An Approach of Semi-numerical Computing Volume of Solids Bounded by Rational Bézier Surfaces

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***Abstract* – The problem of free-form object modelling in CAGD is an actual task in computer graphics and solving it is important for practical use. This paper describes a new approach of semi-numerical computing volume of solids bounded by rational Bézier surface set. There is an approach for volume calculation of solids bounded by simple Bézier surface set. In this paper, the proposed approach is based on the combination of analytical integration for volume integral computation and numerical integration for rational function integral computation.**

***Keywords* – Bézier, integral, rational, volume.**

I. Introduction

The problem of free-form 3D object modelling is an actual task for various sciences and technical areas. Its solution is important for practical application in many industries, such as biomedical engineering and technical design. One of the most efficient 3D modelling methods is the use of rational Bézier curves and surfaces. They play an important role in Computer Aided Geometric Design (CAGD); therefore, their specification, modification and different properties are of interest. Specification and modification of Bézier objects are well-explored fields of CAGD but there are some special problems to be solved, such as finding the optimal way of computing the volume of solids bounded by rational Bézier surfaces. The computation of the volume of solids bounded by simple Bézier surfaces in [2], [5] is proposed in this paper. For volume computation for B-spline case the transformation method [9] can be used.

Bounded volumes plays an important role in rendering tasks ([11], [12] and [15]) and for geometrical analyses [13].The use of bounding volumes is discussed in [10] and [14].

A variational approach to computing an optimal segmentation of a 3D shape for computing a union of tight bounding volumes in [3] was described. Based on an affine invariant measure of e-tightness, the resemblance to ellipsoid, a novel functional is formulated that governs an optimization process to obtain a partition with multiple components. Some ideas of this method are described in [16] and [17].

Refinement of segmentation is driven by application-specific error measures, so that the final bounding volume meets pre-specified user requirement.

An optimized closed formula for the volume of the solid, bounded by a simple Bézier surface and the cones determined by its boundary curves and the origin in [2] was proposed.

Basic definitions and properties of parametric curves, surfaces and volumes on Bézier and B-spline form are presented in [8].

Some properties and constructions of rational Bézier volume are presented and two applications of free-form volumes are discussed in [7].

In this paper, the author proposes an exact semi-numerical approach that can substitute approximating numerical procedures.

The input data for the volume computation algorithm is an array of rational Bézier patches (mesh). The patches are aligned in such a way that the O*z* axis passes the modelled object. Thus, the volume computation formula is described as a sum of curvilinear prism volumes:

The proposed approach was implemented for its verification on employment opportunities to the modelled object example. Free-form objects (ball and cylinder) were taken for simulating this approach. Before the tests, the volumes of the selected objects were known.

Experimental results show that the proposed approach provides correct representation of the results.

II. The Proposed Approach

In this paper we consider rational Bézier surfaces of the following definition [4]:

$$S(u,v)=\frac{\sum_{i=0}^{n}\sum_{j=0}^{m}P_{i,j}\cdot w_{i,j}\cdot B_{i,n}(u)\cdot B_{j,m}(v)}{\sum_{i=0}^{n}\sum_{j=0}^{m}w_{i,j}\cdot B_{i,n}(u)\cdot B_{j,m}(v)}, \quad (1)$$

where: $S(u,v)$ – the rational Bézier surface;
$P_{i,j}$ – the array of control points;
$w_{i,j}$ – the array of weight coefficients;
u,v – parameters, $u\in[0,0;1,0]\,\&\,v\in[0,0;1,0]$;
n,m – the degrees of polynomials;
$B_{i,n}(u), B_{j,m}(v)$ – Bernstein polynomials.

The *i-th* Bernstein polynomial of degree *n* can be calculated as follows:

$$B_{i,n}(u)=\binom{n}{i}\cdot u^{i}\cdot(1-u)^{n-i}, \quad (2)$$

where $\binom{n}{i}$ is a binomial coefficient and can be calculated as follows:

$$\binom{n}{i} = \frac{n!}{i!(n-i)!}. \quad (3)$$

One of the most actual problems of the geometric analysis of objects is the volume calculation. In this part of paper the proposed method is described. This approach is based on the object volume calculation using integral computation.

