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RELIABILITY OF FLEET OF AIRCRAFT AND AIRCRAFT REPLACEMENT PROBLEM

Summary of Doctoral Thesis

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DECLARATION OF ACADEMIC INTEGRITY

I hereby declare that the Doctoral Thesis submitted for the review to Riga Technical University for the promotion to the scientific degree of Doctor of Engineering Sciences is my own and does not contain any unacknowledged material from any source. I confirm that the present Doctoral Thesis has not been submitted to any other university for the promotion to any other scientific degree.

Sergejs Tretjakovs …………………………….(Signature)

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The Doctoral Thesis has been written in English. The Doctoral Thesis comprises an introduction, 4 chapters, conclusions, bibliography with 61 reference sources and appendices of 46 pages. It has been illustrated by 49 figures and 6 tables. The total volume of the present Doctoral Thesis is 140 pages.

CONTENT

GENERAL DESCRIPTION OF THE PRESENT RESEARCH

Topicality of the Research

The airframe of modern aircraft is mostly made of metallic elements in spite of the fact that modern composite materials are used more frequently. Service of such aircraft is connected with metallic element, especially different aluminium alloys, features that are influenced by external stress.

As a result of stress cycles fatigue cracks are developing in such fatigue-prone airframes, reducing its residual strength. Huge number of aviation accidents happened because of this problem. Durability of fatigue-prone airframe and provision of required reliability level are a topical issue since 1950s.

Different problem solution methodology was developed to obtain the required reliability level. Safe-life approach requires nomination of structurally significant item (SSI) service life. During this nominated service life, required reliability level has to be ensured. Another fail-safe approach requires carrying out inspections with intention to discover a fatigue crack before it reaches its critical size.

Nominated service life should assume the worst service conditions but a large number of airframes are not in the worst condition and it results in economical loses. MSG-3 document describes methodology of inspection program development for fatigue-prone airframe. MSG-3 development is a huge step of problem solving but some statistical parameters are still not taken into account. The existing methodologies take into account durability distribution law which is considered to be known, thus enabling probability calculations. This particular methodology considers fatigue crack growth parameters, emergence of crack and its development. Crack parameters are estimated during acceptance fatigue tests, where it is decided either to allow aircraft for service or to send it for redesign. Minimax approach then can be applied to obtain the required level of reliability in service for random cracks.

Problem of when to replace a piece of capital equipment that deteriorates with time is also considered. It is generalization of economical safe-life nomination problem. Safe-life is uneconomical approach by its nature and it is important to find the most profitable service time of an aircraft. On the other hand, reliability requirements set strong limitation of the levels of fatigue failure rate for fatigue-prone airframe. Both conditions should be respected for the correct safe-life nomination.

Existing solutions do not take into account information exchange about the discovery of any fatigue crack in fleet and aircraft service beginning rate. The second task solution, opposed to the solution offered, for example by Howard, is optimization of economic effectiveness of the system (conditionally called as airline) considered under the limitation of system failure rate (number of failures per time unit).

The Aim of the Research

The aims of the research are to develop a methodology for development of inspection programs of fatigue-prone airframes of the aircraft fleet and to develop a methodology for determining when to replace a piece of capital equipment that deteriorates with time.

The Tasks of the Research

In order to achieve the aims, it is necessary to perform the following tasks:

- To develop the method for aircraft fleet failure probability calculation considering:
    a) information exchange about the fatigue crack discovering;
    b) aircraft service beginning rate;
    c) human factor;
    d) crack detection probability function on crack size.
- To perform similar mathematical statistics task analysis when instead of parameters that define the reliability of aircraft fleet estimations of those parameters are used. Parameter estimations are obtained during processing of acceptance fatigue test data.
- To develop the method for inspection program planning for the fleet of aircraft based on previous task solution results.
- To develop the method of airline service gain and fatigue failure rate calculation.
- To develop the method of choice of the most effective economical decision under the limitation of fatigue failure rate.
- To create modular software for developed method implementation.
- To calculate and analyse numerical examples.

Methods of the Research

The Doctoral Thesis has been developed applying the following analytical methods:

- The theory of fatigue crack growth in airframe constructions;
- The theory of probability;
- Mathematical statistics;
- Monte-Carlo method;
- Controlled Markov chain with rewards and Howard's algorithm;
- Visually oriented programming

Thesis Statements

In the present research, the aircraft fleet inspection program planning and replacement time nomination methods have been proposed. The fatigue crack growth parameters have been calculated using data of acceptance full-scale fatigue tests of airframe.

The methodology is based on the results of Monte-Carlo simulations, controlled Markov chains with reward, Howard's algorithm and minimax approach.

Numerical computing environment Matlab has been used for creation of computer program for necessary calculations. This program can be used to conduct scientific research and implement university study process.

Scientific Novelty

- Existing methods of fatigue-prone aircraft reliability provision have several disadvantages:
  a) Failure probability of aircraft fleet is considered to be failure probability of aircraft with the smallest fatigue durability. At the same time, fatigue crack detection in the set of aircraft in fleet is considered to be independent events. In existing solutions it is not taken into account that after detection of at least one crack in the whole fleet appropriate maintenance is ensured to each aircraft in order to prevent a fatigue failure.
  b) Aircraft service beginning rate is not taken into account. This means that existing calculations assume simultaneous aircraft service beginning.
  c) It has not been considered that planned inspection of fatigue-prone airframe is made with some probability. The human factor is not considered.
  d) Crack detection probability function on a crack size is step-like.

  In the present research, all of the mentioned disadvantages have been solved.
- The task of choosing the optimal time of outdated aircraft replaced with the aircraft with smaller operating time with the aim of infinite system (airline) service has been considered. In the literature solutions maximum economic effectiveness is obtained but failure rate limitation is not considered. Task solution has been presented considering such limitation.

Research Results

- The method of developing the inspection program for a fleet of aircraft under reliability requirements has been proposed taking into account the result of acceptance full-scale fatigue test. This solution provides required reliability of aircraft fleet independently of unknown parameter of fatigue life distribution.

- The task of choosing the optimal time to replace outdated aircraft with the aircraft whose operating time is shorter with the aim of infinite system (airline) service has been given.
- A set of Matlab scripts that can be used for further scientific investigations of considered problems and for the educational purposes has been developed.

Practical Relevance of the Research

The developed models of aircraft fleet reliability fail-safe approach and aircraft safe-life approach can be used to develop an inspection program of aircraft fleet and to determine a service life or replacement time of an aircraft. The following aspects have been taken into account:

- Results of acceptance full-scale fatigue tests;
- The required reliability of aircraft fleet or allowed fatigue failure rate of aircraft;
- Economic parameters: fatigue failure cost, service expenses, gain of successful service, cost of new aircraft and others.

The Approbation of Research Results

The main results of the research have been presented in the following

International Scientific Conferences:

1. Reliability and Statistics in Transportation and Communication (RelStat 12), Latvia, Riga, 17–20 October 2012, "Reliability of Fleet of Aircraft", S. Tretyakov, Yu. Paramonov.
2. Fifth International Conference on Scientific Aspects of Unmanned Mobile Objects (SAUMO), Poland, Deblin, 15–17 May 2013, "Reliability of Aircraft Fleet and Airline", S. Tretyakov, M. Hauka, Yu. Paramonov.
3. Seventh International Workshop on Simulation, Italy, Rimini, 21–25 May 2013, "Minimax Decision for Reliability of Aircraft Fleet and Airline", Yu. Paramonov, M. Hauka, S. Tretyakov.
4. 54th International Scientific Conference of Riga Technical University, Latvia, Riga, 14–16 October 2013, "Minimax Decision for Reliability of Aircraft Fleet with and without Information Exchange", S. Tretyakov, Yu. Paramonov.
5. Reliability and Statistics in Transportation and Communication (RelStat 13), Latvia, Riga, 16–19 October 2013, "Planning of Inspection Interval to Provide Reliability of Fatigue-Prone Aircraft Using Result of Acceptance Fatigue Test", Yu. Paramonov, M. Hauka, S. Tretyakov.

6. Eighth International Conference on Modelling in Industrial Maintenance and Reliability (MIMAR), England, Oxford, 10–12 July 2014, “Minimax Inspection Program for Reliability of Aircraft Fleet and Airline”, M. Hauka, S. Tretyakov, Yu. Paramonov.
7. 55th International Scientific Conference of Riga Technical University, Latvia, Riga, 14–17 October 2014, “Planning of Inspection Interval for Aircraft Fleet”, S. Tretyakov, Yu. Paramonov.
8. Eighth International Workshop on Simulation, Austria, Vienna, 21–25 September 2015, “Modelling of Reliability of Aircraft Fleet and Airline. P-set and λ-Set Functions”, Yu. Paramonov, S. Tretyakov, M. Hauka.
9. 56th International Scientific Conference of Riga Technical University, Latvia, Riga, 14–16 October 2015, “Modelling of Reliability of Aircraft Fleet and Airline”, S. Tretyakov, Yu. Paramonov.

The research results (papers) have been published in

Scientific and Technical Proceedings:

1. Tretyakov S., Paramonov Yu. Reliability of Fleet of Aircraft // Proceedings of the 12th International Conference on “Reliability and Statistics in Transportation and Communication” (RelStat 12), 17–20 October 2012, Riga, Latvia, pp. 116–121. ISBN 978-9984-818-49-8.
2. Paramonov Yu., Tretyakov S. Reliability of Fleet of Aircraft Taking into Account Information Exchange about the Discovery of Fatigue Cracks and the Human Factor // AVIATION. 2012, Vol. 16(4), pp. 103–108. ISSN 1648-7788 DOI: 10.3846/16487788.2012.753680.
3. Tretyakov S., Hauka M., Paramonov Yu. Reliability of Aircraft Fleet and Airline // Proceedings of the 5th International Conference on Scientific Aspects of Unmanned Mobile Objects, 15–17 May 2013, Deblin, Poland, pp. 86–88. ISBN 978-83-63792-28-2.
4. Paramonov Yu., Hauka M., Tretyakov S. Minimax Decision for Reliability of Aircraft Fleet and Airline // Book of Abstracts of Seventh International Workshop on Simulation, 21–25 May 2013, Rimini, Italy, pp. 285–286. ISSN 1973-9346.
5. Paramonov Yu., Hauka M., Tretyakov S. Planning of Inspection Interval to Provide Reliability of Fatigue-Prone Aircraft Using Result of Acceptance Fatigue Test // Proceedings of the 13th International Conference on “Reliability and Statistics in

Transportation and Communication" (RelStat 13), 16–19 October 2013, Riga, Latvia, pp. 39–47. ISBN 978-9984-818-58-0.

