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Strength Ensuring of Cylindrical Parts Set by Interference

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***Abstract* – Researches of assembly processes of cylindrical parts set by interference fit resulted in defining the admissible failure on mutual layout of detail axes in the position of assembly, which provides the range of basing schema needed for accuracy and the choice of methods to reach assembly accuracy. In this article a mathematical model for defining the durability of the contact of cylindrical details set by interference was developed, and that provides the possibility to determine the admissible external force, which in practice is mainly connection-oriented eccentrically to its axes.**

***Keywords* – cylindrical parts, mathematical model, pressed connection, strength.**

I. Determination of Stresses and Deformations for the Cylindrical Parts Set by Interference

In this article the mathematical model for defining the strength of contact of cylindrical details set by interference is developed. Strength condition for the admissible external force working on the contact of details is elaborated. An expression for defining the admissible external force is obtained, which in practice is mainly oriented eccentrically to the connection. Usually cylindrical connections transferring the torsion moment caused by some cross-force F_{sk} (Fig. 1) are loaded eccentrically. For example, it occurs in transmission elements of rotation movement (gearwheel transmissions).

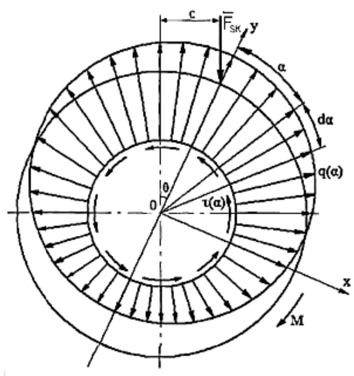

Fig. 1 Schema of the calculation of external force

An expression is obtained, which defines the admissible value of F_{skp} to eccentric force F_{sk}:

$$F_{skp} = \pi r l \sqrt{1 + f^2}\left([p] - p_0\right), \qquad (1)$$

where: p_0 – load in contact before application of force F_{sk};
$[p]$ -admissible load in contact;
F - friction coefficient.

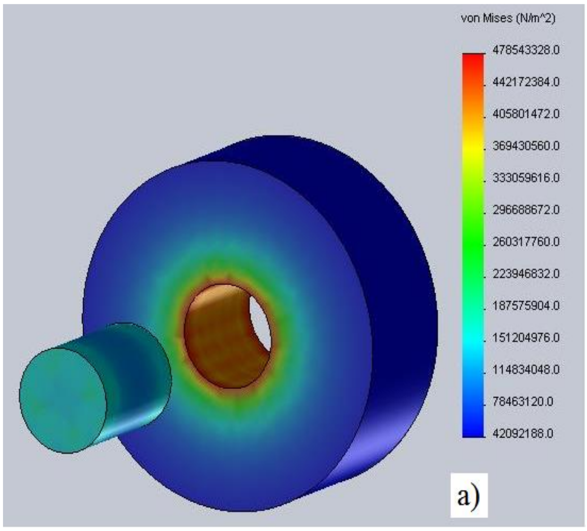

Fig. 2. Changes in total tension

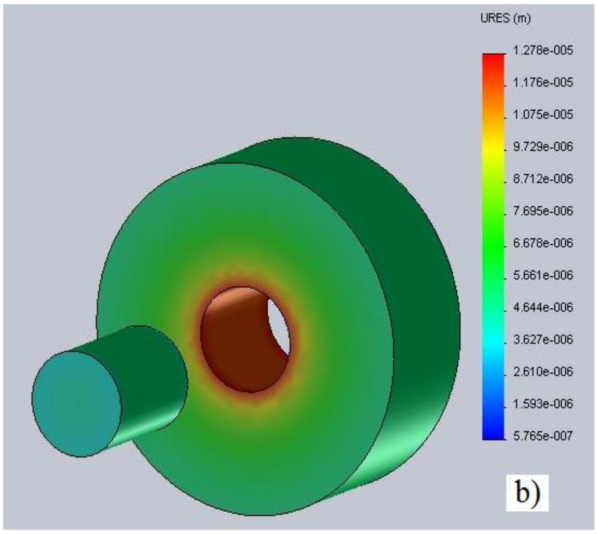

Fig. 3. Summary changes in dimensions of details

Defining the tensions and deformations of the contact of cylindrical details allows effectuating research of the processes in the contact and prognosticating the service of the connection already at the stage of designing. Using the software SolidWorks Simulation Premium 2010 (method of final elements), the character of tensions and deformations of mating details at perfect conditions, namely, without failures of assembly and processing, is defined (Fig. 2, 3).