The input data of this approach is a 3D object formed by a set of Bezier surfaces. Let us suppose that the volume of the object is the sum of curved prism volumes:

$$V_{obj} = \sum V_{prism}, \quad (4)$$

where: V_{obj} – the volume of object;

V_{prizma} – the volume of curved prism.

The example of curvilinear prism is shown in Fig. 1.

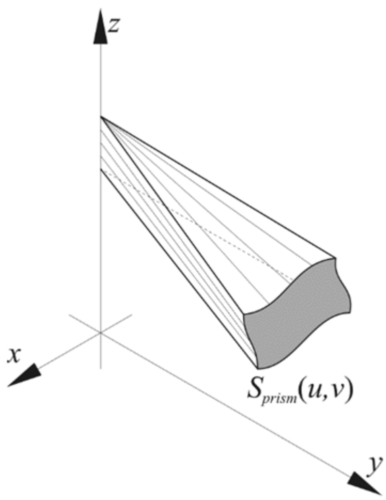

Fig.1. The example of curvilinear prism.

a. Volume Integral

Let us find a curvilinear prism differential volume dV_{prism}. Figure 2 illustrates this problem.

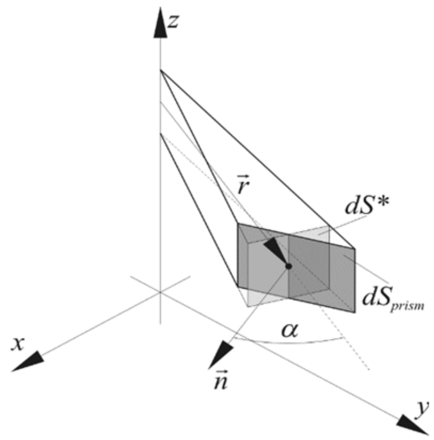

Fig. 2. Calculation of volume differential.

In this case dS_{prism} is a differential area in the arbitrary point of surface $S_{prism}(u,v)$, and $dS*$ is the projection of differential area to the direction of vector $\vec{r}$.

It is obvious that in the case when the vector $\vec{r}$ is perpendicular to $dS*$ the differential volume can be calculated using the following formula:

$$dV_{prism} = \frac{1}{2} \cdot |\vec{r}| \cdot dS*, \quad (5)$$

where: $\vec{r}$ – the vector from Oz to a surface point. The vector $\vec{r}$ is perpendicular to Oz.

Given that infinitely small variables are used in computing, it can be assumed that the following expression is valid:

$$dS* = \cos(\alpha) \cdot dS_{prism}, \quad (6)$$

where: α – the angle between vector $\vec{r}$ and normal $\vec{n}$ to dS_{prism}

From (5) and (6) differential equation may be found

$$dV_{prism} = \frac{1}{2} \cdot |\vec{r}| \cdot \cos(\alpha) \cdot dS_{prism}. \quad (7)$$

To solve these differential equations, it is necessary to integrate both parts of (7) with a surface integral of scalar fields. As a result, we obtain

$$V_{prism} = \frac{1}{2} \iint_{(S_{prism})} |\vec{r}| \cdot \cos(\alpha) dS_{prism}. \quad (8)$$