6. Hauka M., Tretyakov S., Paramonov Yu. Minimax Inspection Program for Reliability of Aircraft Fleet and Airline // Proceedings of 8th International Conference on Modelling in Industrial Maintenance and Reliability (MIMAR), 10–12 July 2014, Oxford, England, pp. 120–124.
7. Paramonov Yu., Tretyakov S., Hauka M. Inspection Program Development for an Aircraft Fleet and an Airline on the Basis of the Acceptance Fatigue Test Result // Transport and Telecommunication, Vol. 16, no 1, 2015, pp. 1–8. DOI 10.1515/ttj-2015-0001.
8. Paramonov Yu., Tretyakov S., Hauka M. Fatigue-Prone Aircraft Fleet Reliability Based on the Use of a P-set Function // Reliability: Theory & Applications, #01 (36), Vol. 10, March 2015, pp. 40–49. ISSN 1932-2321.
9. Paramonov Yu., Tretyakov S., Hauka M. Binary Lambda-set Function and Reliability of Airline // Reliability: Theory & Applications, #03 (38), Vol. 10, September 2015, pp. 37–42. ISSN 1932-2321.
10. Paramonov Yu., Tretyakov S., Hauka M. Modelling of Reliability of Aircraft Fleet and Airline. P-set and λ-set Functions // Submitted to Proceedings of Eighth International Workshop on Simulation, 21–25 September 2015, Vienna, Austria.

The Structure of the Doctoral Thesis

The Doctoral Thesis consists of the introduction, 4 chapters, conclusions, bibliography with 61 reference sources and appendices (46 pages). It has been illustrated by 49 figures and 6 tables. The total volume of the Doctoral Thesis is 140 pages.

- Introduction — the summary of fatigue failure accidents and main methods to prevent them.
- Chapter 1 — metal fatigue history is observed. Problem solution methodology is developed in parallel with air crashes caused by metal fatigue.
- Chapter 2 — understanding of failure and development of maintenance strategies to prevent failures. Non-destructive testing and Maintenance Steering Group development are observed. Reliability of maintained systems is studied. Maintenance concepts and strategies, such as clock based, age based or condition based, are studied. All of the methods have advantages and disadvantages. Exploration of this field helps make a better choice of maintenance strategy for fatigue-prone airframe.

- Chapter 3 — it examines model of fatigue crack growth and estimates this model parameters. The methodology is developed for inspection frequency planning for known parameters of the crack. Minimax approach is used to obtain required level of reliability for unknown crack parameters. Numerical examples are calculated to illustrate the influence of different parameters on a reliability of fleet of aircraft.
- Chapter 4 — development of methodology for the most profitable safe-life determination under the limitation of fatigue failure rate. Numerical example is calculated to illustrate the operation of the method.
- Conclusions — main results of research are observed and main directions of further investigations are proposed.
- Appendices — probability of any fatigue failure in the fleet of aircraft for the case of part durability normal distribution is considered. Printout of Matlab scripts of the developed modular software.

THE SCOPE OF RESEARCH

Chapter 1. History of the Problem

Chapter 1 makes an overview of fatigue problems in aviation and proposes problem solutions. According to research “A Survey of Serious Aircraft Accidents Involving Fatigue Fracture” by G. S. Campbell and R. Lahey: “A total of 1885 accidents since 1927 to 1981 were identified as having fatigue fractures as a related cause, and these accidents resulted in 2240 deaths”. According to information provided by the National Transportation Safety Board (NTSB), on the territory of the United States there were 1114 accidents caused by fatigue which resulted in 536 deaths from 1982 to 2015.

The first important investigation was made after the Comet (during 1953 and 1954) catastrophes. The first approach to the fatigue problem, which developed, was called a safe-life approach. In general, this requires that all the parts of the structure, the failure of which could result in loss of the aircraft, are to be able to remain safely in use for a predetermined retirement life. Later fail-safe and damage tolerance approaches were developed. Some structurally significant items have to be inspected with the aim to discover a fatigue crack. Unfortunately, none of those approaches is ideal and fatigue failures still are a part of aviation, as an example fatigue cracks were discovered on the wing ribs of Airbus in 2011. There are many studies devoted to metal fatigue problem and development of inspection programs. For example, studies by B. Lundberg, F. H. Hooke, J. N. Yang, E. L. Zimont, V. Ya. Senik, V. V. Nikonov, V. S. Strelayev, N. N. Smirnov, V.S. Shapkin, V. M. Baykov, G. I. Nestrenko, H. B. Kordonsky, J. A. Martinov, G. S. Locmanov and others.

All these studies take into account the random nature of fatigue crack growth model parameters and inspection techniques; nonetheless, final decision-making procedure and, chiefly, possibility of project redesign may effect a failure probability. The development of mathematical model in a given direction can be found in research by Yu. M. Paramonov and P. M. Sobolev. Investigation was continued by N. M. Kimlik, A. Kuznetsov, K. Nechval and M. Hauka.

The present study continues the research on the modernisation of the model by including a possibility of developing an inspection program not just for a single aircraft or airline but also for the fleet of aircraft with different aircraft service beginning rate. Possibility of the most profitable safe-life calculation under the limitation of fatigue failure rate has also been developed.

Chapter 2. Reliability of Maintained Systems and Maintenance Strategies

Chapter 2 explains the development of equipment failure and development of hard time on the basis of condition monitoring maintenance policies. It is clear that structurally significant items of aircraft should be inspected. It is necessary to use non-destructive tests to discover possible fatigue crack. Maintenance and examination regulations were necessary and Maintenance Steering Group was developed (MSG-1, MSG-2 and MSG-3). MSG-3 process generally produces high safety standard due to the intelligent approach to maintenance in terms of effective task selection.

Next part describes preventive maintenance policies and their mathematical models. Replacement policy requires preventive replacement due to different reasons or corrective replacement due to item failure. It is assumed that the replaced item has the same failure distribution function as a new one.

Age replacement policy assigns replacement of an item (component, system) when specified operational age t_0 is reached or in the case of failure. This policy makes sense if the failure rate of item is increasing with operational time and failure replacement cost is noticeably higher than cost of planned replacement. Value t_0 could be determined by the cost criterion or by the availability criterion.

Block replacement policy assigns replacement of item (component, system) at regular time intervals ($t_0, 2t_0, 3t_0, ...$) regardless of age. The main advantage of block replacement policy is administrative simplicity. Calendar time since last replacement should be monitored only as a substitute for the operational time since last replacement for age replacement policy. On the other hand, new items may be replaced at planned replacement times. This policy makes sense if a large number of the same type items in service should be maintained. Age replacement policy cost criterion can be used for maximum gain acquisition or availability criterion can be used to maximise the stationary availability.

Condition-based maintenance also known as predictive maintenance is a maintenance policy where the maintenance action is decided based on the results of item degradation predictors. Predictor is a variable that determines the condition of an item (component, system). It can be physical variables such as pressure, temperature, electrical resistance and so forth, system performance variables or variables related to the residual life of the item. This policy makes sense if a failure rate of an item is increasing with operational time.

Condition-based maintenance requires a mathematical model of system deterioration process. Item monitoring with required variable measurement is necessary to predict failure of an item. It is necessary to understand the nature of failure to select the correct type of maintenance action and the date of the action. A decision is based on a measured variable value. An action is required when a variable value passes a predefined threshold limit. Different predefined threshold values correspond to different maintenance decisions and this kind of policy is called a control limit policy.

Chapter 3. Inspection Program Development

Chapter 3 describes the methodology of inspection program development in order to ensure the required reliability level.

3.1 Initial Data

It is assumed that in the interval when crack can be detected till moment when it reaches its critical size the fatigue crack growth can be approximated as follows

$$a(t) = \alpha \exp(Qt), \tag{3.1}$$

where

α — equivalent initial crack size;

Q — crack growth speed in logarithmic scale.

Estimations of those parameters can be obtained from acceptance full-scale fatigue test results using the regress analysis. In the present research, traditional assumptions are used that can be found in the research by M. Hauka.

3.2 Calculation of Probability of Fatigue Failure of One Aircraft for the Known θ

For the known θ, there are two decisions: 1) new type of the aircraft is good enough and the operation of this aircraft type can be allowed, 2) the operation of the new type of aircraft is not allowed and the redesign of aircraft should be made. In the case of the first decision, the vector $t = (t_1, ..., t_n)$, where t_i is the time moment of *i*-th inspection, should also be defined. If θ is known the different rules can be offered for the choice of structure of the vector t: 1) every interval between the inspections is equal to the constant $d_t = t_{SL} / (n + 1)$, where t_{SL} is the aircraft specified life (SL) (the retirement time), n is a number of inspections, 2) the conditional probabilities of a failure (under condition that the fatigue failure does not take place in the previous interval) in every interval are equal to the same value $P(T_c < t_{SL}) / (n + 1)$… In the present research, it is supposed that the first type of the choice and the vector t are defined by the fixed t_{SL} and the choice of n.

For the substantiation of the choice of the inspection number we should know fatigue crack detection probability as a function of crack size a. We suppose that this probability is defined by the equation

$$p_d\left(a\right)=w_0 w(a),\quad w(a)=\begin{cases}0, & \text{if } a\le a_{d0},\\ \dfrac{a-a_{d0}}{a_{d1}-a_{d0}}, & \text{if } a_{d0}<a<a_{d1},\\ 1, & \text{if } a\ge a_{d1}.\end{cases}\tag{3.2}$$

where a_{d0}, a_{d1} are some constants,

w_0 can be considered to be probability to carry out the planned inspection (human factor).