Nevertheless, this method provides the engineer with the possibility to experiment with geometric dimensions of details and to choose them considering material, friction coefficient, pressure in contact of surfaces of cylindrical details, interference, and pressing force.

Surface roughness is one of the main values characterizing normal functioning of the rigid connection; and this value defines the division of normal and tangential tensions in the borders of the contact area. For defining the impact of such parameters on contacting surfaces of assembly details, contact model of cylindrical details with interference fit is elaborated using 3D regular case field theory. Each type of mechanical processing and process of surface forming has its own typical topography of surface roughness. Since the pressing process is related to irregular changes in the surface roughness of assembly details during the entire process of pressing in contacting areas, the paper investigates a model of irregular rough surface corresponding to regular causality. It was defined that deformation type of connection is affected by standardized parameters of surface roughness *Ra* and *Sm*. The critical approximation of details was determined, at which the elastic contact transforms into the elastic – plastic one.

In order to define which contact surface of cylindrical details deforms more, coherence for determination of average value of volume of material deformed during the contact was obtained:

$$E\{V\} = \left\{ \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}r^2} - \gamma[1-\Phi(\gamma)] \right\} \sigma^* A_c \ , \qquad (2)$$

where: A_c- contour area of contact;

$\gamma = \frac{u}{\sigma}$ - relative level of deformation, until which the rough surface is deformed,

where: u – deformation level subtracted from the average value of roughness;

σ – average quadratic deviation of roughness of contacting surface;

$$\Phi(\gamma) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\gamma} \exp\left(-\frac{t^2}{2}\right) dt \text{ - Laplace function.}$$

In the majority of cases, in equipment design it is significant to predict, which of the contacting details is most subjected to deformations . It provides the possibility to choose materials and geometric parameters appropriate for mating details as well as the type of processing of the contacting surfaces in order to obtain the desirable mate with the needed durability at definite conditions of exploitation.

II. Development of Method of Engineering Calculations for Cylindrical Parts Set by Interference Fit Using 3D Regular Case Field Theory

By applying the 3D regular case field theory in describing of surfaces, new method of engineering calculations for cylindrical parts set by interference fit was developed. The obtained coherences permit a more accurate determination of the force needed for pressing and parameters of surface roughness for connection at definite conditions, which is approved by the results acquired during the experiments. It is possible to define the required proportion of surface roughness parameters *Ra/Sm* characterized by contact load. It was not included in the known methods of fit calculation.

The obtained results allow viewing the assembly process of pressed connections of cylindrical details set by interference in a new perspective and give rise for further researches related to cylindrical parts set by interference fit.

Following the previously assumed conditions, the value of force was found out, which works on each of cam peaks of contacting surfaces. Conditions are the following:

1) cam peaks of surface roughness are situated rather densely and even over the whole cylindrical surface; thereby, radial pressure on contact surfaces caused during the assembly will be viewed as evenly situated radial load;

2) total deformation of assembly details is elastic and, consequently, subjects to the Lama law;

3) surfaces in the areas of actual contact are subjected to elastic-plastic deformations;

4) solidity of a rough surface exceeds the one of an even surface; thus, by performing pressing-in of the detail set by interference, peaks of roughness cams deform elastically;

5) there is no impact of external forces on the contact;

6) mutual influence of peaks is insignificant;

7) there is no deviance in surface form;

8) and nominal contacting surfaces during the whole period of contacting remain mutually parallel.

If heights of cam peaks contacting the even surface are situated in n^{th} levels, the amount belonging to defining the force P_{ij} (Fig. 4) equals $\gamma = \sum_{i=1}^{K} \sum_{j=1}^{n} P_{ij}$ where K_i- number of cams and number of its peaks located in the j^{th} level. P_{ij}- force working on i^{th} group of cams in the j^{th} level.

Average load in the contact can be defined in the following way:

$$p_k = \frac{\sum_{i=1}^{K} \sum_{j=1}^{n} P_{ij}}{A_a} = \frac{\sum_{i=1}^{K} \sum_{j=1}^{n} P_{ij}}{\pi d l} \ , \qquad (3)$$

where: A_a – nominal contacting area;

d – nominal dimension of mate;

l – length of contact.

