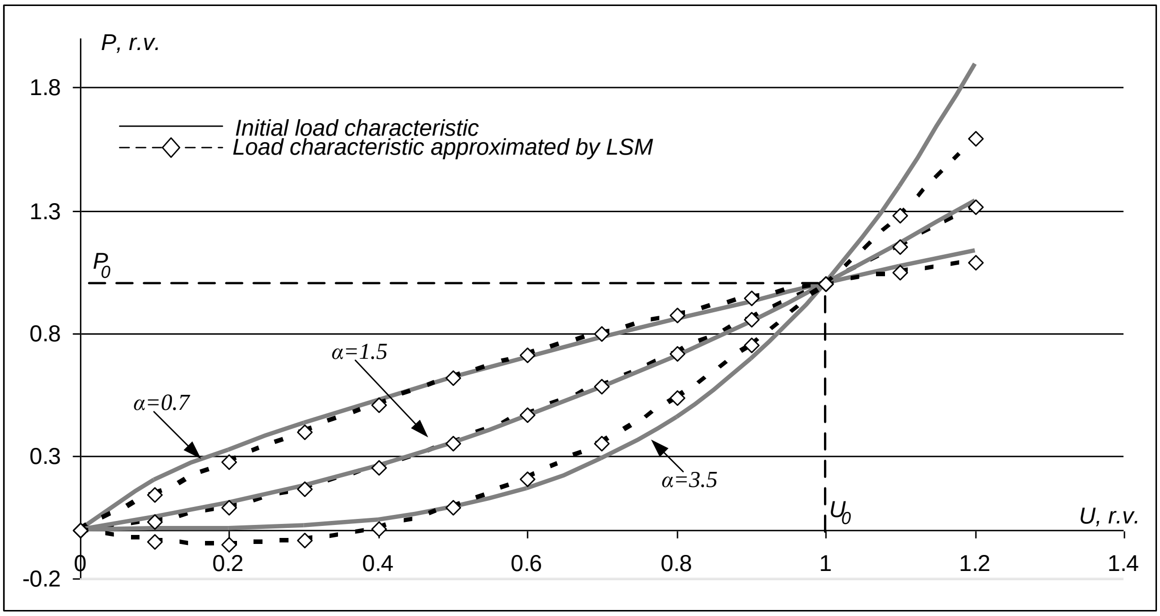


Fig. 4.1. Comparison of initial load characteristic with the approximated by LSM load characteristic at the different quantities α

$I_{act.}$ in (4.5) is represented as two constitutes, where the coefficient a represents the value of current, which flows through equivalent active constant admittance g_0, and coefficient b - constant constituting of drive load current $I_{act.}$, which doesn't depend from voltage, fig. 4.2. At this, it is necessary to remember that g_0 depend from exponential quantity α and numerically differ from earlier found at the $\alpha = 2$ [11,12,14-16].

Thereby, the equation 4.4. can be written in the following form:

$$P = P_0 \cdot \left(\frac{U}{U_0} \right)^{\alpha} \approx g_0 \cdot U^2 + I_{act} \cdot U , \qquad (4.6)$$

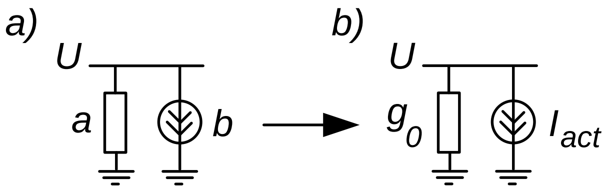


Fig. 4.2. Cirquit of load replacement: a-through coefficients a,b; b-through admittance and drive current

However at such load representation, as a result of nodal admittance matrix increase on the one additional column to the right, which forms at the taking into account of load current I_{act} (further the free term b)the system 2.2 will obtain the new form [11-15]:

$$\boldsymbol{Y \cdot U + I_{act} = 0} \qquad (4.7)$$

Important is the addition of load current value I_{act} (next the free term b), doesn’t introduce any changes in the formulas of Jordan’s elimination rules given at the 2nd chapter. That was the main idea of the exponential load representation change exactly by the polynomial of second degree.