If the number of cracks observed is sufficiently large, it is possible to accept the part of missed cracks among all the cracks in the series as the estimate of probability of failure for a particular inspection program.

In the present research, human factor w_0 and probability are taken into account to detect a crack according to (4.2). The corresponding value of fatigue failure probability for fixed inspection number n for specific aircraft is

$$p_f\left(n\right)=\prod_{i=1}^{n}\left(1-p_{di}\left(a_i\right)\right),\tag{3.3}$$

where p_{di} is crack detection probability of i-th inspection.

Then we can calculate the mean value of failure probability as, for example, a function of inspection number n

$$\widehat{p}_f\left(n\right)=\frac{\sum_{i=1}^{N}p_{fi}\left(n\right)}{N}.\tag{3.4}$$

where p_{fi} is failure probability of i-th aircraft.

3.3 Probability of Any Fatigue Failure in the Fleet of Aircraft for the Known θ

Let us consider the case when the operation of all N aircraft will be stopped if any fatigue crack is detected. In order to limit the probability of fatigue failure in the fleet it is enough to find at least one fatigue crack before the failure of any aircraft in the fleet takes place.

For the case when the probability of fatigue crack detection is defined by (3.2) and $a_{d0}=a_{d1}$ the corresponding probability is equal to the expected value of random variable

$$P_{fNW} = (1 - w_0)^R, \quad (3.5)$$

where R is the total random number of inspections before the first failure in the whole fleet. Let t_k^+, $t_{k-1}^+ < t_k^+$, $t_0^+ = 0$ be the "calendar" time moment when k-th aircraft begins the service, $T_{dk}^+ = t_k^+ + T_{dk}$, $T_{ck}^+ = t_k^+ + T_{ck}$, $k = 1, 2, ..., N$ be the random calendar time moments when fatigue crack can be discovered and fatigue failure of aircraft takes place respectively, (Fig. 1). And let $K_{SL} = \{k : T_{ck} < t_{SL},\ k = 1, 2, ..., N\}$ be a set of indices of aircraft, the failure of which can take a place, if an inspection will not take the place, $T_f^+ = \min\{T_{fk}^+ : k \in K_{SL}\}$, $T_{fk}^+ = \min\{T_{ck}^+,\ T_f^+\},\ k \in K_{SL}$,

$$R = \sum\nolimits_{k \in K_{SL}} R_k, \quad (3.6)$$

where $R_k = \max(\{[(T_{fk}^+ - t_k^+) / d_t] - [(T_{dk}^+ - t_k^+) / d_t]\}, 0),\ k \in K_{SL}$, is the random inspection number of k-th aircraft from the set K_{SL} if inspection interval $\Delta = t_{SL} / (n+1)$ (it is supposed to be a specific "calendar" schedule of the inspections for each aircraft: $i = 1, 2, ..., n+1$, $k \in K_{SL}$).

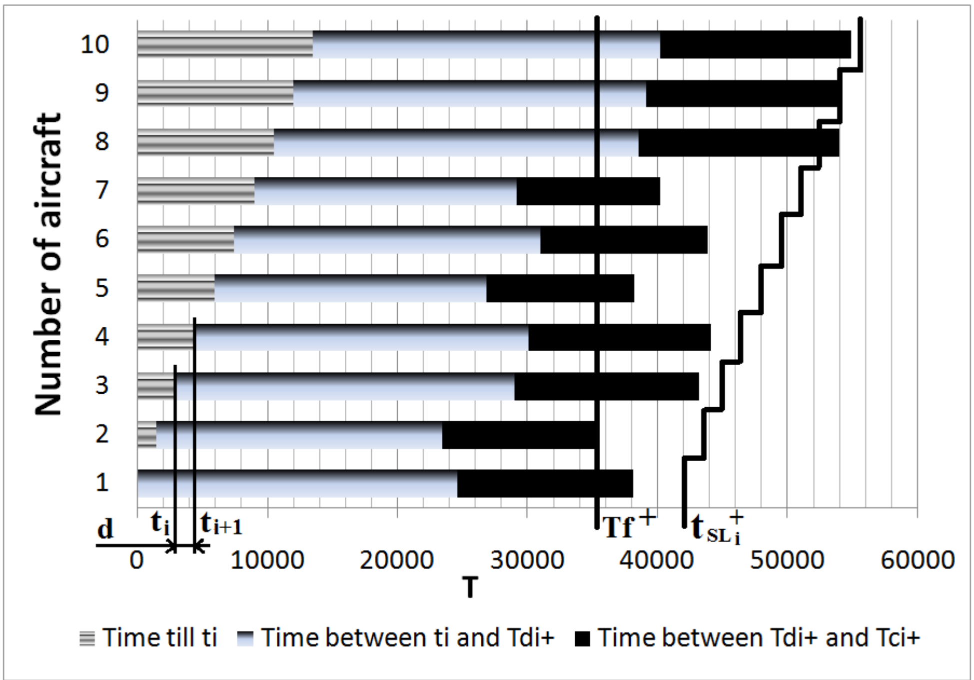

Fig. 1. Operation of N aircraft $a_{d0} = a_{d1}$.

Random variable Q is a speed of fatigue crack growth in logarithmic scale. It has specific realisation for each aircraft and $Q_1, \ldots, Q_N$ are independent random variables. Thus, a mean value of random probability of failure in the fleet is

$$E(P_{fNW}) = p_{fNW}(n, \theta) = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} (1 - w)^{r(q)}\, dF_{Q_1}(q_1) \ldots dF_{Q_N}(q_N), \quad (3.7)$$

where $q=(q_1,\ldots,q_N)$, $r(q)$ is realisation of random variable *R*.

For large number *N* the Monte-Carlo method is appropriate for the calculation of p_{fNW}. If this function is known then the number of inspections, $n(p,\theta)$, required to limit the aircraft fleet fatigue failure probability by value p is defined by the function $n(p,\theta)=\min(r: p_{fNW}(r,\theta)\le p \text{ for all } r>n(p,\theta),\ r=1,2,\ldots)$.

For general definition of the function $p_d(a)$ it is necessary to make a more detailed analysis. Operation of N aircraft is shown on Fig. 2.

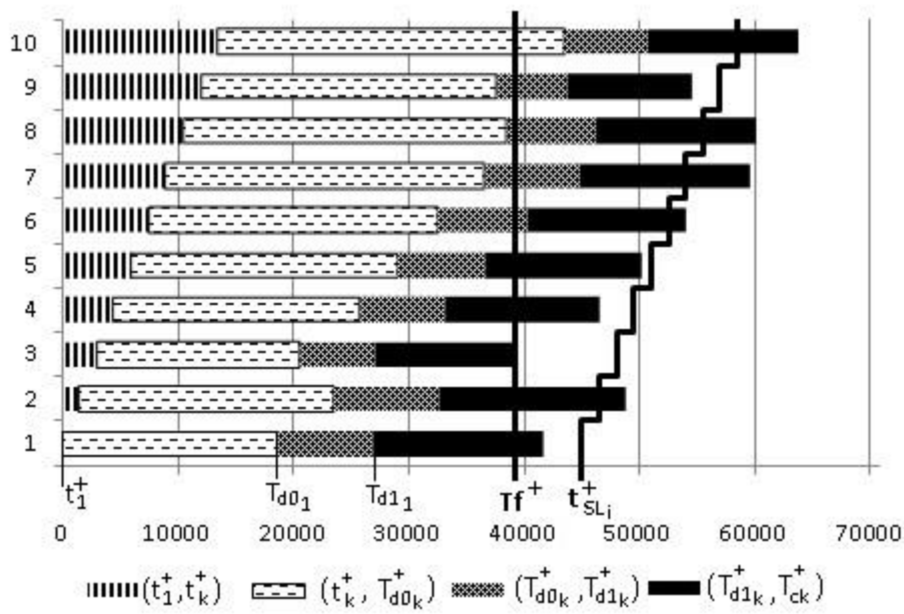

Fig. 2. Operation of N aircraft $a_{d0} \neq a_{d1}$.

Let us denote by t_{kj}^+ the calendar time of j-th inspection of k-th aircraft $k\in K_{SL}$, $j\in J_k$, $k\in K_f$, where $J_k=\{j: t_{kj}^+ < T_{fk}^+, j=1,2,\ldots\}$ is the set of indices of inspections of k-th aircraft; $t_{kj}^+=t_k+jd_t, j=1,2,\ldots$, $K_f=\{k: k\in K_{SL}, t_k^+ < T_f^+\}$.

Let $q=(q_1,\ldots,q_N)$ be realisation of random vector $Q=(Q_1,\ldots,Q_N)$ and let $a_{jk}=\alpha\exp(q_k d_t j)$ be a size of fatigue crack at j-th inspection of k-th aircraft for which the growth of fatigue crack is defined by specific value of q_k which is the realisation of random variable Q_k. The respective probability of fatigue crack discovery is equal to $p_{dkj}=p_d(a_{jk})$. For a specific value of vector q probability that fatigue crack will not be discovered is equal to

$$p_f(q)=\begin{cases}0, \text{ if } K_f=\varnothing,\\ \prod_{k\in K_f}\prod_{j\in J_k}\left(1-p_{dkj}\right).\end{cases} \quad (3.8)$$

By modelling random vector Q using Monte-Carlo method, we can calculate a mean value of this probability: $p(n,\theta)=E_\theta\left(p_f(q)\right)$. Now we can choose the number of the inspection $n(p,\theta)$ in such a way that the failure probability is equal to p.

3.4 Solution to Unknown θ, p-set Function

In real circumstances, it is difficult to obtain estimation of θ parameter with more or less reliable confidence interval. A small number of items can be tested in laboratory but it is not enough. In this chapter, a solution will be found for the case when θ is considered to be an unknown parameter.