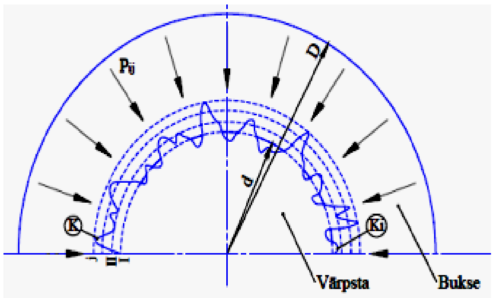

Fig. 4. Contact schema of rough (spindle) and even (bush) surfaces of cylindrical surfaces:
I, II, j- contacting levels of cam peaks; $K_1, K_2..K_i$- number of cams located in the j^{th} level; P_{ij} – force working on one i^{th} cam in the j^{th} level

After pressing-in, cams deform under the influence of the other surface; consequently, the initial interference will reduce for value *S*. Presuming that there is no initial deformation of roughness, there is no load on the contacting surfaces before their pressing; therefore, we take thickness of roughness level $Rmax=6\sigma$ as the point of reference. Consequently, it was calculated that the approximation is:

$$S=(3-\gamma)\sigma \quad , \tag{4}$$

The maximal interference is defined in the following way:

$$\delta_{\max\ j}=\frac{cd}{A_a}\sum_{i=1}^{K}\sum_{j=1}^{n}P_{ij}+2S\ , \tag{5}$$

where: $c=(C_1/E_1+C_2/E_2)^{*}k$;

where: E_1 and E_2 – elasticity modules of material of assembly details;
C_1 and C_2 – constants of materials;
k – coefficient, which follows changes in contact pressure on the surface of the detail longitudinally.

The minimal interference is defined in the following way:

$$\delta_{\min\ j}=p_s d\times c+2S\ , \tag{6}$$

where: p_s – load on connection under the impact of external forces.

This article has proven that the relative level of deformation can be defined applying [1, 2] contact theory to an elastic contact, in accordance to which the relative load of the contact can be defined as follows:

$$p_i=\frac{k_q^{el}Ra}{Sm\theta}F(\gamma)\ ,\quad \gamma>1, \tag{7}$$

where: k_q^{el}- coefficient that depends on anisotropy of roughness;
$F(\gamma)$ - function that depends on the level of deformation;
θ - elasticity constant of the material of details.

The equation components are solved in accordance with the following schema: following the given load, parameters of surface roughness, and properties of material, $F(\gamma)$ is defined and then γ is found in tables.

Thus, it was determined that smoothening of micro-roughness depends on the proportion of surface roughness parameters Ra/Sm and physical-mechanical properties of the material of details.

III. Method and Example of Engineering Calculation of Interference Fit for Cylindrical Parts

In this article, the method and example of engineering calculations of interference fit is provided.

Source data: nominal diameter of connection d=15 mm, external diameter of bush D=50mm, length of connection contact l=20 mm. The spindle and bush have the same material S355J2G3, where: E=2*10^{-5} MPa, μ=0.3.

By calculations, values of minimal interference following [3] method and the developed method providing the reduced value were obtained. By satisfying conditions of connection strength at axial force *Fa*=2 kN, the minimal interference is $\delta_{min\ v}$=7μm [3]; and in accordance with the developed method: $\delta_{min\ j}$=9.5 μm.

Calculation schema of interference fit is given in Figure 5.