For the task of solving the surface integral of scalar fields (8), if the surface is described in parametric form, we can use the property [1]

$$\iint_{(S)} f(x,y,z) dS = \int_{v\_min}^{v\_max} \int_{u\_min}^{u\_max} f(x(u,v), y(u,v), z(u,v)) \cdot \sqrt{A^2 + B^2 + C^2}\, du dv, \quad (9)$$

where:

$$A = \begin{vmatrix} \frac{\partial S_Y(u,v)}{\partial u} & \frac{\partial S_Z(u,v)}{\partial u} \\ \frac{\partial S_Y(u,v)}{\partial v} & \frac{\partial S_Z(u,v)}{\partial v} \end{vmatrix}, \quad (10)$$

$$B = \begin{vmatrix} \dfrac{\partial S_Z(u,v)}{\partial u} & \dfrac{\partial S_X(u,v)}{\partial u} \\ \dfrac{\partial S_Z(u,v)}{\partial v} & \dfrac{\partial S_X(u,v)}{\partial v} \end{vmatrix} \tag{11}$$

and

$$C = \begin{vmatrix} \dfrac{\partial S_X(u,v)}{\partial u} & \dfrac{\partial S_Y(u,v)}{\partial u} \\ \dfrac{\partial S_X(u,v)}{\partial v} & \dfrac{\partial S_Y(u,v)}{\partial v} \end{vmatrix}. \tag{12}$$

In equations (10)-(12) the function $S_X(u,v)$, $S_Y(u,v)$ and $S_Z(u,v)$ is x, y and z coordinates of rational Bezier surface (1).

The normal to surface point with parameters (u,v) can be calculated as follows:

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dfrac{\partial S_X(u,v)}{\partial u} & \dfrac{\partial S_Y(u,v)}{\partial u} & \dfrac{\partial S_Z(u,v)}{\partial u} \\ \dfrac{\partial S_X(u,v)}{\partial v} & \dfrac{\partial S_Y(u,v)}{\partial v} & \dfrac{\partial S_Z(u,v)}{\partial v} \end{vmatrix}. \tag{13}$$

Taking into account A, B and C values and the normal's calculation equation (13) it can be concluded that the normal may also be described as follows:

$$\vec{n} = (A, B, C)\cdot \tag{14}$$

In this case the length of the vector $\vec{n}$ can be calculated as follows:

$$|\vec{n}| = \sqrt{A^2 + B^2 + C^2}\ . \tag{15}$$

Now, take into account that in the rational Bezier surface case the parameters u and v vary in the range [0, 1] and by inserting equation (15) into integral (8) we have

$$V_{prism} = \frac{1}{2}\cdot\int_0^1\int_0^1 |\vec{r}|\cdot\cos(\alpha)\cdot \cdot\sqrt{A^2 + B^2 + C^2}\,dudv\ . \tag{16}$$

Value $\cos(\alpha)$ can be calculated in the following form:

$$\cos\alpha = \frac{\vec{r}\cdot\vec{n}}{|\vec{r}|\cdot|\vec{n}|} = \frac{r_X\cdot n_X + r_Y\cdot n_Y + r_Z\cdot n_Z}{|\vec{r}|\cdot|\vec{n}|}\ . \tag{17}$$

After inserting equation (17) into integral (16) we have:

$$V_{prism} = \frac{1}{2}\cdot\int_0^1\int_0^1 |\vec{r}|\cdot\sqrt{A^2 + B^2 + C^2}\cdot \cdot\frac{r_X\cdot n_X + r_Y\cdot n_Y + r_Z\cdot n_Z}{|\vec{r}|\cdot|\vec{n}|}\,dudv\ . \tag{18}$$

After transformation and using equation (14) the volume integral can be described as follows:

$$V_{prism} = \frac{1}{2}\int_0^1\int_0^1 (r_X\cdot n_X + + r_Y\cdot n_Y + r_Z\cdot n_Z)\,dudv\ . \tag{19}$$

The vector $\vec{r}$ definition can be described in the following way:

$$r = (S_X(u,v), S_Y(u,v), 0). \tag{20}$$

Using equations (14) and (20) curvilinear prism volume integrals, the final computation formula can be described as follows:

$$V_{prism} = \frac{1}{2}\cdot\int_0^1\int_0^1 (A\cdot S_X(u,v) + + B\cdot S_Y(u,v))\,dudv\ . \tag{21}$$