In this way, for example, for the previously circuit of 3 nodes (fig. 2.1.), the nodal admittance matrix will have the following view:

Table 4.1

	U1	**U2**	**U3**	**I_{act}**
J1	Y_{11}	$-Y_{12}$	0	b_1
J2	$-Y_{21}$	Y_{22}	$-Y_{23}$	b_2
J3	0	$-Y_{32}$	Y_{33}	b_3

Y load
$Y_{sl1} + 1/2\omega c_1$
$Y_{sl2} + 1/2(\omega c_1 + \omega c_2)$
$Y_{sl3} + 1/2\omega c_2$

After the second node elimination for the nodes 1 and 3 we will receive:

1-new values of driving current $b_1^{'}$ and $b_3^{'}$

2 – new values of nodal admittances $Y_1^{'}$ and $Y_3^{'}$

Thereby receiving the new values of loads $Y_1^{'} + b_1^{'}$ and $Y_3^{'} + b_3^{'}$ or new parameters $a_1^{'} = Y_1^{'}$, $a_3^{'} = Y_3^{'}$ и $b_1^{'}$, $b_3^{'}$, with the help of which we define the new load characteristics (LC) for the each saved node of the scheme:

$$a^{'} \cdot U_0 + b^{'} \Rightarrow P_0^{'} \cdot \left(\frac{U}{U_0} \right)^{\alpha^{'}} , \qquad (4.8)$$

The point of crossing of two characteristics depends from value of α. Thereby, in regard of load characteristic approximation by LSM, there were defined three points:

1. $U = 0$ which corresponds to maximal possible range, as in the emergency state always $U \geq 0$ and optimal result of approximation may be reach by choosing a maximal narrow range of voltage changes $U_1 > 0$;

2. $U = U_0$ that allows to save the initial operation;

3. $U = 0{,}5 \cdot (U_1 + U_0)$ as both parabolas (initial and approximated) always intersect at this point [11-15].

As a result it is possible to go to the interpolation method and do the interpolated curve already through 3 defined points. Thereby we will receive 2 equations with two unknown g_0 and I_{act} [11-15].

$$P = P_0 \cdot \left(\frac{U}{U_0}\right)^{\alpha} \rightarrow P_0 = g_0 \cdot U_0^2 + I_{act} \cdot U_0, \tag{4.9}$$

where, changing U to $U = \dfrac{U_0}{2}$ we will receive 2 formulas for the defining of unknown parameters $g_0{}'$ and I_{act}' [14,16]:

$$\begin{aligned} g_0' &= \frac{P_0 / U_0^2 \left(1 - (1/2)^{\alpha-1}\right)}{1/2} \\ I_{act}' &= -\frac{P_0}{U_0} \cdot \left(1 - (1/2)^{\alpha-2}\right) \end{aligned}. \tag{4.10}$$

Return to the load representation in power form is realized by means of obtaining values $g_0{}'$ and I_{act}' transformation, received in the (4.10).

From equations 4.10 define the load power:

$$P_0' = g_0{}' \cdot U_0^2 + I_{act}' \cdot U_0. \tag{4.11}$$

Exponential quantity α' [11,12,14-16]:

$$a' = \ln\left(1 - U_0^2 \cdot a' \cdot 0{,}5 / P_0'\right) / \ln(0{,}5) + 1 \tag{4.12}$$

4.2. Elimination of load nodes with frequency dependent characteristics

More complex and accurate is load representation by its frequency dependent characteristics, for example in power form [7,25-27]:

$$P = P_0 \cdot \left(\frac{U}{U_0}\right)^{\alpha} \cdot \left(\frac{f}{f_0}\right)^{\gamma}, \qquad (4.13)$$

Power system frequency during transient processes can change for the maximal range of $46{,}5 \div 53\ Hz$ [30,31].