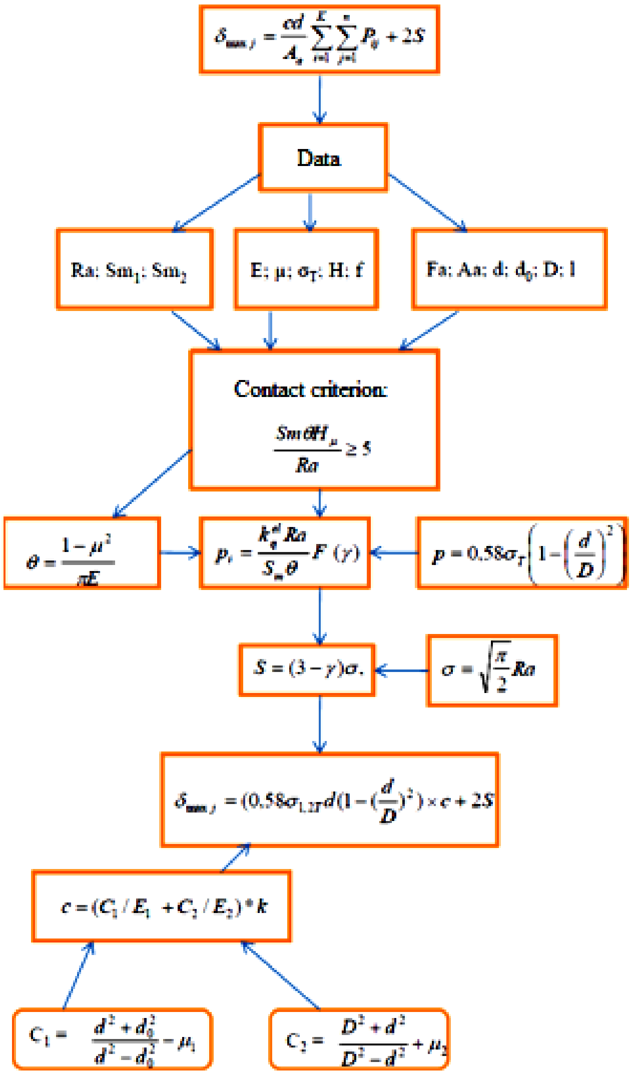

Fig. 5. Calculation schema of interference fit.

TABLE I

Comparison of Engineering Calculations for Cylindrical Parts Set with Interference Fit

Interference δ_{maxj}, µm	Sm/ Ra, µm	Contact load p_j,MPa	Interference [2] δ_{maxv}, µm	Ra, µm	Contact load p_v,MPa
30,2	100/1,25	186	31,3	1,25	186
27		169	28,6		169
24		145	25		145

In Table I the results of calculations for cylindrical parts set with interference fit are summarized. At equal loads, interference calculated by the method of S. P. Timoshenko is a little higher. Difference in results is explained by the following:
1) By approximating the dimensions of contacting surfaces. In accordance with the method of Timoshenko, it is 1.5 times bigger. In the second case, approximation of surfaces is calculated by (4) formula, where relative level of deformation of surface roughness, until which the roughness is deformed, is defined. It depends on the pressure in the contact and surface roughness parameters Ra and Sm as well as material properties E and µ;
2) In the developed method for calculation of interference fit, surface roughness proportion Ra/Sm is included, which was not considered in the method of S. P. Timoshenko and which is an important value characterizing the roughness of the surface profile and deformation type of the contact of rough surfaces. This proportion characterizes also the choice of the needed type of processing. As more accurate processing technologies have entered modern manufacturing, they permit obtaining the details with desirable parameters of surface roughness. In addition, the method of engineering calculation for cylindrical parts set with interference fit offered in this article allows defining the proportion of Ra/Sm needed for definite requirements of mate. Characteristic values of the contact such as load, approximation of surfaces, and dimension of interference depend not only on a separately taken surface roughness parameter Ra but also on step parameter Sm and their proportion.

The abovementioned confirms that the developed method for calculation of interference provides more accurate and more realistic calculation results in comparison with method [3]. The need for a more accurate method of calculation is promoted by the development and progress of mechanical engineering, which demands manufacturing of more and more accurate equipment and long-time resistant parts with high durability.

IV. Impact of Roughness Parameters Ra and Sm of Contacting Surfaces on Capacity of Interference Fit and Load in Contact Research

With increase of Ra/Sm proportion (Fig. 6), approximation of contacting surfaces of details increases. At the maximal interference 30.2 µm (Table I), approximation S=8.7 µm was obtained; and at the minimal interference 9.5 µm - S=5.1 µm. Analytically it was defined that by increasing Ra and maintaining the value of Sm, approximation S increases while the load decreases respectively. It provides the possibility to view changes in two rough surfaces during contact more comprehensively and to obtain more accurate values of interference depending on forces working on the contact because value S is characterized by surface roughness proportion Ra/Sm and load in contact.