3.4.1 Solution to Unknown θ for One Aircraft

First, we consider the problem of limitation of fatigue failure probability in the operation of one aircraft if the probability of detection is defined by (3.2), $a_{d0}=a_{d1}$, and the human factor $w_0=1$. This means that if there is a detectable fatigue crack, then during the inspection after T_d we discover it with probability 1 and the limitation of fatigue failure probability of aircraft is provided by the choice of the specific p-set function. Let us take into account that the operation of a new type of aircraft will not take place if the result of acceptance fatigue test in a laboratory is "too bad" (previously, the redesign of the new type of aircraft should be made). We say that in this case the event $\hat{\theta}\notin\Theta_0$, $\Theta_0\subset\Theta$ takes place (for example, $\hat{\theta}\notin\Theta_0$ if fatigue life T_c is lower than some limit; or $n(p,\hat{\theta})$ is too large,…). Let us define some set function:

$$S\left(\hat{\theta},\Theta_0,n\right)=\begin{cases}\bigcup_{i=1}^{n+1}S_i(n) \text{ if } \hat{\theta}\in\Theta_0\\ \varnothing \text{ if } \hat{\theta}\notin\Theta_0\end{cases}, \quad (3.9)$$

where $S_i=\left\{\left(t_d,t_c\right):t_{i-1}<t_d,t_c\le t_i\right\}$,

$t_i=\dfrac{it_{SL}}{n+1}$, $i=1,2,\ldots,n+1$,

$\varnothing$ is an empty set.

This function is called a binary p-set function if

$$\sup_\theta\sum_{i=1}^{n+1}P\left(Z\in S_i(n)\bigcap\hat{\theta}\in\Theta_0\right)=p. \quad (3.10)$$

Here it is taken into account that if $\hat{\theta} \notin \Theta_0$ operation of aircraft is not allowed, the failure probability is equal to 0. Example of value of binary p-set functions is shown in Fig. 3.

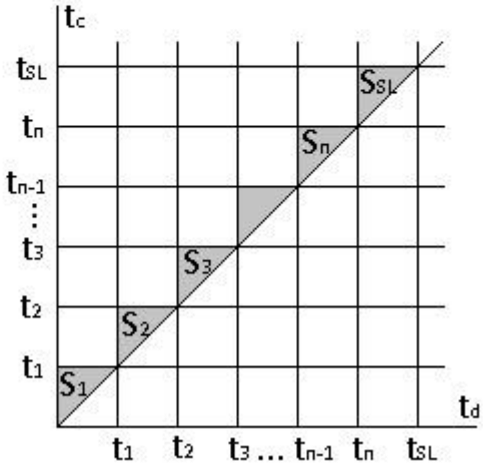

Fig. 3. Example of a "value" p-set function for one aircraft.

It can be shown that for a very wide range of the definition set Θ_0 and the requirements to limit aircraft fatigue failure probability by the value p^*, where $(1-p^*)$ is required reliability, there is a preliminary "designed" choice of allowed aircraft fatigue failure probability, p_{fD}, such that the corresponding set function $S\left(\Theta_0, n\left(p_{fD}, \hat{\theta}\right)\right)$ is p-set function of the level p^* for the vector $Z = (T_d, T_c)$. The value of p_{fD} is defined as follows

$$\sup_\theta \sum_{i=1}^{n+1} P\left(Z \in S_i\left(n\left(p_{fD}, \hat{\theta}\right)\right) \bigcap \hat{\theta} \in \Theta_0\right) = p. \tag{3.11}$$

For this p_{fD} the aircraft failure probability will be limited by the value p^* for any unknown $\theta \in \Theta$.

The domain of a p-set function is a set of possible samples $(x_1, x_2, ..., x_n)$ that are used for estimation of $\hat{\theta}$ parameter. The range of p-set functions is a set of samples (triangles): $\{t_i \le t_d, t_c < t_{i+1}, i = 1, 2, ..., n\}$. There is a failure if (T_d, T_c) vector hits one of those triangles. Possibility of such an event does not exceed the allowable failure probability p^*.

3.4.2. Solution to Unknown θ for the Fleet of Aircraft

Now we consider the reliability of the fleet of N aircraft when there is an information exchange and the operation of all aircraft will be stopped if fatigue crack is found during the inspection of any aircraft and, as it has already been discussed, in order to prevent the failure in the fleet, it is enough to find at least one fatigue crack before the failure of any aircraft in the fleet takes place. Let us define some multiple set function:

$$S^{+}\left(\hat{\theta},\Theta_0,n\right)=\bigcup\nolimits_{k\in K_{SL}} S_k^{+}\left(\hat{\theta},\Theta_0,n\right), \qquad (3.12)$$

where

$$S_k^{+}\left(\hat{\theta},\Theta_0,n\right)=\begin{cases}\bigcup_{i=1}^{n+1} S_{i,k}\left(n\right) \text{ if } \hat{\theta}\in\Theta_0 \\ \varnothing, \, if \;\hat{\theta}\notin\Theta_0\end{cases}, \qquad (3.13)$$

where $S_{i,k}^{+}\left(n\right)=\left\{\left(t_{dk}^{+},t_{ck}^{+}\right): t_{(i-1)k}<t_{dk},t_{ck}\le t_{ik}\right\}$,

$t_{ik}^{+}=t_k^{+}+t_i$,

$t_i=\dfrac{it_{SL}}{n+1}$, $i=1,2,...,n+1$, $k=1,2,...,N$.

Again, it can be shown that for a very wide range of the definition set Θ_0 and the requirements to limit aircraft fleet fatigue failure probability by the value p^*, there is a preliminary “designed” choice of allowed aircraft fleet fatigue failure probability, p_{fD}, such that corresponding multiple set function $S^{+}\left(\Theta_0,n\left(p_{fD},\hat{\theta}\right)\right)$ is p-set function of the level p^* for the set of vectors $\left\{Z_k^{+},k\in K_{SL}\right\}$, where $Z_k^{+}=\left(T_{dk}^{+},T_{fk}^{+}\right)$:

$$v\left(p_{fD}\right)=p^*, \qquad (3.14)$$

where

$$v\left(p\right)=\sup\nolimits_{\theta} v\left(\theta,p\right), \qquad (3.15)$$

$$v\left(\theta,p\right)=E\left\{\sum_{k\in K_{SL}}\sum_{i=1}^{n+1}P\left(Z_k^{+}\in S_{ik}^{+}\left(n\left(p,\hat{\theta}\right)\right)\bigcap\hat{\theta}\in\Theta_0\right)\right\}. \qquad (3.16)$$

That means that fatigue failure of aircraft fleet will be limited by the value p^* for any unknown θ. An example of set of “values” of a p-set function for N aircraft with different starting periods of operation is shown in Fig. 4.

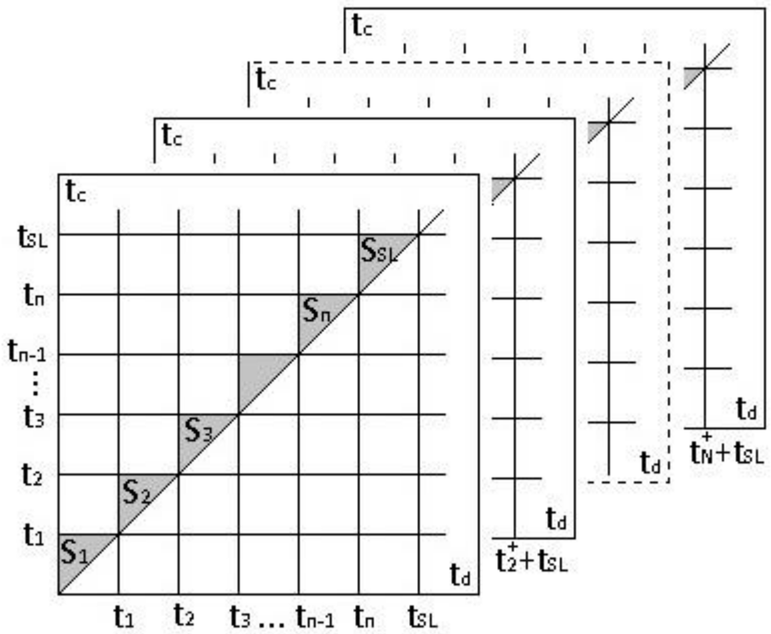

Fig. 4. Set of "values" of a p-set function for N aircraft with different starting periods of operation.

3.5. Probability of Any Fatigue Failure in the Fleet of Aircraft for the Unknown θ

First, let us consider the problem of limitation of fatigue failure probability in the operation of one aircraft if the probability of detection is defined by (4.2), $a_{d0} = a_{d1}$ and the human factor $w_0 = 1$. This means that if there is a detectable fatigue crack, then during the inspection after T_d it is discovered with probability 1 and the limitation of fatigue failure probability of aircraft is provided by the choice of the specific p-set function. Let us repeat that the operation of a new type of aircraft will not take place if the result of acceptance fatigue test in a laboratory is "too bad" (previously, the redesign of the new type of aircraft should be made). It is assumed that in this case the event $\hat{\theta} \notin \Theta_0$, $\Theta_0 \subset \Theta$ takes place (for example, $\hat{\theta} \notin \Theta_0$ if fatigue life T_c is lower than some limit; or $n(p,\hat{\theta})$ is too large,…). Let us define some set function

$$S(\Theta_0, n) = \begin{cases} \bigcup_{i=1}^{n+1} S_i(n) \text{ if } \hat{\theta} \in \Theta_0, \\ \varnothing, \ if \ \ \hat{\theta} \notin \Theta_0. \end{cases} \tag{3.17}$$

where $S_i = \{(t_d, t_c): t_{i-1} < t_d, t_c \le t_i\}$, $t_i = it_{SL}/(n+1)$, $i = 1,\dots,n+1$; $\varnothing$ is an empty set. Examples of p-set functions are shown in Fig. 4.