Thereby at the condition of frequency change, the elimination of load nodes through Jordan's elimination completes considering values of lines admittances and nodes power dependent of its changes:

$$g = \frac{R}{R^2 + X^2} = \frac{R}{R^2 + (\omega \cdot L)^2}, \qquad (4.14)$$

All process of frequency dependent loads elimination represented in power form, conditionally is allowed to be divided in 3 steps, at each of them it is necessary to consider the same steady state, but at the different values of frequency ($f_0 = 50\ Hz$, $f_1 = 48\ Hz$, $f_2 = 52\ Hz$) [13-16]:

1-use of the given methodic in the paragraph 4.1, at the frequency $f_0 = 50\ Hz$. Whereupon completes the elimination of load nodes and defines the values of power $P_0^{'}$ and $Q_0^{'}$, as well as current $I_{act}^{'}$, which are necessary for the defining of exponential quantities $\alpha^{'}$ and $\beta^{'}$ for the remaining nodes.

2-Correction of lines reactive and capacitive admittances completes for the full scheme in the computation table of nodal admittances matrix at the frequency $f_1 = 48\ Hz$ [13-16]:

$$X^{'} = X \cdot \frac{f_1}{f_0} = X \cdot \frac{48}{50},\ Bc^{'} = Bc \cdot \frac{f_1}{f_0} = Bc \cdot \frac{48}{50} \qquad (4.15)$$

Correction of initial powers at the nodes for the initial scheme in accordance with its frequency characteristics [13-16]:

$$P = P_0 \cdot \left(\frac{f}{f_0}\right)^{\gamma}, \qquad (4.16)$$

Let's complete the elimination of same nodes as at the estimation of powers $P_0^{'}$ and $Q_0^{'}$ and define the value of powers $P_{f1}^{'}$ for the remained nodes. To complete this it is necessary to complete all following operations for the defining of values $P_{f1}^{'}$ in accordance (4.3 till 4.11), but only at the values of α and β equal 2. Running to the exponential quantities values $\alpha, \beta = 2$ at the defining of powers at the new value of frequency explains the possibility to avoid the buffering of error, which generates at the definition of the exponential quantities α, β [13-16].

New value of exponential quantity γ'_{f1} is possible to define out of 4.13:

$$\ln\left(P'_{f1}\right) = \ln\left[P'_0 \cdot \left(\frac{U}{U_0}\right)^{\alpha'}\right] + \gamma'_{f1} \cdot \ln\left(\frac{f_1}{f_0}\right), \tag{4.17}$$

$$\gamma'_{f1} = \frac{\ln\left(P'_{f1}\right) - \ln\left(P'_0\right) - \alpha' \cdot \left(\ln U - \ln U_0\right)}{\ln\left(\frac{f_1}{f_0}\right)}, \tag{4.18}$$

3-The computation of the similar to the point 2, but for the frequency $f_2 = 52\ Hz$.

Thereby for tracing the still unknown frequency dependent for each saved node of examined power system model were obtained by three points (at the frequency $f_0 = 50\ Hz$, $f_1 = 48\ Hz$, $f_2 = 52\ Hz$ accordingly), using which it is possible to do the numerical approximation of the given dependence in the range of $f = 46{,}5 \div 53\ Hz$ for each node, what is sufficient for any practical researches [13-16].

Final values for the exponential quantities γ' define as arithmetic mean of its sum defined at the frequency $f_1 = 48\ Hz$ and $f_2 = 52\ Hz$ [13-16]:

$$\gamma' = \frac{\gamma'_{f1} + \gamma'_{f2}}{2}, \tag{4.19}$$

5. POWER SYSTEM NETWORK REDUCTION BY MEANS OF NODES WITH NONLINEAR LOADS ELIMINATION, REPRESENTED BY ITS STATIC AND DYNAMIC CHARACTERISTICS.

In this chapter are presented practical examples on power system network reduction by means of nonlinear nodes elimination, represented as static and/or dynamic load characteristics with the use of the method proposed in the chapter 4. Individual attention is gives to consideration of influence of real loads characteristics on the transient processes behavior character.