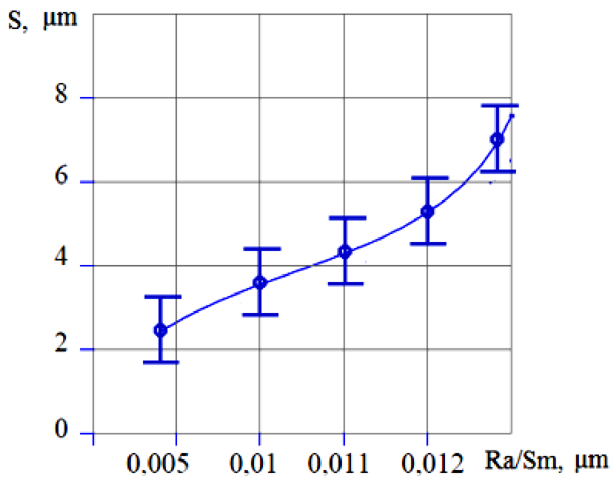

Fig. 6. Changes in approximation S depending on the proportion of surface roughness parameters Ra/Sm

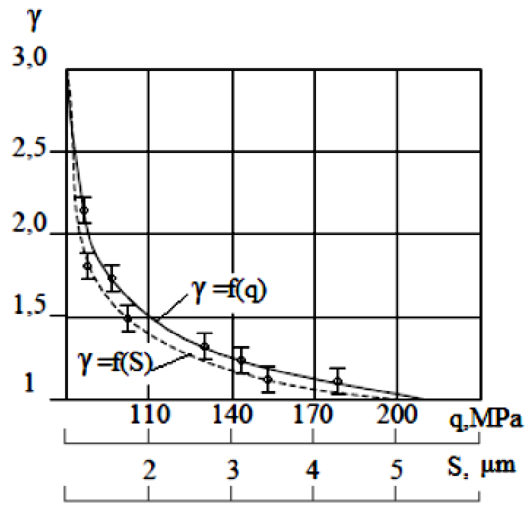

Fig. 7. Changes in deformation level of surface roughness depending on load in contact and approximation of surfaces

Figure 7 shows changes in deformation level depending on the load in contact and approximation of surfaces. Values are obtained in accordance with formulas (4) and (7). With increase of load, the level of deformation reduces. With increase of surface approximation, the level of deformation reduces. Similarly to researches of J. Rudzitis [1; 4], higher deformations of surface roughness correspond to lower values of level. By applying the developed formulas in this article, several coherences between characterizing values of contact were defined; thus, different regularities with real processes of contacting were confirmed.

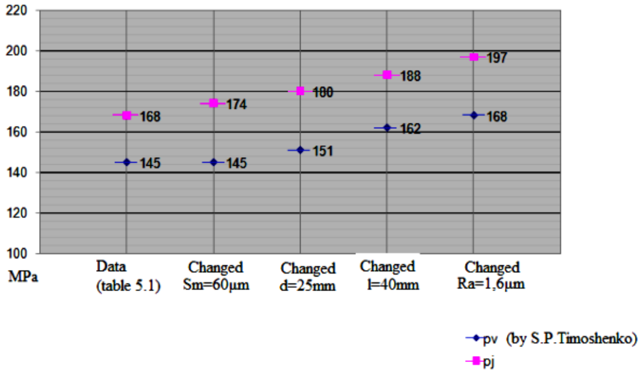

Fig. 8. Changes of contact load depending on geometrical and surface roughness parameters

Figure 8 shows changes of load in contact depending on the geometrical and surface roughness parameters. By changing the surface step parameter 60 µm (initial 100 µm), contact load determined by the developed method will increase. Similar to the research of J. Rudzitis [4, 5], decrease of surface roughness step gives increase of contact load. Contact load increases if Ra increases but Sm is constant. As it has already been studied, S. P. Timoshenko method of calculation of interference fit does not include step parameter of surface roughness.

Calculation results obtained following the elaborated method show higher compatibility with real detail contacting processes than the already known method. The method provides the option to obtain more precise mate with interference and excludes possibilities to choose too high values of interference to provide durability of the connection.

V. Conclusions

1) A mathematical model for defining the durability of cylindrical detail contact set by interference for the admissible external cross-force, under the impact of which the connection is mainly loaded eccentrically in practice, has been developed. Expressions of durability condition depending on the admissible external force working on the contact of details are obtained. It permits effectuation of researches of processes in the contact and prognostication of connection services already at the stage of its design.

2) A new method of engineering calculations for cylindrical parts set by interference fit by using 3D regular case field theory in description of surfaces is elaborated, which provides possibility to define the force needed for pressing and parameters of surface roughness for mate at definite conditions more accurately; and it was confirmed by the results obtained in the experiment. It is possible to determine the needed proportion of surface roughness parameters Ra and Sm, which is not included in the previously known calculation methods with mating character (fit).