Rewriting equation (1) as follows:

$$S(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{m} P_{i,j}\cdot w_{i,j}\cdot R_{i,j}(u,v)\ , \tag{22}$$

where:

$$R_{i,j}(u,v) = \frac{B_{i,n}(u)\cdot B_{j,m}(v)}{w(u,v)}\ , \tag{23}$$

where:

$$w(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{m} w_{i,j}\cdot B_{i,n}(u)\cdot B_{j,m}(v)\ . \tag{24}$$

In this case the function from integral (21) can be calculated

$$f_1 = A\cdot S_X(u,v) = = S_X(u,v)\cdot\frac{\partial S_Y(u,v)}{\partial u}\cdot\frac{\partial S_Z(u,v)}{\partial v} - - S_X(u,v)\cdot\frac{\partial S_Z(u,v)}{\partial u}\cdot\frac{\partial S_Y(u,v)}{\partial v} \tag{25}$$

and

$$f_2 = B \cdot S_Y(u,v) = \\ = S_Y(u,v) \cdot \frac{\partial S_Z(u,v)}{\partial u} \cdot \frac{\partial S_X(u,v)}{\partial v} - \\ - S_Y(u,v) \cdot \frac{\partial S_X(u,v)}{\partial u} \cdot \frac{\partial S_Z(u,v)}{\partial v} \quad . \tag{26}$$

After inserting equation (22) functions (25) and (26) can be described as follows:

$$f_1 = \left(\sum_{i1=0}^{n} \sum_{j1=0}^{m} X_{i1,j1} \cdot w_{i1,j1} \cdot R_{i1,j1}(u,v) \right) \cdot \\ \cdot \left(\sum_{i2=0}^{n} \sum_{j2=0}^{m} Y_{i2,j2} \cdot w_{i2,j2} \cdot \frac{\partial R_{i2,j2}(u,v)}{\partial u} \right) \cdot \\ \cdot \left(\sum_{i3=0}^{n} \sum_{j3=0}^{m} Z_{i3,j3} \cdot w_{i3,j3} \cdot \frac{\partial R_{i3,j3}(u,v)}{\partial v} \right) - \\ - \left(\sum_{i1=0}^{n} \sum_{j1=0}^{m} X_{i1,j1} \cdot w_{i1,j1} \cdot R_{i1,j1}(u,v) \right) \cdot \\ \cdot \left(\sum_{i2=0}^{n} \sum_{j2=0}^{m} Z_{i2,j2} \cdot w_{i2,j2} \cdot \frac{\partial R_{i2,j2}(u,v)}{\partial u} \right) \cdot \\ \cdot \left(\sum_{i3=0}^{n} \sum_{j3=0}^{m} Y_{i3,j3} \cdot w_{i3,j3} \cdot \frac{\partial R_{i3,j3}(u,v)}{\partial v} \right) \tag{27}$$

and

$$f_2 = \left(\sum_{i1=0}^{n} \sum_{j1=0}^{m} Y_{i1,j1} \cdot w_{i1,j1} \cdot R_{i1,j1}(u,v) \right) \cdot \\ \cdot \left(\sum_{i2=0}^{n} \sum_{j2=0}^{m} Z_{i2,j2} \cdot w_{i2,j2} \cdot \frac{\partial R_{i2,j2}(u,v)}{\partial u} \right) \cdot \\ \cdot \left(\sum_{i3=0}^{n} \sum_{j3=0}^{m} X_{i3,j3} \cdot w_{i3,j3} \cdot \frac{\partial R_{i3,j3}(u,v)}{\partial v} \right) - \\ - \left(\sum_{i1=0}^{n} \sum_{j1=0}^{m} Y_{i1,j1} \cdot w_{i1,j1} \cdot R_{i1,j1}(u,v) \right) \cdot \\ \cdot \left(\sum_{i2=0}^{n} \sum_{j2=0}^{m} X_{i2,j2} \cdot w_{i2,j2} \cdot \frac{\partial R_{i2,j2}(u,v)}{\partial u} \right) \cdot \\ \cdot \left(\sum_{i3=0}^{n} \sum_{j3=0}^{m} Z_{i3,j3} \cdot w_{i3,j3} \cdot \frac{\partial R_{i3,j3}(u,v)}{\partial v} \right) \quad , \tag{28}$$

where: $X_{i,j}$; $Y_{i,j}$; $Z_{i,j}$ – coordinates of point $P_{i,j}$.