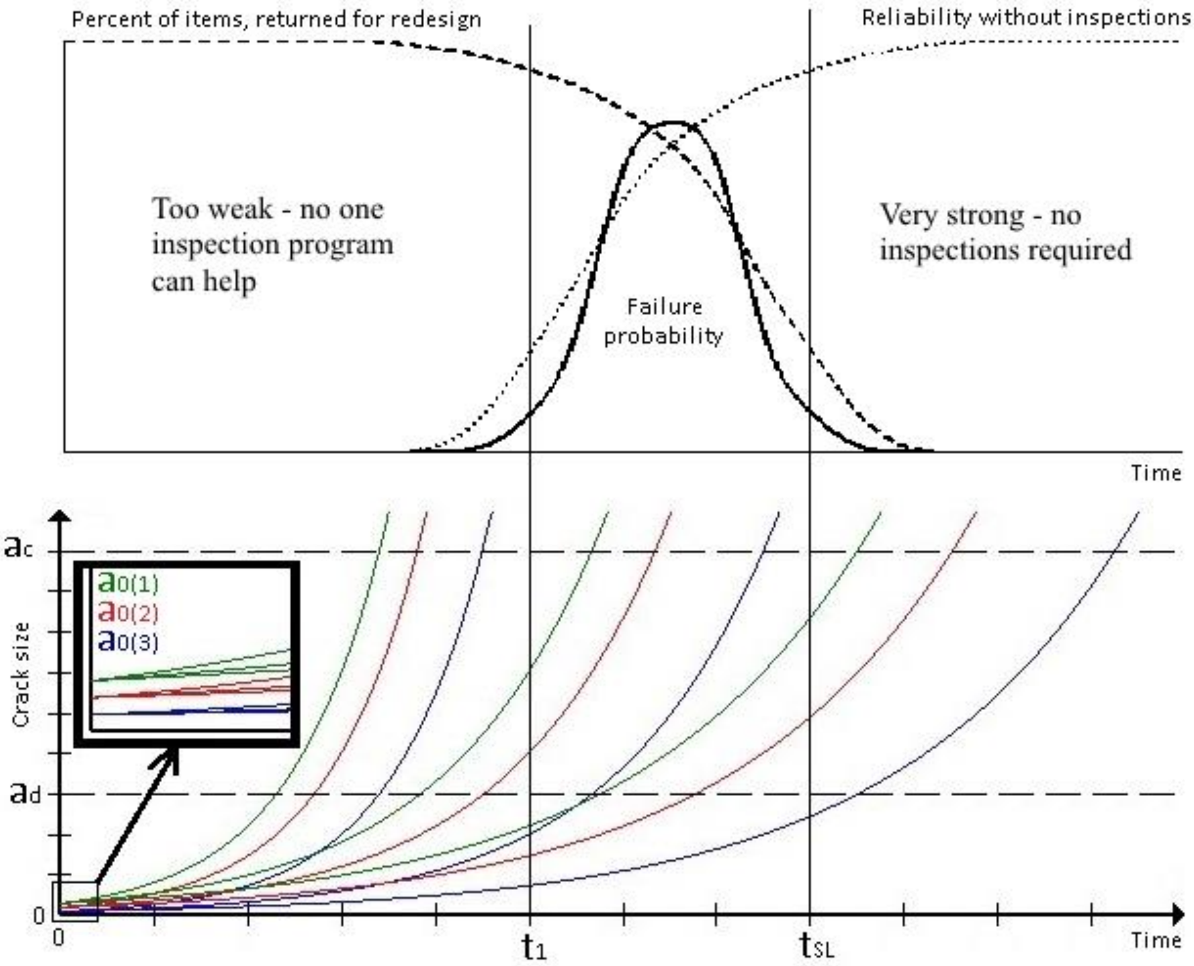

Fig. 5. Minmax approach.

It can be shown that for a very wide range of the definition set Θ_0 and the requirements to limit aircraft fatigue failure probability by the value p^*, where $(1-p^*)$ is a required reliability, there is a preliminary "designed" choice of allowed aircraft fatigue failure probability, p_{fD}, such that corresponding set function $S(\Theta_0, n(p_{fD}, \hat{\theta}))$ is p-set function of the level p^* for the vector $Z=(T_d, T_c)$. The value of p_{fD} is defined as follows

$$\sup_\theta \sum_{i=1}^{n+1} P(Z \in S_i(n(p_{fD}, \theta)) = p^* \quad (3.18)$$

For this p_{fD} the aircraft fatigue failure probability will be limited by the value p^* for any unknown $\theta \in \Theta$.

Now let us consider the reliability of the fleet of N aircraft when there is an information exchange and the operation of all aircraft will be stopped if fatigue crack is found during the inspection of any aircraft and, as it has already been discussed, in order to prevent the failure in the fleet, it is enough to find at least one fatigue crack before the failure of any aircraft in the fleet takes place. Let us define some multiple set function:

$$S^+(\Theta_0, n) = \bigcup_{k \in K_{SL}} S_k^+(\Theta_0, n) \ , \tag{3.19}$$

where

$$S_k^+(\Theta_0, n) = \begin{cases} \bigcup_{i=1}^{n+1} S_{i,k}^+(n) \ \text{ if } \ \hat{\theta} \in \Theta_0, \\ \varnothing, \ \ if \ \ \hat{\theta} \notin \Theta_0, \end{cases} \tag{3.20}$$

$S_{ik(n)}^+ = \{(t_{dk}^+, t_{ck}^+) : t_{(i-1)k} < t_{dk}, t_{ck} \le t_{ik}\}$, $\quad t_{ik}^+ = t_k^+ + t_i$, $\quad t_i = it_{SL} / (n+1)$, $\quad i = 1, ..., n+1$, $k = 1, 2, ..., N$. Again, it can be shown that for a very wide range of the definition set Θ_0 and the requirements to limit aircraft fleet fatigue failure probability by the value p^*, there is a preliminary "designed" choice of allowed aircraft fleet fatigue failure probability, p_{fD}, such that corresponding multiple set function $S^+(\Theta_0, n(p_{fD}, \hat{\theta}))$ is p-set function of the level p^* for the set of vectors $\{Z_k^+, \ k \in K_{SL}\}$, where $Z_k^+ = (T_{dk}^+, T_{fk}^+)$:

$$v(p_{fD}) = p^* \tag{3.21}$$

where

$$v(p) = \sup_{\theta} v(\theta, p) \tag{3.22}$$

$$v(\theta, p) = E\left\{ \sum_{k \in K_{SL}} \sum_{i=1}^{n+1} P(Z_k^+ \in S_{ik}^+(n(p, \hat{\theta}))) \right\} \tag{3.23}$$

That means that aircraft fleet fatigue failure probability will be limited by the value p^* for any unknown θ.

Numerical Example

In this numerical example we assume that $t_{SL} = 50000$, $w_0 = 0.95$, processing the result of full-scale fatigue test we get the estimate of fatigue crack parameters $\hat{\theta} = -8.5885$, $\alpha = 0.286$ mm, the standard deviation of log(Q) is equal to 0, and for considered inspection technology the detection probability p_d is defined by (3.5) with $a_{d0} = 10$ mm, $a_{d1} = 20$ mm, $a_c = 237$ mm. There are 10 aircraft in the fleet, the interval between the aircraft put into operation $\Delta = 1000$; allowed failure probability $p^* = 0.01$, the set Θ_0 is defined by the condition: if $\hat{n} = n(0.01, \hat{\theta}) > 20$ then the redesign of aircraft should be made. Using the Monte Carlo calculation we get $\hat{n} = n(0.01, \hat{\theta}) = 6$ (Fig. 6).

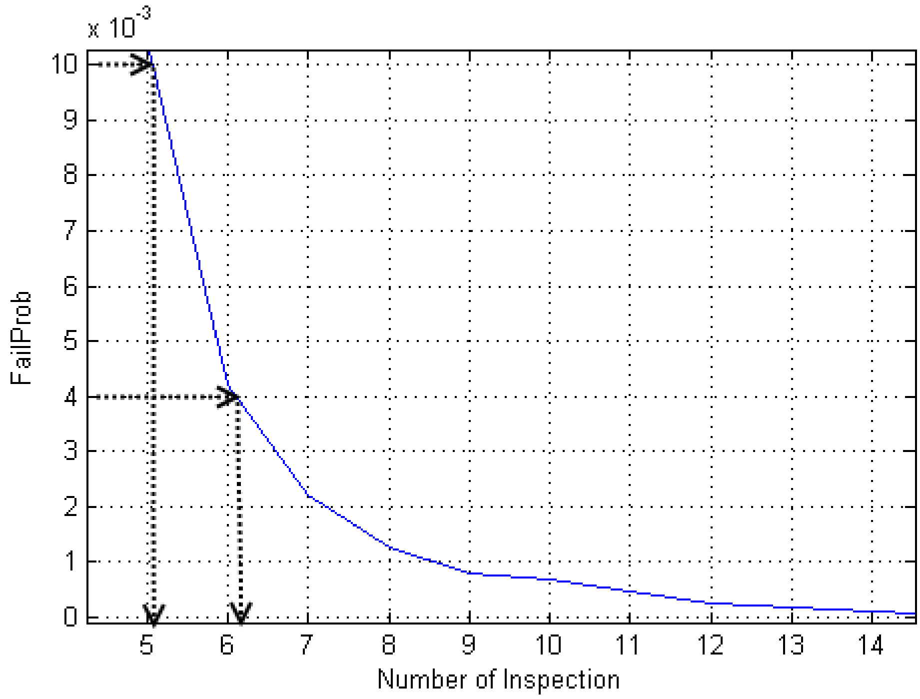

Fig. 6. Number of inspections.

But this calculation is correct only if in the service the same value of $\theta_0 = -8.5885$ takes place. In reality, we do not know the θ_0. If θ_0 value is changed, then the selected number of inspections to provide the required reliability level will be changed as well. This effect could be seen in Fig. 7.