3) The developed method of engineering calculations for cylindrical parts set by interference fit allows defining the changes in approximation value of contacting surfaces of mating details at load in contact depending on proportion of surface roughness parameters Ra/Sm. Changes in interference value, contact pressure, and approximation of contact surfaces depending on the proportion of surface roughness parameter Ra/Sm were analytically defined. In this article, coherences between values characterizing contact were obtained by applying the developed formulas; such coherences show regularities with real processes of contacting. Similarly to researches of J. Rudzitis [4, 5], load on detail increases due to reduction of the surface roughness step. Increasing Ra and maintaining Sm value, pressure in the contact increases. With increase of load, deformation level of surface roughness decreases. With increase of approximation of surface roughness, level deformation reduces. Higher deformations of surface roughness correspond to lower values of level.

References

1. **Rudzitis J.** *Surface roughness topography investigations*// VIIth Int. Oberflächen Kolloquium Technical University of Chemnitz.-Chemnitz: TU Chemnitz, 1992.-65.-70. p.
2. **Rudzitis J.** *„Kachestvo poverhnosti i ekspluatacioonie svoistva detalej mashin i priborov"*, Riga: RPI, 1978. – 101 str.**P**
3. **Timoshenko S. P.**, Goodier J. N. *Theory of Elasticity*.- New York: McGraw-Hill,1970.-50 p.
4. **Rudzitis J.** *„Mikrogeometrija i kontaktnoe vzaimodejstvie poverhnostej"*, Riga: Zinatne, 1975. – 216 str.
5. **Rudzitis J.** *„Kontaktnaja mehānika poverhnosti, 1-aja chastj. Parametri profilja sherohovatosti poverhnosti"*, ISBN 978-9984-9964-1-7, Riga: RTU, 2007. – 193 str.

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rotary assembly machines.

Zita Dreija, Oskars Liniņš, Natālija Mozga. Cilindrisko detaļu mezglu ar uzspīlējumu izturības nodrošināšana

Cilindrisko detaļu mezglu ar uzspīlējumu salikšanas procesu pētījumu rezultātā noteikta detaļu asu savstarpējā novietojuma pieļaujamā kļūda salikšanas pozīcijā, kas nodrošina precizitātei nepieciešamo bāzēšanas shēmas izvēli un savienojuma precizitātes sasniegšanas metodes izvēli. Izstrādāts cilindrisko detaļu ar uzspīlējumu kontakta izturības noteikšanas matemātiskais modelis, kas dod iespēju noteikt pieļaujamo ārējo spēku, kas praksē lielākoties uz savienojumu vērsts ekscentriski. Rakstā tiek izstrādāts cilindrisko detaļu mezglu ar uzspīlējumu kontakta modelis, izmantojot virsmu aprakstam 3D normālo gadījuma lauka teoriju. Izstrādātā cilindrisko detaļu mezglu sēžas ar uzspīlējumu inženieraprēķina metode dod iespēju noteikt salāgojamo detaļu kontaktējošo virsmu tuvinājumu un tā izmaiņas pie kontakta slodzes atkarībā no virsmu raupjuma parametriem Ra un Sm. Iegūtie rakstā rezultāti ļauj aplūkot jaunā aspektā cilindrisko detaļu ar uzspīlējumu presēto savienojumu kontaktu. Un to var skaitīt par pamatu tālākajiem cilindrisko detaļu mezglu ar uzspīlējumu saistītiem pētījumiem.

Зита Дрейя, Оскарс Лининьш, Наталия Мозга. Обеспечение прочности узлов цилиндрических деталей с натягом

В результате исследования сборочных процессов узлов цилиндрических деталей с натягом определена допустимая ошибка относительного расположения осей деталей на сборочной позиции, которая обеспечивает для точности необходимый выбор схем базирования и выбор метода достижения точности соединения. Разработана математическая модель определения прочности контакта цилиндрических деталей с натягом, которая дает возможность определить допустимую внешнюю нагрузку, которая на практике к соединению направлена преимущественно эксцентрично. В статье разработана модель контакта узлов цилиндрических деталей с натягом, используя для описания поверхности теорию 3Д нормального случайного поля. Разработанный инженернорасчетный метод посадки с натягом узлов цилиндрических деталей дает возможность определить приближение контактирующих поверхностей соединяемых деталей и его изменения при контактной нагрузке в зависимости от параметров щероховатости поверхностей Ra и Sm. Полученные в статье результаты позволяют рассмотреть в новом аспекте контакт запрессованных соединений цилиндрических деталей с натягом.