After simplification, these functions can be described as follows:

$$f_1 = \sum_{i1,i2,i3=0}^{n} \sum_{j1,j2,j3=0}^{m} \{ (w_{i1,j1} \cdot w_{i2,j2} \cdot w_{i3,j3}) \cdot \\ \cdot \left(R_{i1,j1}(u,v) \cdot \frac{\partial R_{i2,j2}(u,v)}{\partial u} \cdot \frac{\partial R_{i3,j3}(u,v)}{\partial v} \right) \cdot \\ \cdot (X_{i1,j1} \cdot Y_{i2,j2} \cdot Z_{i3,j3} - X_{i1,j1} \cdot Z_{i2,j2} \cdot Y_{i3,j3}) \} \tag{29}$$

and

$$f_2 = \sum_{i1,i2,i3=0}^{n} \sum_{j1,j2,j3=0}^{m} \{ (w_{i1,j1} \cdot w_{i2,j2} \cdot w_{i3,j3}) \cdot \\ \cdot \left(R_{i1,j1}(u,v) \cdot \frac{\partial R_{i2,j2}(u,v)}{\partial u} \cdot \frac{\partial R_{i3,j3}(u,v)}{\partial v} \right) \cdot \\ \cdot (Y_{i1,j1} \cdot Z_{i2,j2} \cdot X_{i3,j3} - Y_{i1,j1} \cdot X_{i2,j2} \cdot Z_{i3,j3}) \} . \tag{30}$$

In this case the function from the volume integral can be described as follows:

$$f_1 + f_2 = \sum_{i1,i2,i3=0}^{n} \sum_{j1,j2,j3=0}^{m} \{ I(u,v) \cdot M \cdot \\ \cdot (w_{i1,j1} \cdot w_{i2,j2} \cdot w_{i3,j3}) \} , \tag{31}$$

where:

$$I(u,v) = \\ R_{i1,j1}(u,v) \cdot \frac{\partial R_{i2,j2}(u,v)}{\partial u} \cdot \frac{\partial R_{i3,j3}(u,v)}{\partial v} \tag{32}$$

and

$$M = \det \begin{vmatrix} X_{i1,j1} & X_{i2,j2} & X_{i3,j3} \\ Y_{i1,j1} & Y_{i2,j2} & Y_{i3,j3} \\ 0 & Z_{i2,j2} & Z_{i3,j3} \end{vmatrix} . \tag{33}$$

Equation (21) in this case can be described as follows:

$$V_{prism} = \frac{1}{2} \int_0^1 \int_0^1 \sum_{i1,i2,i3=0}^{n} \sum_{j1,j2,j3=0}^{m} \{ I(u,v) \cdot \\ \cdot M \cdot (w_{i1,j1} \cdot w_{i2,j2} \cdot w_{i3,j3}) \} dudv. \tag{34}$$

After transformation from integral of sum to sum of integrals, equation (32) can be described as follows:

$$V_{prism}=\frac{1}{2}\cdot\sum_{i1,i2,i3=0}^{n}\sum_{j1,j2,j3=0}^{m}\left\{M\cdot\left(\int_0^1\int_0^1 I(u,v)dudv\right)\cdot\left(w_{i1,j1}\cdot w_{i2,j2}\cdot w_{i3,j3}\right)\right\}. \quad (35)$$

As seen from equation (35) the volume calculation task is transformed to double integral. The value of this integral is independent on control point coordinate.