An algorithm, represented at the fig. 5.1, describes the process of elimination of load nodes of power system scheme given as static and/or dynamic load characteristics. Only those nodes are liable to the eliminations that were indicated in the special block. Nodes' elimination is performed by using Jordan's elimination method in a complex form. This is a step by step method - elimination of the one node at each step. To ease the process, all activities are performed in relative units.

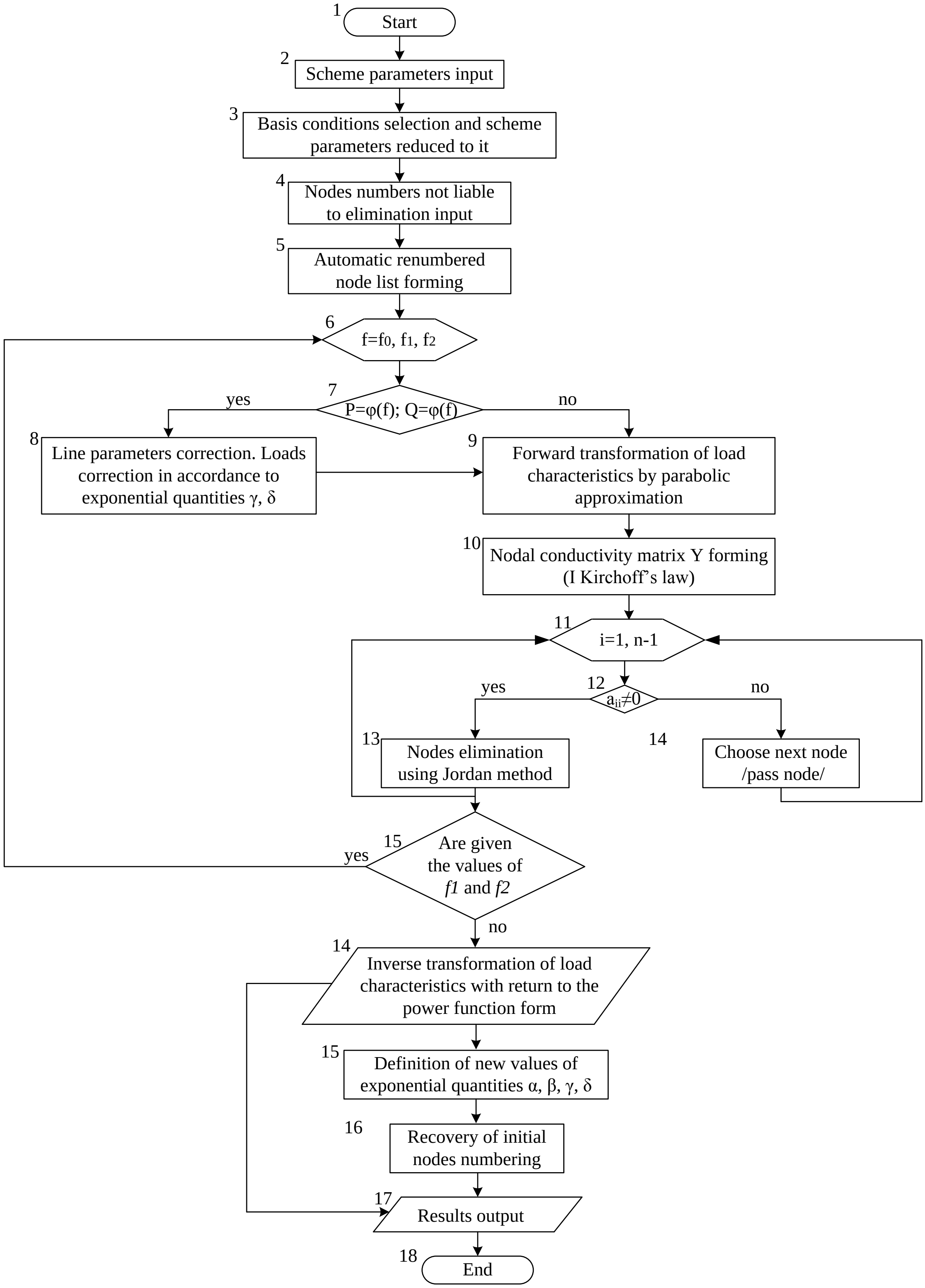


Fig. 5.1. Algorithm block scheme

The functionality of the algorithm of the first five blocks is presented at the fig. 3.1 and 5.1 and fully coincides. The detailed description of those blocks is given in the chapter 3.

