# Modeling of Water Utilization in Hydroelectric Power Plants on the Daugava River

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Abstract – In this paper the task of modeling water utilization in the hydroelectric power plants on the Daugava River is outlined. Difficulties caused by the complexity of the task are discussed and the necessity to decompose it is considered. Unit commitment proves to be especially important subtask as it introduces several constraints and limitations. A possibility to solve it using dynamic programming approach has been offered and the veracity of the mathematical model of reservoirs has been tested in this paper.

Keywords - Water resources, reservoirs, hydroelectric power, optimization

#### I. Introduction

Water resources have a significant role in the energy sector of Latvia. Most of the electric energy produced in the country is generated in the three hydroelectric power plants (HPP) situated on the Daugava River. However, these stations work at their full capacity only during the annual spring flood period which is no longer than a month. For the rest of the year their power production capabilities are limited by small natural water inflow. Thus it is necessary to develop power generation planning methods that allow for more rational and effective utilization of the limited water resources.

The solution to this problem must provide both, optimal distribution of water resources for power generation in time and distribution between dispatched generation units. The latter is achieved by solving the unit commitment (UC) subtask. Improved management of hydro-energetic resources would let the producer achieve increased revenue when electrical energy is sold in Nord Pool energy market.

However, the complexity of this task increases if the stochastic nature of its input parameters – energy price and inflow – is considered. Another complication is the requirement to account for reservoir constraints due to environmental and safety concerns.

Some assumptions have to be made in order to simplify the optimization procedure. Firstly, it is presumed that, since the producer under consideration provides relatively minor part of the total energy in Nord Pool, it does not have significant enough impact on the market clearing price for it to be taken into account. As the producer in question operates not only hydroelectric power plants, but thermoelectric plants as well, normally it would have been required to optimize their hydrothermal dispatch. The previous decision to assume electricity market prices as exogenous variables, however, allows hydroelectric power cascade to be optimized independently

from other types of generation. It has to be noted that validity of this presumption is disputable and in future work the producer's influence on energy price in local bidding area should be studied more thoroughly.

Another step to simplify optimization procedure is decomposition of the task. There have been many methodologies proposed on how to decompose the problem of water utilization optimization in hydroelectric power plants.

For instance, [1] studies the use of search procedures like progressive optimality algorithm to decompose the problem into multiple two stage decision tasks. [2] and [3] offer techniques on how to separate the complex task into parts, where the master problem can be solved using linear programming, but the secondary tasks — dynamic programming (DP).

The main contribution of the paper is determining how to apply DP in solving the UC task and developing and validating the mathematical models of the reservoirs for HPPs on the Daugava River.

The remainder of this paper is structured as follows. In section II we present mathematical formulation of the optimization problem. Section III briefly explains the structure of optimization algorithm. In section IV we define the UC problem and offer a solution technique using DP. In section V we validate the mathematical model and in section VI we compare results from DP to actual data. Section VII concludes the paper and discusses future work.

#### II. MATHEMATICAL DESCRIPTION OF THE PROBLEM

## A. Optimization object

The object of optimization studied in this paper is the HPPs that operate in storage basin of the Daugava River. These power plants are Plavinas HPP, Kegums HPP and Riga HPP. Their installed active power is 893.5 MW, 264 MW and 402 MW respectively.

Unlike in [4] the comparatively smaller reservoir volumes create circumstances where changes in water level are more immediate and have bigger effect on efficiency of water turbines. Therefore, if the goal is HPP short-term operation planning to build hourly generation bids for day-ahead trading, it is crucial to calculate water head changes within the planning interval for each power plant.

Due to limited natural inflow HPPs cannot generate power constantly. They can work when prices are high, but must limit or stop production eventually to refill the reservoirs.

Furthermore, only Plavinas reservoir is filled by natural inflow in Daugava, the lateral inflow in the other two stations is negligible. This means that water level in Kegums and Riga reservoirs rises only when the water discharged by upstream HPP reaches them. Techniques on how to calculate the time it takes for discharged water to travel to and have an impact on a downstream reservoir are offered in [5]. In order to simplify the mathematical model it is assumed in this paper that the travel time of water between two Daugava HPPs is equal to 1 hour and is not dependant on any other variables, such as elevation of tailwater or volume of discharge.

The factors mentioned above demonstrate a necessity to develop a model capable of simulating the changes in reservoir parameters, most importantly – upstream and downstream water levels, which are required to calculate the water head as described by (1).

$$h = l_{fb} - l_{fw}, \tag{1}$$

where h is the water head (m),  $l_{fb}$  - forebay elevation (m),  $l_{tw}$  - tailwater elevation (m).