b. Numerical Integration

Function $R_{i,j}(u,v)$ in equation (23) is given. Partial averages can be calculated as follows:

$$\frac{\partial R_{i,j}(u,v)}{\partial u}=\frac{B_{j,m}(v)}{[w(u,v)]^2}\cdot\left\{\frac{dB_{i,n}(u)}{du}\cdot w(u,v)-B_{i,n}(u)\cdot\frac{dw(u,v)}{du}\right\} \quad (36)$$

and

$$\frac{\partial R_{i,j}(u,v)}{\partial v}=\frac{B_{i,n}(u)}{[w(u,v)]^2}\cdot\left\{\frac{dB_{j,m}(v)}{dv}\cdot w(u,v)-B_{j,m}(v)\cdot\frac{dw(u,v)}{dv}\right\}. \quad (37)$$

The function $I(u,v)$ can be transformed as follows:

$$I(u,v)=\frac{B_{i1,n}(u)\cdot B_{i3,n}(u)\cdot B_{j1,m}(v)\cdot B_{j2,m}(v)}{[w(u,v)]^5}\cdot\left\{\frac{dB_{i2,n}(u)}{du}\cdot w(u,v)-B_{i2,n}(u)\cdot\frac{dw(u,v)}{du}\right\}\cdot\left\{\frac{dB_{j3,m}(v)}{dv}\cdot w(u,v)-B_{j3,m}(v)\cdot\frac{dw(u,v)}{dv}\right\}. \quad (38)$$

To calculate this integral in a program, numerical integration is used. In this paper trapezoidal quadric equation [1] is used

$$\int_0^1\int_0^1 I(u,v)dudv\approx I'=\frac{1}{p\cdot q}\cdot\left\{\frac{1}{4}\cdot\left(I(u_0,v_0)+I(u_{p-1},v_0)+I(u_0,v_{q-1})+I(u_{p-1},v_{q-1})\right)+\frac{1}{2}\cdot\left(\sum_{k=1}^{p-2}\left(I(u_k,v_0)+I(u_k,v_{q-1})\right)+\sum_{l=1}^{q-2}\left(I(u_0,v_l)+I(u_{p-1},v_l)\right)\right)+\sum_{k=1}^{p-2}\sum_{l=1}^{q-2}I(u_k,v_l)\right\}, \quad (39)$$

where p,q –the number of steps in parametric directions u and v.

Values of parameters can be calculated as follows:

$$u_k=\frac{k}{p-1} \quad (40)$$

and

$$v_l=\frac{l}{q-1}. \quad (41)$$

The final equation for curvilinear prism volume calculation can be described as follows:

$$V_{prism}=\frac{1}{2}\cdot\sum_{i1,i2,i3=0}^{n}\sum_{j1,j2,j3=0}^{m}\left\{M\cdot I'\cdot\left(w_{i1,j1}\cdot w_{i2,j2}\cdot w_{i3,j3}\right)\right\}. \quad (42)$$

III. Experimental Results

In this paper, the proposed method has been implemented. For its validation two objects have been chosen: cylinder and ball. These objects have been chosen for the following reasons:

1) Both objects can be accurately modelled by rational Bézier surfaces;

2) Their volumes can be precisely calculated before starting the experiment.

The size of objects is follows:

- The radius of cylinder is 10 and height is 20. Therefore its volume is $2000 \cdot \pi \approx 6283, 1853\ldots$
- The radius of ball is 10. Therefore its volume is $(4000/3) \cdot \pi \approx 4188, 7902\ldots$

For object modelling bi-quadric rational Bézier patches are used.

Table I shows results of object volume computation obtained by the proposed method.

TABLE I

EXPERIMENTAL CALCULATION RESULTS

Approach	Cylinder	Ball
Object's volume	6283,1853	4188,7902
Numerical integration (12×12)	5877,8525	3822,6742
Numerical integration (40×40)	6257,3786	4145,9062
Numerical integration (100×100)	6279,0520	41691,1072
	Error, %	
Numerical integration (12×12)	6.451	8.740
Numerical integration (40×40)	0.411	1.024
Numerical integration (100×100)	$6.578 \cdot 10^{-2}$	$7.195 \cdot 10^{-2}$

IV. CONCLUSIONS

The main conclusions of this paper:

- In this paper a new approach of free-form object volume calculation has been proposed, which is based on analytical and numerical integration combination;
- The experimental results show the efficiency and precision of the proposed approach.