Cycle block 6 contains the value of nominal frequency $f_0 = 50\,Hz$ and new values of frequency f_1 and f_2. On the values of f_1 and f_2 together with given constant of f_0, entrust role of interpolated nodes. They must be held away from each other so that the interpolation by frequency in the best rate envelopes the expected rate of frequency changes that depends from specific type of delivered tasks [13-16].

In the conditional block 7 verification of availability of frequency-dependent load in the block number 2 is completed. If in the second block values of exponential quantities γ, δ, are given, then blocks from 7 till 15 will be completed 3 times and at this only in two last completions the program will come to the block nr.8, in other case to the block nr. 9.

In block 8 the correction of lines parameters (X, Bc) at the new values of frequency f_1 and f_2 is completed, in accordance to the formula 4.15, where values of f_1 and f_2 are in the range of 46.5 till 53 Hz, such range of frequency is sufficient for examination of any tasks of transient stability [13-16]. The corrections of loads $P = (U/U_0)^{\gamma}$ and $Q = (U/U_0)^{\delta}$ according to exponential quantities γ, δ are also performed in this block.

The 9th block performs the forward transformation of load characteristics by parabolic approximation with maintenance of all necessary rules given in the chapter 4 with the following definition of unknown g_0 and I_{act} in accordance to (4.10), [11-16].

The functionality of block 9 by analogy is also valid for the definition of reactive power Q.

In the block 10 is formed the nodal conductivity matrix Y in accordance to the I Kirchoff's law with one auxiliary column addition to the nodal conductivity matrix, which contains the value of drive load current $YU+I_{act}=0$. [11-16].

The functionality of blocks 11, 12 and 14 fully corresponds to the functionality of blocks 7, 8 and 10 described in the chapter 3.

On each step of the elimination, in the block 13 is completed availability check of node number liable to the elimination in the block 4, in the nodes' list not liable to the elimination. If this node is missing in the list, then its elimination by the Jordan's method occurs, used over all elements of extended nodal admittance matrix with one additional column of driving load current in accordance to Jordan's elimination rules [11-16]. If the number of node liable to the elimination is in the block number 4, then this node will be saved and program will transfer to the next node. After the checking of all nodes of network with the following elimination of its part, block forms new extended nodal admittance matrix corresponding to the obtained reduced scheme model which consist only of nodes given in the block 4 with updated column of driving load currents I_{act} ,[11-16].

In the block 15 the check frequency is completed.

The 16th block completes inverse transformation for the new obtained scheme model of received new values of admittance g_0' and driving current I_{act}' received at (4.10), in accordance with (4.11).

New values of exponential quantities $\alpha', \beta', \gamma', \delta'$ are calculated in the 17th block, using formulas (4.12, 4.18), but resultant value defines by (4.19) [13-16].

The functionality of blocks 18, 19 and 20 fully corresponds to the functionality of blocks 11, 12 and 13 described in the chapter 3.

Case studies, presented in this chapter, were completed on the scheme of electrical power grid “БРЭЛЛ”, that is a content of power system IPS/UPS (see chapter 3).

Research of the different kinds of transient processes and detailed analysis of obtained results for the initial scheme and its reduced model, authenticates that the new proposed methodic of load nodes elimination represented by its characteristics dependent only from voltage, allows to receive reduced model of power system which has lesser dimensionality but at this time it saves and represents the behavior of initial power system scheme at the different kinds of short-circuits with the minimal relative error 0,2% - 0,6%.