Upstream and downstream water levels can be calculated using the relationship curves of water storage versus forebay elevation and tailwater elevation versus outflow release. Thus it can be concluded that change in upstream level is a function of inflow and outflow (2), whereas downstream level is a function of outflow (4).

$$\Delta l_{fb} = f(\Delta q) = f(q_{in} - q_{out}), \tag{2}$$

$$l_{fb}^{t} = l_{fb}^{t-1} + \Delta l_{fb}, \tag{3}$$

where  $\Delta q$  is the difference between water inflow and outflow in the reservoir (m<sup>3</sup>/s).

$$l_{tw} = f(q_{out}) \tag{4}$$

These functions are nonlinear because of the relatively small size and irregular form of the reservoirs.

The power generated by a particular hydroelectric set depends from discharge through it  $q_{out}$  (m³/s), effective water head h (m), hydro turbine efficiency  $\eta_t$  and generator efficiency  $\eta_g$ :

$$p = 9.81 \cdot h \cdot q_{out} \cdot \eta_t \cdot \eta_g \tag{5}$$

However, hydroelectric set efficiency itself is a function of power and water head.

$$q_{out} = f(p, h) \tag{6}$$

The relationship curves between these parameters and water discharge through a generation unit can be obtained experimentally.

#### B. Main objective function

If we ignore generator start-up costs the profit maximization task can be substituted by revenue maximization.

$$R_{\Sigma} = \sum_{t=1}^{T} p_{\Sigma}^{t} \cdot c^{t} \to \max, \tag{7}$$

where  $R_{\Sigma}$  is the total revenue in the planning horizon, which is T hours long,  $p_{\Sigma}^t$  - total generated power in the cascade in t hour,  $c^t$  - electrical energy price in t hour.

The main objective function can be expanded to reveal the secondary problem - unit commitment.

$$R_{\Sigma} = \sum_{t=1}^{T} \left( \left( \sum_{n=1}^{10} p_{PHPP,n}^{t} + \sum_{n=1}^{7} p_{KHPP}^{t} + \sum_{n=1}^{6} p_{RHPP}^{t} \right) \cdot c^{t} \right), (8)$$

where 10, 7 and 6 is the number of hydroelectric units in Plavinas, Kegums and Riga HPP respectively.

#### C. Constraints

Restrictions imposed by environmental and safety requirements limit permissible water level variation in reservoirs. The upper and lower limits of upstream and downstream elevation are summarized in table 1 [6].

TABLE I
RESERVOIR CONSTRAINTS

		Plavinas HPP	Kegums HPP	Riga HPP
Upstream level, m	Upper limit	72	32	18
	Lower limit	69	30.4	17
Downstream level, m	Upper limit	35.9	18.5	3.9
	Lower limit	30.5	17	-1.5

Displayed values for lower limit of upstream water level are correct for most of the year, however, there are seasonal differences, e.g. before spring flooding or during summer.

An HPP cannot utilize its water resources from the upper to the lower limit within a single day as they are not allowed to decrease reservoir level for more than 0.3 meters an hour or 0.75 - 1.6 meters (depending on the season) within 24 hours.

To summarize, the main objective function is subject to the following reservoir constraints:

$$l_{\text{fb.min}} \le l_{\text{fb}} \le l_{\text{fb.max}} \tag{9}$$

$$l_{tw.\min} \le l_{tw} \le l_{tw.\max} \tag{10}$$

$$\Delta l_{fb.hour} \le \Delta l_{fb.hour.max}$$
 (11)

$$\Delta l_{fb.24hours} \le \Delta l_{fb.24hours.\,\text{max}}$$
 (12)

There are also constraints limiting the power output of a generation unit. Generally, hydroelectric set characteristics forbid operation in vibration zone, where it is allowed only transitionally. In a simplified manner unit constraints can be represented as

$$p_{n,\min} \le p_n \le p_{n,\max} \tag{13}$$

Alternatively, given known value of water head, power constraints can be expressed as restrictions on permissible water discharge through a particular hydro turbine as

$$q_{out,n,\min} \le q_{out,n} \le q_{out,n,\max} \tag{14}$$

# III. OPTIMIZATION ALGORITHM

Even though the purpose of the model is forming hourly generation bids for the next day, a 24 hour optimization interval is insufficient. It is proposed to develop a model for a week (168 hours) or more.