REFERENCES

[1] G. Farin, Curves and Surfaces for Computer Aided Geometric Design. Morgan Kaufmann, 2001.

[2] I. Juhász, "Computing Volume of Solids Bounded by Bézier Surfaces" Mathematical Notes, Miskolc, Vol. 1., No. 2., pp. 127-133., 2000.

[3] L. Lu, Y.-K. Choi, W. Wang and M. S. Kim, "Variational 3D Shape Segmentation for Bounding Volume Computation" in Computer Graphics Forum, vol.26, no. 3; pp. 329-338, 2007.

[4] L. A. Piegl, W. Tiller, The Nurbs Book (Monographs in Visual Communication), Springer, 1997.

[5] A. Sisojevs, A. Glazs, *Volume estimation of 3D object modelled by parametrical iso-surfaces*, Biomedical Engineering Proceedings. – Kaunas: Kaunas University of Technology, pp. 220 – 223, 2010.

[6] A. Sisojevs, K. Krechetova, A. Glazs, *3D Modelling of Free-Form Object*, The International Conference WSCG'2009 proceedings. – Plzen: University of West Bohemia, pp. 125 - 128 p. 2009.

[7] D. Lasser, "Rational tensor product Bézier volumes", Computers & Mathematics with Applications, Vol. 29, Iss. 8, pp. 95–108, April 1995.

[8] M. Samueleik, "Bézier and B-spline volumes", Project of Dissertation, Physics and Informatics Comenius University, Bratislava, Slovakia, 2005.

[9] W. Böhm, "Generating the Bézier points of B—spline curves and surfaces", Computer Aided Design, 13(6), pp. 365—366, 1981.

[10] J. Munkbergy, J. Hasselgren, R. Toth, T. Akenine-Möller, Efficient Bounding of Displaced Bézier Patches: In Proceedings of the Conference on HPG '10. pp. 153-162. 2010.

[11] K. Miura, A. Pasko, V. Savchenko, "Parametric patches and volumes in the functional representation of geometric solids", CSG 96 Information Geometers, pp.217-231, 1996.

[12] A. Nguyen, "Implicit Bounding Volumes and Bounding Volume Hierarchies", PhD theses, Stanford University, USA, 2006.

[13] A. A. Farag, H. Hassan, R. Falk and S. G. Hushek, "3D Volume Segmentation of MRA Data Sets Using Level Sets", Academic Journal of Radiology, pp. 419-435, 2004.

[14] G. Bradshaw, C. O'Sullivan, "Adaptive medial-axis approximation for sphere-tree construction", ACM Trans. Graph., Vol. 23(1), pp. 1-26, 2004.

[15] A. Kalaiah, A. Varshney, "Statistical geometry representation for efficient transmission and rendering", ACM Trans. Graph., Vol. 24(2), pp. 348–373, 2005.

[16] R. Wang, K. Zhou, J. Snyder, X. Liu, H. Bao, Q. Peng, B. Guo, "Variational sphere set approximation for solid objects" The Visual Computer, Vol.22(9-11), pp. 612–621, 2006.

[17] W. Wang, Y. K. Choi, B. Chan, M. S. Kim, J. Wang, "Efficient collision detection for moving ellipsoids using separating planes. Computing" Vol. 72(1-2), pp. 235–246, 2004.