Comparative analysis of transient processes at the different kinds of active power deficiency on the initial scheme and its reduced model, which was obtained by means of elimination load nodes dependent from voltage and frequency and represented with independent load characteristics in power form, showed that the maximal relative error doesn’t exceed 0,01% of the frequency and 1,31% of the voltage. An example of comparison of results transient process curves for the initial scheme and its reduced model at arise of iterative active power deficiency is presented at the fig. 5.2.

With the help of the proposed methodic also is possible to define required static and dynamic characteristics for the aggregated load using the given LC on the lower voltage in the distribution network with the maximal relative error which doesn’t exceed 0,1% of the frequency and 0,7% of the voltage.

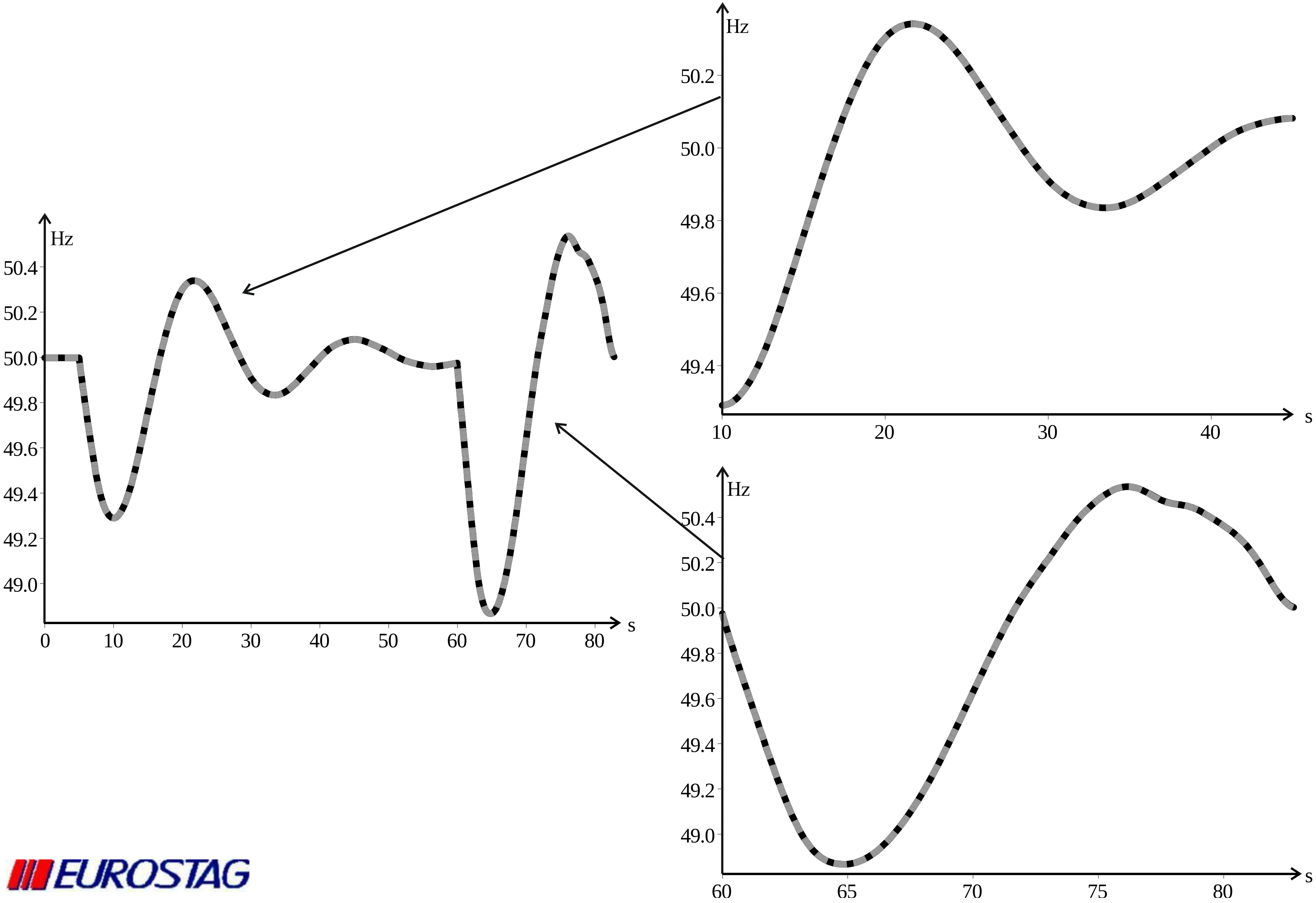


Fig. 5.2. Frequency changes in the examined power system: solid line-initial scheme, doted line-reduced scheme