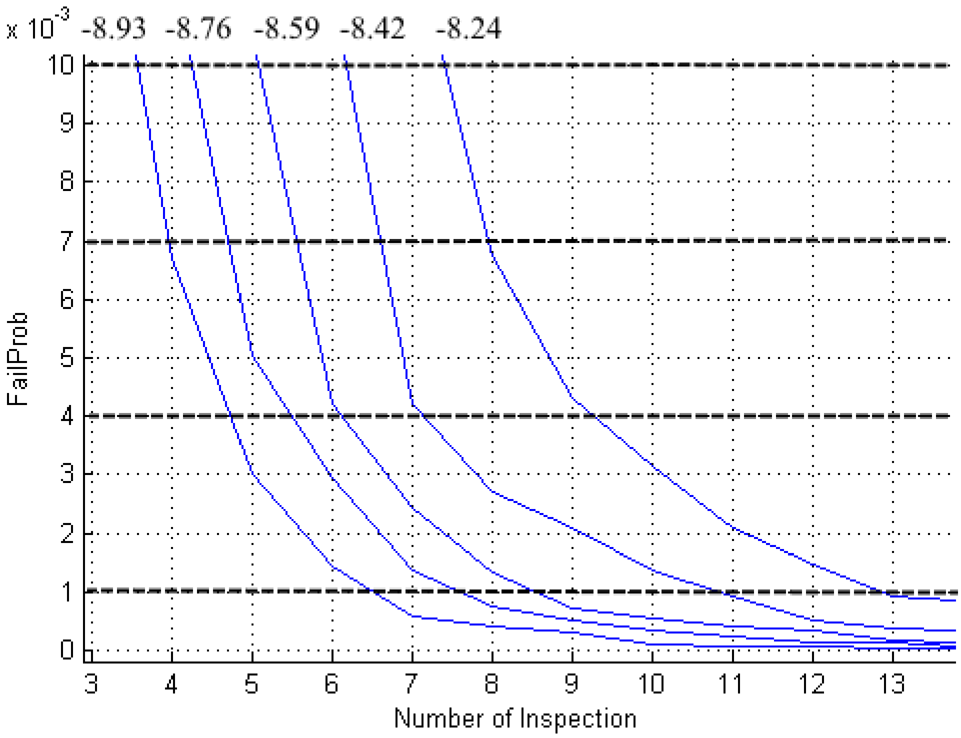

Fig. 7. Example of failure probability function for different θ_0 values.

This means that selected inspection program $\hat{n} = n(0.01, \hat{\theta}) = 6$ will not be optimal for all possible θ_0 values. In service it is possible that some fatigue cracks require a higher number of inspections. Figure 8 demonstrates the dependence of number of inspections on θ_0 value for different p^*.

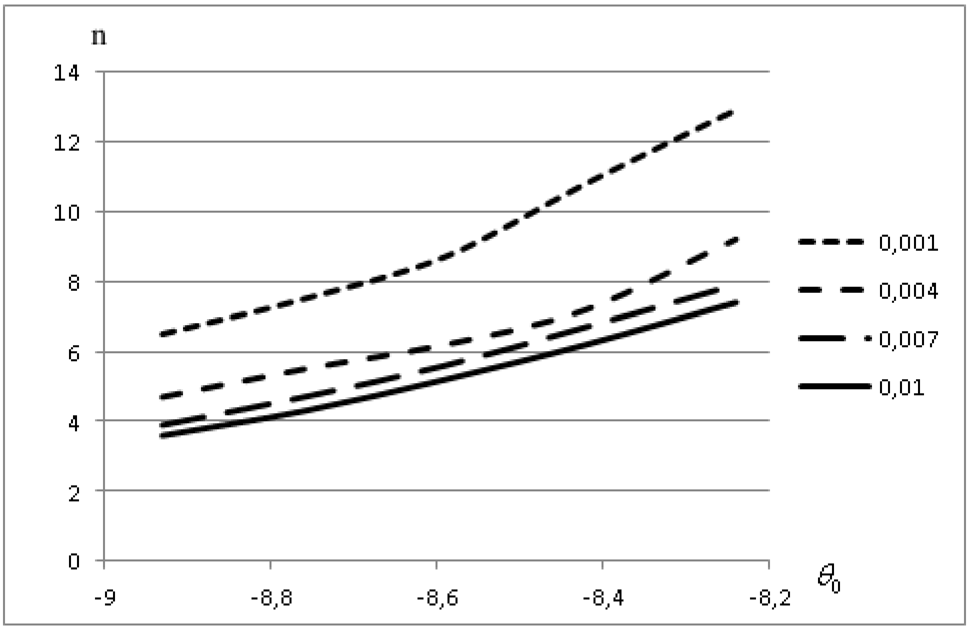

Fig. 8. Example of function *n*(*p, *θ*) for different *p**.**

We should limit the maximum possible failure probability for any θ_0. It can be done by choosing specific "designed" failure probability, p_{fD}. The family of the functions $v(\theta, p)$ for different p is shown in Fig. 9, where the corresponding calculations for parallel axis are made for corresponding "mean durability" $= \frac{C_c}{Q}$ for $C_c = \ln a_c - \ln \alpha$, $Q = \exp(\theta_0)$.

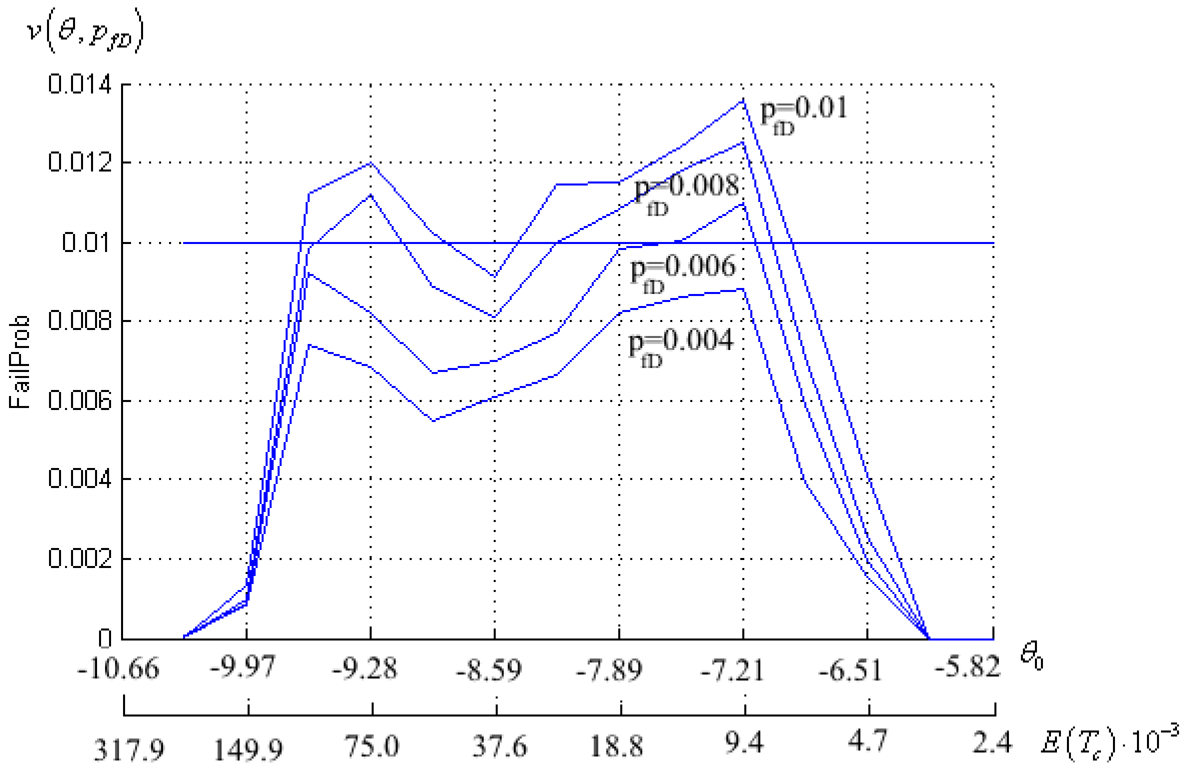

Fig. 9. The function $v(\theta, p)$ for different p_{fD}.

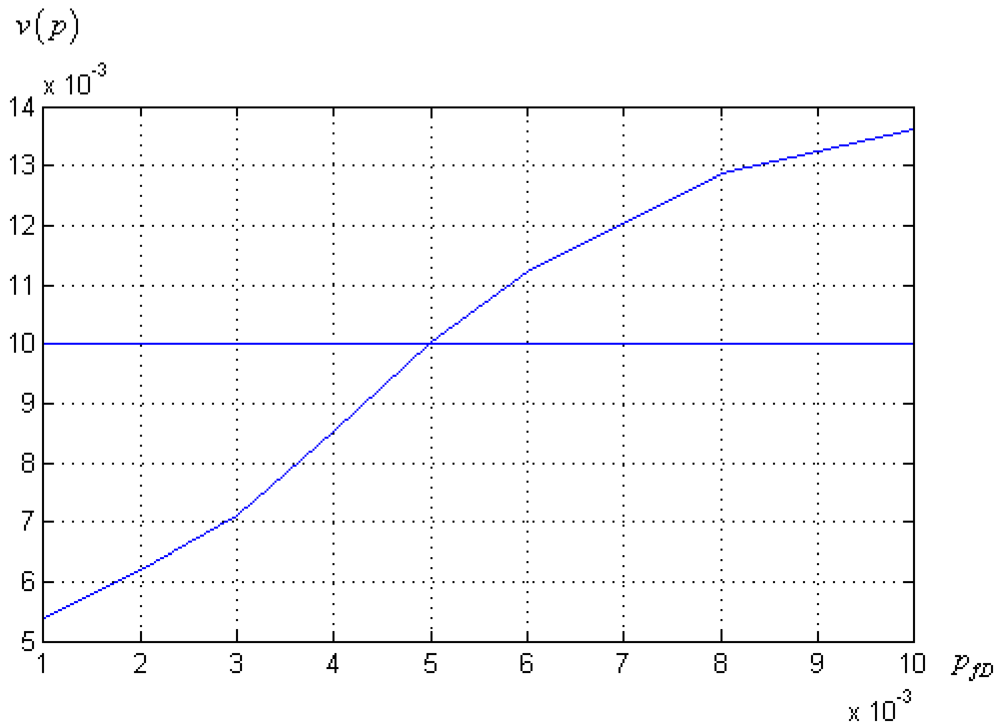

Fig. 10. The function $v(p)$.

In Fig. 10 the function $v(p)$ is shown for the considered example data. In order to limit failure probability of aircraft fleet by value $p^* = 0.01$ the value $p_{fD} = 0.004$ should be chosen.