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Aleksandrs Sisojevs. Pusskaitliska pieeja ķermeņa, kurš ierobežots ar racionālo Bezjē virsmu, tilpuma aprēķināšanai
Patvaļīgas formas objektu 3D modelēšana ir aktuāls uzdevums dažādās zinātnes un tehnikas jomās. Šī uzdevuma risinājums ir svarīgs praktiskai lietošanai daudzās nozarēs, piemēram, biomedicīnas inženierijā un tehniskajā projektēšanā. Viena no efektīvām 3D modelēšanas metodēm ir racionālo Bezjē līkņu un virsmu izmantošana. Šī metode ļoti bieži tiek pielietota datorizētas ģeometriskas projektēšanas sistēmās. Bezjē objektu specifikācija ir labi izpētīta datorizētas ģeometriskas projektēšanas sistēmās, bet ir dažas problēmas, kuras līdz šim vēl nav atrisinātas. Piemēram, kā izskaitļot ķermeņa, kas ir ierobežots ar racionālo Bezjē virsmu, kopu tilpumu. Tilpuma aprēķināšanas metode ķermenim, kas ir ierobežots ar Bezjē virsmām, ir aprakstīta zinātniskajā literatūrā. Šī metode izmanto tilpuma aprēķināšanu, balstoties uz integrālrēķiniem un to risināšanu analītiskā veidā. Dotajā darbā autors piedāvā pusskaitlisko pieeju tilpuma aprēķināšanai ķermenim, kas ir ierobežots ar racionālo Bezjē virsmu kopu. Ieejas dati tilpuma aprēķināšanas algoritmam ir racionālo Bezjē gabalvirsmu masīvs. Gabalvirsmas ir sakārtotas tādā veidā, lai Z ass iet cauri modelētajam objektam. Piedāvātā pieeja sastāv no divām daļām. Pirmajā daļā aprakstīta tilpuma integrāļa sastādīšana un tā risināšana analītiskā veidā, izņemot racionālas funkcijas integrāļa aprēķināšanu. Šīs funkcijas integrāļa aprēķināšanai ir domāta otrā daļa, kur tiek piedāvāta skaitliskās integrēšanas izmantošana.
Piedāvātā pieeja tika realizēta, izmantojot izmodelētu objektu, lai pārbaudītu tās funkcionalitāti. Par modelētiem patvaļīgas formas objektiem tika pieņemti lode un cilindrs. Pirms eksperimentālās pārbaudes izvēlēto objektu tilpumi jau bija zināmi. Eksperimenta gaitā iegūtie rezultāti parādīja piedāvātās pieejas izmantošanas iespējas un korektu rezultātu sasniegšanu.

Александр Сысоев. Получисленный подход вычисления объёма тела, ограниченного рациональными поверхностями Безье.
Проблема моделирования 3D объектов свободной формы является актуальной во многих областях науки и техники. Её решение является важным для практического использования в различных сферах, например, в биомедицинском инженеринге и техническом проектировании. Одним из эффективных методов 3D моделирования является использование рациональных поверхностей Безье. Это играет значительную роль в системах геометрического автоматизированого пректирования. Спецификация объектов Безье хорошо исследована в системах геометрического автоматизированого проектирования, однако существует несколько проблем, не решенных в настоящее время. Например, как вычислить объем тела, которое ограничено множеством рациональных поверхностей Безье. Метод вычисления объема тела, ограниченного обычными поверхностями Безье описан в научной литературе. Этот метод базируется на вычислении объема с помощью интегрального исчисления и решения этой задачи в аналитическом виде. В данной работе автор предлагает получисленный подход к количественной оценке объема тела, которое ограничено множеством рациональных поверхностей Безье. Входные данные для алгоритма расчета – массив рациональных патчей Безье. Патчи расположены так, что ось *Z* проходит через моделируемый объект. Предлагаемый подход состоит из двух частей. В первой части составляется интеграл вычисления объема и дается решение в аналитическом виде за исключением вычисления интеграла рациональной функции. Для этого во второй части подхода предлагается использование численного интегрирования.

Предложенный подход был реализован для проверки работоспособности на примере смоделированных объектов. В качестве примеров моделируемых объектов свободной формы были взяты сфера и цилиндр. Перед экспериментальной проверкой известны объёмы выбранных объектов. Полученные в ходе эксперимента результаты показывают возможность использования предлагаемого подхода и получения корректных результатов.