[7] proposes to separate the HPP operation planning optimization problem into subtasks. Firstly, a linear HPP model is used to plan water utilization for 168 hours. Afterwards, parameters at the end of the first day from the linear model are used as input data for nonlinear model and a solution for 24 hours is acquired. This results in hourly generation plan for each power plant that the market operator could use in bidding. Finally, the last step is to distribute the planned generation among hydroelectric units. The order of these subtasks is illustrated in Fig. 1.

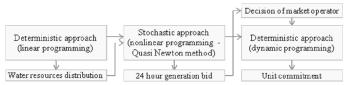


Fig. 1. Optimization algorithm of HPP operation planning

However, no planning can be done if expected hourly prices and inflow are not known. Several authors consider artificial neural networks (ANN) the best option to predict these stochastic variables, especially if there is sufficient statistical data recording their trends. This is true for prices in Nord Pool and historic inflow values in the Daugava River.

#### IV. UNIT COMMITMENT

#### A. Vibration zone avoidance

There are several factors that have to be considered when solving the unit commitment problem. The most important requirement is for a hydroelectric set to work within its operational zone. Fig. 2 illustrates characteristic of a unit. It displays operational and vibration zones.

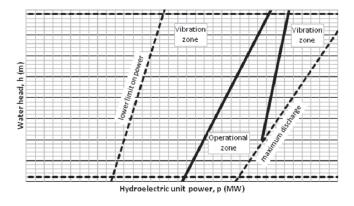


Fig 2. Example of a hydroelectric set characteristic

If unit power constraints are defined as in (13) or (14), then to use them in calculations it is necessary to approximate the experimental relationship curves of water discharge versus effective water head and generated power. An example of these curves is given in Fig. 3.

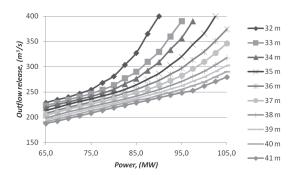


Fig. 3. Relationship curves of water discharge versus effective water head and generated power for Plavinas HPP units 2, 5, 7 and 8

Characteristic can be approximated in polynomial form as

$$q_{out} = a_1 \cdot p + a_2 \cdot h + a_3 \cdot p \cdot h + a_4 \cdot p^2 + a_5 \cdot h^2 + a_0$$
(15)

As according to the optimization algorithm the value of water head is calculated for every hour before solving the UC problem, its value can be assumed as constant for a given hour. This enables us to perform approximation of the relationship curve like this

$$q_{out} = a_1 \cdot p^3 + a_2 \cdot p^2 + a_3 \cdot p + a_0 \tag{16}$$

where  $a_0$ ,  $a_1$ ,  $a_2$ ,  $a_3$  - polynomial coefficients obtained by interpolation.

(16) is the relationship of water discharge (m³/s) versus active power (MW) for a specific water head (m). For instance, if the characteristic is defined by water head values in the interval from 33 to 40 meters and the chosen step is 0.1 meters, than we have to interpolate 71 functions. This can be easily done by creating a looping script in any scripting environment.

The above example is a polynomial of 3<sup>rd</sup> order. By increasing the rank we could achieve greater precision of interpolation. Precision can be evaluated by comparing the curves obtained in the interpolation to the source data. In this paper polynomials of 3<sup>rd</sup> order were used and resulting relationship equations were deemed sufficiently accurate as the greatest error value calculated by (17) was 3.59%.

$$\varepsilon_{\text{max}} = \max \left( \frac{q_{out.real.i,j} - q_{out.polyn.i,j}}{q_{out.real.i,j}} \right)$$
(17)

#### B. Other factors to consider when solving UC problem

Apart from power and discharge limits there are certain other demands that have to be taken into account when planning hydro turbine scheduling. They are:

- · uniformity of operation;
- excessive use;
- · positions of turbines.

Frequent start-ups and shut-downs of a turbine have to be avoided, because it increases wear and thus the maintenance needs of the unit.

On the other hand, it is unadvisable to continuously operate the same units for long periods of time, especially if there are other units in shut-down state at the same time. Uneven wear can cause costly maintenance issues.

Evidently, the requirements of uniform load of a hydroelectric unit and even distribution of up-time between all units can be conflicting. Some authors [2], [8] propose introducing decision variables describing the state of units. Some additional requirements can be introduced, such as minimum downtime and maximum number of start-ups to avoid frequent connections and disconnections, maximum uptime to avoid overworking a given unit.