Now the function $p_{fNW}(n,\theta)$ which is shown in Fig. 6 (for N=10) for the estimate of fatigue crack parameters $\hat{\theta}_0 = -8.5885$, the number of inspections should be chosen equal to $\hat{n} = n(0.004,\theta) = 7$.

Chapter 4. Reliability Replacement Problem

The present chapter considers the provision of system (conditionally called airline) with infinite service reliability. This is the development of task of service life selecting considered in the literature. This task considers that aftermath of fatigue failure has cost valuation and it is necessary to choose the optimal time of outdated aircraft replacement with the aircraft with smaller operating time. This option is not provided by the previous solution of the present task. The second solution does not consider limitation of fatigue failure rate. It could be considered that development of Howard's task is presented.

If we know the cumulative distribution function of aircraft fatigue life and the price function on aircraft operating time, we can solve the following problem: if we have an aircraft with a certain operating time, should we continue service or trade it in? Further, if we trade it in, what would be the operating time of aircraft which we should buy? Service time of aircraft is limited with specified life t_{SL} for technical reasons. Interval $[0,t_{SL}]$ is divided into n intervals, so one interval is equal to operating time $\Delta = \frac{t_{SL}}{n}$ (Fig. 11).

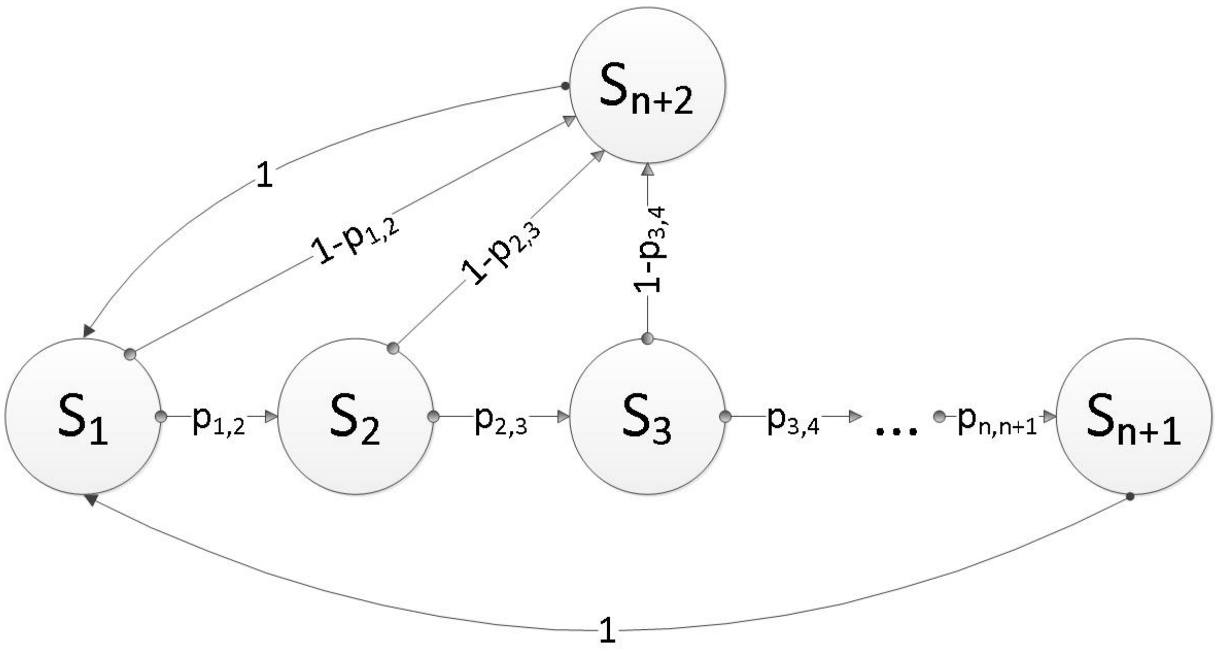

Fig. 11. Transition diagram.

We will start solving this problem using the Markov chain policy-iteration method. Let us consider the Markov chain with $n+2$ states. For $1 \le i \le n+1$ it is assumed that chain is in state i, if operating time of aircraft is $\Delta(i-1)$. The first state corresponds to new aircraft with no operating time in service. Next state corresponds to aircraft used for one interval with corresponding operating time Δ. Chain is in state $n+1$ if operating time of aircraft is Δn. Transition from n state to state $n+1$ means ending of aircraft servicing because of expiration of overhaul period (reaching the specified service life t_{SL}). There is one more state in Markov chain — $n+2$ state. Fatigue failure is possible during the aircraft service. In the case of fatigue failure of aircraft during service the Markov chain goes to the state $n+2$, i.e. the failure state.

There are different policies available in each state. The alternatives available in states $i = 1, 2, ..., n$ are the following: the zeroth policy, $k = 0$, is to continue aircraft service for the other step. The other policies, $0 < k \le n$, are to trade in existing aircraft and buy an aircraft of state k. In the case of t_{SL} reaching (MC is in state $n+1$) or fatigue failure (MC is in state $n+2$) further service of aircraft is impossible and the zeroth policy cannot be used. The alternatives available in states $i = n+1$ and $i = n+2$ are to buy an aircraft of state k ($0 < k \le n$). For each state we can choose a specific policy. Set of policies chosen in each state $k_{1:(n+1)} = \{k_1, k_2, ..., k_i, ..., k_{n-1}, k_n, k_{n+1}\}$ require a strategy. We need to find an aircraft optimal strategy from the point of view of reliability and economic effectiveness. Solution to this problem could be found using the controlled Markov chain and Howard's algorithm.

For each policy and each state transition probabilities:

$$p_{ij}^k = \left\{ \begin{array}{cc} p_{i,i+1} & j = i+1 \\ 1 - p_{i,i+1} & j = n+2 \\ 0 & \text{other} \end{array} \right\} \text{ for } k = 0,\ 1 \le i \le n. \tag{4.1}$$

$$p_{ij}^k = \left\{ \begin{array}{l} 1 \ \ j = k \\ 0 \ \text{other} \end{array} \right\} \text{ for } 0 < k \le n,\ i = n+1 \text{ or } i = n+2. \tag{4.2}$$

	S_1	S_2	S_3	...	S_n	S_{n+1}	S_{n+2}
S_1	0	$p_{1,2}$	0	...	0	0	$1-p_{1,2}$
S_2	0	0	p_{23}	...	0	0	$1-p_{23}$
S_3	0	0	0	...	0	0	$1-p_{3,4}$
...	...	...	...	...	...	...	...
S_{n-1}	0	0	0	...	$p_{n-1,n}$	0	$1-p_{n-1,n}$
S_n	0	0	0	...	0	$p_{n,n+1}$	$1-p_{n,n+1}$

Fig. 12. Transition probability matrix if in every state *i* = 1, 2, ..., *n* the policy *k* = 0 is selected.

	S_1	...	S_k	...	S_{n+1}	S_{n+2}
S_1	**0**	...	**1**	...	**0**	**0**
...	...	...	...	...	...	...
S_i	**0**	...	**1**	...	**0**	**0**
...	...	...	...	...	...	...
S_{n+1}	**0**	...	**1**	...	**0**	**0**
S_{n+2}	**0**	...	**1**	...	**0**	**0**

Fig. 13. Transition probability matrix if in every state *i* = 1, 2, ..., *n* + 2 the same policy *k* > 0 is selected.

For the state S_{n+1} or S_{n+2} zeroth policy ($k=0$) cannot be used and one of other policies ($k>0$) should be selected. It is because we cannot continue aircraft service when its service life is equal to the specified life t_{SL} or when it is in the failure state. For each transition from state i to state j we have rewards $r_{i,j}^{k}$:

For $k=0$, $i=1,2,...,n+1$ — transition from state i to $i+1$: $r_{i,i+1}^{1}=-e_i+b_s$;

For $k=0$, $i=1,2,...,n$ — transition from state i to state $n+2$: $r_{i,n+1}^{1}=-e_i-t_i-f$;

For $k > 0$, $i = 1, 2, ..., n+1$ — transition from state i to state k : $r_{i,k}^{k} = t_i - c_k$;

For $k > 0$, $i = n+2$ — transition from state i to state k : $r_{i,k}^{k} = -c_k$

where b_s is income from successful service in one interval

e_i — service expenses;

t_i — trade-in value of aircraft of operating time $i-1$;

f — loses caused by failure;

c_k — price of aircraft of operating time $k-1$.

Then we select the policy with highest expected reward for each state. Expected reward is calculated according to (4.3)

$$q_i^k = \sum_{i=1}^{n+2} p_{ij}^k r_{ij}^k . \tag{4.3}$$

According to the selected policies, an equation is written for each state. Let us denote by v_i the relative values of the policies and by g the expected gain of the system for one step of the selected strategy. The basic equations governing the system when it is in state i, are the following:

$$g + v_i = q_{i,i+1}^k + p_{i,i+1}^k v_{i+1} + \left(1 - p_{i,i+1}^k\right) v_{n+2} \tag{4.4}$$

if $k = 0$ is chosen (continuation of aircraft service),

$$g + v_i = q_{i,i+1}^k + v_k \tag{4.5}$$

if $k > 0$ is chosen (we trade in aircraft of existing state i and buy an aircraft of the state k).

Then we solve those equations assuming that $v_{n+2} = 0$, and calculate test quantity values $g_i^k + \sum_{i=1}^{n+2} p_{ij}^k v_j$ for each state and each policy. Here we are looking for k corresponding to the highest quantity values for each state to find a new better policy that will maximise this quantity. We repeat those iterations until the best policies are found.