Comparison of behavior of initial scheme and its reduced model in the case of using emergency automation devices underfrequency load shedding (UFLS) showed that it fully coincidences at the deficiency of active power. Maximal relative error doesn't exceed 0,001% of the frequency and 0,33% of the voltage.

The comparison of transient process behavior on the two models of one power system that differ from each other by the loads representation was completed:

1- all loads of power system are given as constant impedance (see chapter 4.3.);

2-all loads of power system are represented in the power form where exponential quantities $\alpha, \gamma, \beta, \delta$ are given in the admissible range given in the [25-27].

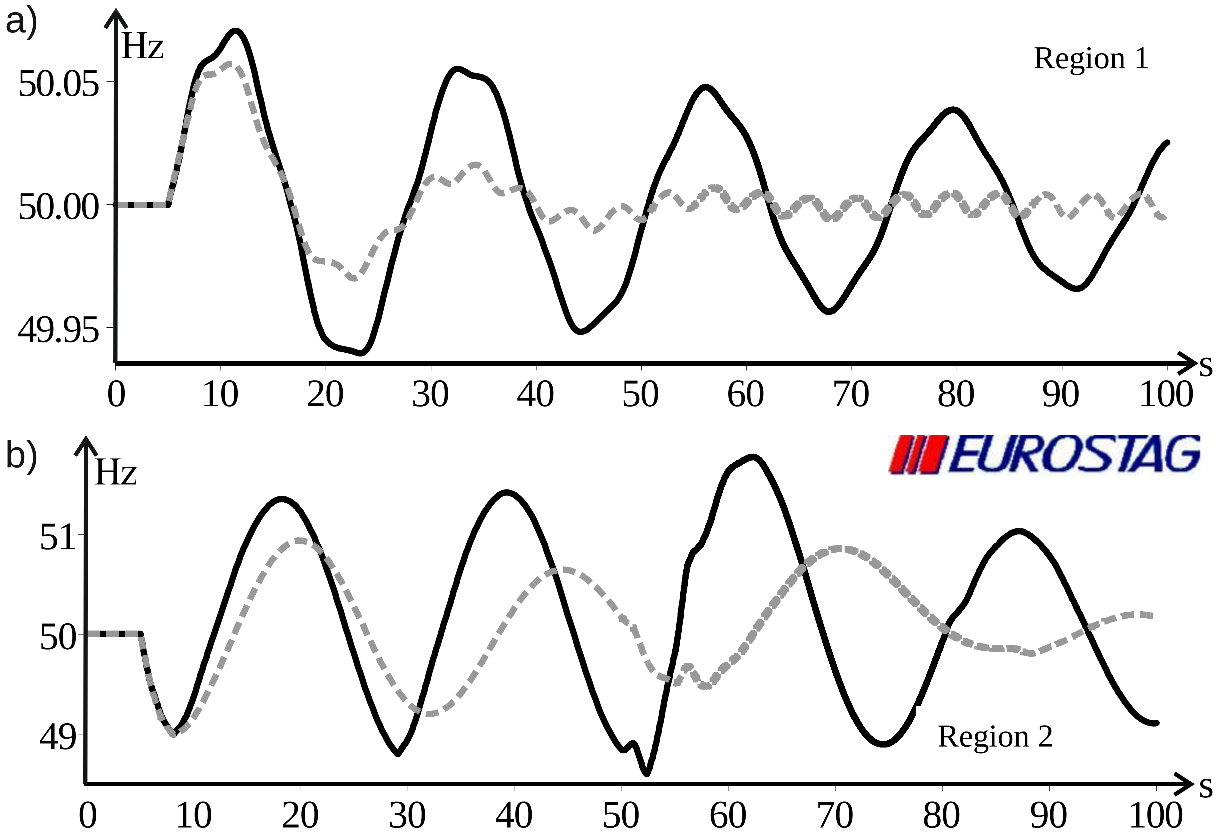
From the obtained results follows that the character of transient process behavior for 2 models of power system has constitutive differences due to the different representation of load characteristics.