Operator experience suggests that positioning of units in generation mode can have an impact on their effectiveness. Because of this, if HPP power output plan is such that it requires 2 or more turbines to be operational at the same time, it is advisable to not activate units that stand next to each other. Another factor to consider is the corrosive effect turbines close to the banks of the basin can have on the shores.

### C. Solving the UC problem with Dynamic Programming

Let's assume that the main optimization algorithm has created a water resources utilization schedule that fulfills reservoir constraints and estimates acquiring maximum revenue. Generally, this could be a power generation plan, but for the purposes of this paper let's consider that the input variables for the UC task is the recommended amount of water that has to be discharged through each HPP every hour in the planning interval (m³) and the value of water head at the beginning of every hour (m). For any given outflow there are several combinations of units that can use the given water resources while producing different amounts of power.

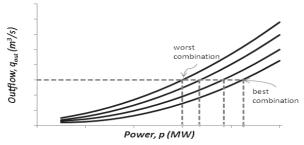


Fig. 4. Example of block characteristics of units

We can define the subtask as maximization of generated power for a given amount of discharge as

$$P_{\Sigma}^{t} = \sum_{n=1}^{N} P_{n}^{t} \to \max$$
 (17)

subject to

$$\sum_{n=1}^{N} W_n^t \le W^t \tag{18}$$

and discharge constraints as previously defined in (14).  $W^t$  is the total volume of dischargeable water through an HPP in a given hour (m³) and  $W_n^t$  is the volume of discharged water through a turbine designated as n.

We can substitute the amount of discharged water in cubic meters with rate of discharge in cubic meters per second, knowing that there are 3600 seconds in an hour:

$$q_{out,n}^t = \frac{W_n^t}{1.3600} \tag{19}$$

When the UC subtask has been defined we can solve it using Dynamic Programming. Traditionally, when DP is applied in optimizing hydroelectric power plant operation it is done in larger planning horizons and decisions are made in different time stages. For instance, in [9] DP is used to obtain water values and hourly bids. In this paper, however, it is employed within each hour solving a UC problem static in time. Solution is obtained by choosing the optimal outflow through each hydroelectric unit in regards to other units as well as deciding on whether a certain unit should be connected at all.

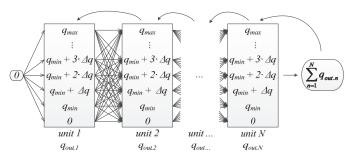


Fig. 5. Application of dynamic programming in solving the UC problem

Given that there are 23 units in the Daugava river HPP cascade, the total amount of information that needs to be stored during calculations can be quite taxing. For instance, if in a given hour Kegums HPP is supposed to have a discharge of 1000 m<sup>3</sup>/s and the calculations are done in increments of 5 m<sup>3</sup>/s then the array where intermediate results are written will have dimensions of 200x7. Though still time consuming, this approach, however, is more effective than evaluating every possible combination.

The recursive equation describing the total power generated in an HPP depending on the outflow through unit n and the units before it is as follows:

$$r_{k}(q_{out}) = \max\{r_{k-1}(q_{out} - q_{out.n.k}) + f_{k}(q_{out.n.k})\}$$

$$(20)$$

Once the matrix is filled, it is time to initialize traceback procedure and output the results, which are the amount of power (MW) each participating hydroelectric unit has to be generating for the given hour.

As described by constraint (18) it is possible for less water resources to be spent in *t* hour than was originally planned. This can happen if it is impossible for the desired value to be achieved by any combination of generating units. In this case the surplus water resources have to be redistributed to different hours, before or after the particular hour for which the UC problem is being solved.

The DP approach described in this section, however, does not address issues of uniform and even operations. This is rectified by introducing a list of priority for each hour. If there are several units with the same or very similar characteristics in an HPP and either of them is chosen to be operational by DP, then it can be replaced according to this preset list. If a turbine's id is omitted from the list, it is not considered in the calculations. That is a way how to set the state of a certain hydroelectric unit as unavailable for production either because of maintenance or any other reasons.

Alternatively, the decision on which turbines from a set of equal units should be in generation mode can ultimately be made by the operators. For this purpose they must be presented with all the viable combinations of generator loading that offer maximum value of the target function.

#### V. VALIDATION OF THE RESERVOIR MODEL

Any results obtained by optimization algorithms are useful only if they reliably fulfill the various reservoir constraints mentioned previously. That largely depends on the veracity of reservoir models.

In order to validate the models used in this paper, they can be tested by feeding them real statistical data and comparing the results. For this purpose the hourly generation data from June 1, 2015, is used as input in the program and the output is water level in the reservoir for each power plant on the Daugava River. The levels calculated by the model are then compared to the actual reservoir levels as they were measured on that date. This comparison is illustrated in figures 6, 7 and 8, showing Plavinas, Kegums and Riga HPP.

The deviations at the end of the day are -0.03 meters, 0.03 meters and -0.05 meters for each reservoir respectively. These deviations are small enough for the model to be considered satisfactory.

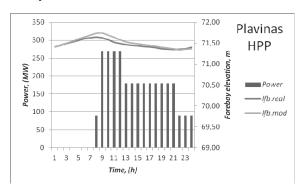


Fig. 6. Comparison of real and modeled upstream water level in Plavinas HPP

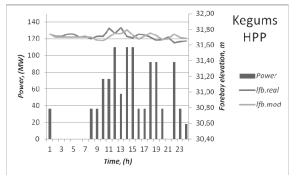


Fig. 7. Comparison of real and modeled upstream water level in Kegums HPP

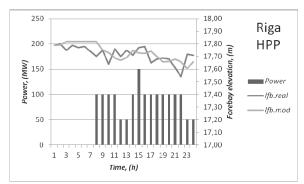


Fig. 8. Comparison of real and modeled upstream water level in Riga HPP

# VI. RESULTS FROM SOLVING THE UC SUBTASK USING DYNAMIC PROGRAMMING

After it has been established that the reservoir model and approximations of unit characteristics are accurate enough to be used in calculations it is also necessary to test the dynamic programming approach discussed in section IV.

As was previously described there are two ways how to employ DP to determine which units in an HPP should be operational and what their power should be within a given hour. If the input variable for each hour is the total amount of water to be used in the particular HPP, then DP solves a power maximization task. On the other hand, if the input variable for each hour is the desired sum of all units' generation, a discharge minimization is performed. Both approaches essentially strive to increase efficiency of operation and thus higher water value.

Let's use the same statistical data from June 1, 2015 we used in reservoir model validation to test both these approaches in solving the UC subtask:

#### A. Power (and by extension – revenue) maximization

Input data is hourly discharge which is obtained by calculations performed in section V. For this example, optimization is performed on Plavinas HPP for each of the 24 hours. Results are compared to the actual statistics in Fig. 9.

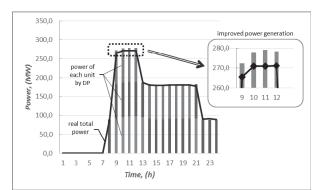


Fig. 9. Comparison of actual power and power obtained by DP

The UC subtask solution offers to produce more electrical energy while using the same amount of water. The improvement constitutes 45.6 MWh which is a 1.49% increase. Taking market prices into account it can be concluded that revenue from Plavinas HPP has thus increased by  $2758.24 \in$ .

#### B. Discharge minimization

Input data is hourly power generation. The objective is to find such a combination of units that produces the same power, but uses less water in doing so.

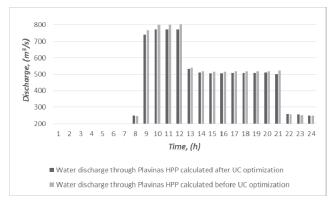


Fig. 10. Comparison of outflow with and without solving the UC subtask

The solver achieves the given desired Plavinas HPP hourly power by finding the most effective combination of units. In this case those turn out to be units 5, 7 and 8. Fig.10 displays the differences in hourly discharge before and after solving the minimization task. The saved water resources amount to 0.72 mill.m³ which constitutes 2.26%.

#### VII. CONCLUSIONS

The task of optimizing the effectiveness of water resources utilization with the objective to increase profit is very complicated. It is necessary to decompose the problem in order to simplify finding solution. The HPP cascade on the Daugava River has proven to be especially hard to model, as the relatively small size of reservoirs calls for a nonlinear description of them. However, the model used in this paper produces relatively small errors and can be considered reliable.

Once the total amount of water that can be used in power generation has been distributed between hours within the planning interval by the main solver, a secondary task arises since it is necessary to determine which units should be dispatched in any given hour and how much power each of them ought to generate.

In this paper a dynamic programming approach was suggested for the unit commitment task. It allows increasing the efficiency of water usage. This was confirmed by comparing calculation results to actual data. However, the improvement is fairly small suggesting that HPP operators already dispatch units quite efficiently. Thereby, it could be beneficial to offer a set of several valuable solutions to the HPP operators, whereas they might choose the best dispatch schedule based on their experience and technological circumstances.

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