While creating the reduced model of power system the chosen way of load representation can have constitutive impact on the emergency automation device's operation.

For example, during the transient process (power system's division with the following short-circuit) the emergency automation UFLS has performed. At this, depending on load representation (as constant impedance or in the power form), differs as the mount of operated thresholds of UFLS as well as thresholds operation time, table 5.1, fig. 5.3.

Table 5.1

Area	Threshold number	Operation frequency, Hz	Disconnected load, %	Operation time of UFLS thresholds when the loads are represented as constant impedance	Operation time of UFLS thresholds when the loads are represented in power form
Region 1	1	49,5	5%	6,0416	6,0350
Region 1	2	49,2	5%	6,9636	6,8754
Region 1	3	49,0	5%	8,0411	8,0714
Region 1	4	48,8	10%	29,0561	-
Region 1	5	48,6	10%	52,3391	-



Fg. 5.3. a-frequency changes in the region 1; b- frequency changes in the region2, where the black line-loads of power system are given as constant impedance, dotted line- loads of power system are given as random characteristics, dependent from frequency and voltage.

COCLUSIONS AND RECOMMENDATIONS FOR THE FUTHER WORK

1. Topical problem of interconnection of power systems is associate with whole row of problems at the input of its detailed model in the computer programs for the preliminary calculation of operations as well as analysis of transient stability, as a result arises the necessity to build an equivalent models of power systems, which are able to correctly present a steady state as well as dynamic characteristic of initial model.

2. In this work the new original methodic improving the main idea of power system reduction by the Jordan's elimination is proposed. It is based on the modification of Jordan's method and allows eliminating of all/part of load nodes of power system taking into account its static and/or dynamic load characteristics, that is, the methodic gives a possibility to eliminate load nodes with arbitrary nonlinear load characteristics, dependent from voltage and/or frequency.

3. Transformation and elimination of nodes out of scheme by means of proposed method doesn't lead to the disturbance of node balance, that is, the voltages, angles and currents in the unaffected nodes and loads as well as voltage, angle and current in the balancing node remains unchangeable.

4. Jordan's elimination method application for the analysis of power system allows operate not obligatory with full scheme, but only with its part, which consists of eliminated nodes as well as adjacent nodes.

5. Power system reduction by the Jordan's method or proposed original methodic based on this method, allows receiving reduced scheme for the whole row of different operations study with the fully correspondence to the initial scheme at the condition that scheme topology and loads values at the all nodes remains unchanged.

него напряжения.

6. With the help of new methodic it is possible also to obtain the value of aggregated loads on the high voltage side and its characteristics, dependent from voltage and frequency taking into account load characteristics on the low and medium voltage side.

7. Proposed reduction methodic requires the completing only of linear operations, which considerably simplify the programming of the methodic.

8. Completed practical experiments at the different transient processes and analysis of obtained results showed that maximal relative error of voltage, voltage angle and frequency for the received reduced scheme doesn't exceed 1%.

9. An algorithm proposed in this work can be combined with any power engineering computer program or complement with converter which allows obtain results in the one of formats compatible with necessary power engineering computer program, what allows significantly force the computational process.

10. In the given work was examined elimination of load nodes given with different load characteristics. In the future with the help of proposed methodic it is necessary to increase research and consider cases of nodes elimination which consist of capacitor batteries that exceeds by power inductive load.

11. For the future development, proposed methodic will be in need of additional theoretical well-founded background algorithm, which allows analyzing short circuits in the eliminated node and/or eliminated line.

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