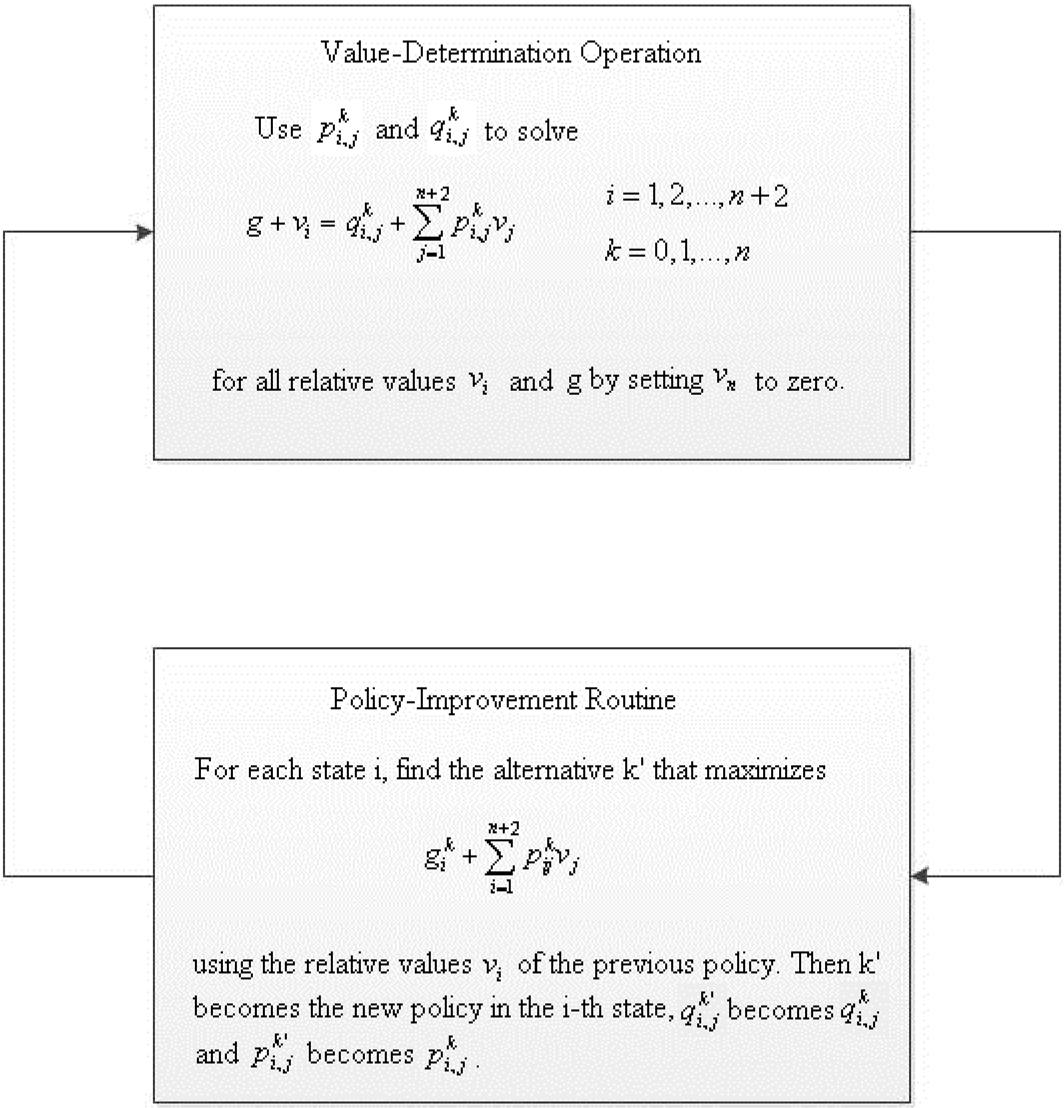

Fig. 14. The iteration cycle.

Using Howard's algorithm we find optimal service ending state S_e and optimal service beginning state S_b.

In order to limit a fatigue failure rate, we should know how to calculate it for the selected strategy (really set of policies in every state $S_b, ..., S_e$). First, we are going to find stationary probabilities of our system, which is defined by the equation system:

$$\pi P = \pi , \tag{4.6}$$

where $\sum_{i=S_b}^{S_f} \pi_i = 1$.

Mean time between failures is

$$L_f = \frac{1}{\pi_{S_f}} \Delta \tag{4.7}$$

(π_{S_f} is failure state probability), and failure rate is

$$\lambda_f = \frac{1}{L_f} = \frac{\pi_{S_f}}{\Delta}. \tag{4.8}$$

Then all allowable solutions should be found for all $S'_b \le S_b$ and $S'_e \le S_e$. From this variety of different solutions we should find a pair (S_b, S_e) that corresponds to a maximum value of $g_{b,e}$, under the limitation $\lambda_f \le \lambda_{fa}$. It can be written in the following way. The pair (λ_f, g) is vector function of vector-argument pair (S_b, S_e),

$$\left(\lambda_f, g\right) = G_\lambda\left(S_b, S_e\right). \tag{4.9}$$

The best pair (S_b^*, S_e^*) is defined by

$$\left(S_b^*, S_e^*\right) = \underset{S_b, S_e}{\arg\max}\left(G_\lambda\left(S_b, S_e\right) : \lambda_f \le \lambda_{fa}\right). \tag{4.10}$$

Numerical Example

For this example we suppose that $\theta_0 = 11.6354$ and $\theta_1 = 0.346$. Those values correspond to expected durability of aircraft $E(X) = \exp\left(\theta_0 + \frac{\theta_1^2}{2}\right) = 120\,000$ flight hours. In this numerical example, specified service life is defined as one third of aircraft expected durability: $t_{SL} = \frac{E(X)}{3} = 40\,000$ flight hours. In this numerical example, we have the following additional data: service expense of aircraft service to state n is higher than that of new aircraft $e_{\text{increase}} = \frac{e_n}{e_1}$ by $e_{\text{increase}} = 2$, gain of trading-in an aircraft is smaller than aircraft price by $t_i = c_i\left(1 - t_L\right)$ with trade loss value $t_L = 0.1$ and acceleration factor that adjusts the depreciation rate $a = 2$, hour cost factor $h_c = 0.3$, hour expense factor $h_e = 0.4$, hour failure factor $h_f = 50$. Markov chain step (interval between states) is $\Delta = 1000$ flight hours, this means that successful service for one step brings $b_s = 1000$ flight hour income. From the data, we calculate the following values: cost of new aircraft $c_1 = h_c t_{SL} = 12\,000$ flight hours, total service expenses $e = h_e t_{SL} = 16\,000$ flight hours, failure loses $f = h_f t_{SL} = 2 \cdot 10^6$ flight hours. Allowable fatigue failure rate is $\lambda_{fa} = 1 \cdot 10^{-10}$.

Using Howard's algorithm, we obtain $S_b = 14$, $S_e = 31$. The average gain of one step is $g = 320$ flight hours for this strategy and it is maximum possible average step gain for the given initial data. It can be seen that we continue service until we reach state 31, then we use policy 14 ($k = 14$) that is to trade in existing aircraft of operating time of 30000 flight hours and to buy an aircraft in state 14 (13000 flight hours of operating time). State 14 is optimal service beginning state s_b and state 31 is optimal service ending state s_e. In the case of failure, we also use policy 14.

Now it is possible to find failure stationary probability $\pi_{S_f} = 5.13 \cdot 10^{-6}$, mean time between failures is $L_f = 1.95 \cdot 10^8$ flight hours and fatigue failure rate is $\lambda_f = 5.13 \cdot 10^{-9}$. We can see that this strategy does not satisfy $\lambda_f \le \lambda_{fa}$ requirement.

In order to decrease failure rate all (s_b, s_e) pairs corresponding to maximum $g_{(b,e)}$ should be found under the limitation $\lambda_{f(b,e)} \le \lambda_{fa}$; those pairs are shown in Figs. 15 and16. Failure rate and gain functions on states are shown in Figs. 17 and 18.

It has been found that the most profitable strategy that satisfies the limitation $\lambda_f \le \lambda_{fa}$ is ($s_b = 10$, $s_e = 22$) $\lambda_{10,22} = 8.63 \cdot 10^{-11}$, $g_{10,22} = 275$ flight hours and $L_{f10,22} = 1.16 \cdot 10^{10}$.

Probabilities to end service in S_e state or in S_f state according to different existing states $(10, 11, ..., 21)$ are shown in Table 1.

Table 1

Probabilities of Failure

	S_e	$S_f \cdot 10^{-7}$
10	0.999998877582	11.22418
11	0.999998877590	11.22410
12	0.999998877627	11.22373
13	0.999998877786	11.22214
14	0.999998878369	11.21631
15	0.999998880240	11.19760
16	0.999998885584	11.14416
17	0.999998899432	11.00568
18	0.999998932392	10.67608
19	0.999999005251	9.94749
20	0.999999156173	8.43827
21	0.999999451361	5.48639

	S_{b1}	S_{b2}	S_{b3}	S_{b4}	S_{b5}	S_{b6}	S_{b7}	S_{b8}	S_{b9}	S_{b10}	S_{b11}	S_{b12}	S_{b13}	S_{b14}
S_{e1}	1	1	1	1	1	1	1	1	1	1	1	1	1	1
S_{e2}	0	1	1	1	1	1	1	1	1	1	1			
S_{e3}	0	0	1	1	1	1	1	1	1	1	1			
S_{e4}	0	0	0	1	1	1	1	1	1	1	1	1	1	1
S_{e5}	0	0	0	0	1	1	1	1	1	1	1	1	1	1
S_{e6}	0	0	0	0	0	1	1	1	1	1	1	1	1	1
S_{e7}	6.E-20	7.E-20	9.E-20	1.E-19	1.E-19	2.E-19	1	1	1	1	1	1	1	1
S_{e8}	1.E-18	1.E-18	2.E-18	2.E-18	2.E-18	3.E-18	5.E-18	1	1	1	1	1	1	1
S_{e9}	1.E-17	2.E-17	2.E-17	2.E-17	3.E-17	3.E-17	4.E-17	6.E-17	1	1	1	1	1	1
S_{e10}	1.E-16	1.E-16	2.E-16	2.E-16	2.E-16	2.E-16	3.E-16	4.E-16	5.E-16	1	1	1	1	1
S_{e11}	8.E-16	8.E-16	9.E-16	1.E-15	1.E-15	1.E-15	2.E-15	2.E-15	3.E-15	4.E-15	1	1	1	1
S_{e12}	4.E-15	4.E-15	5.E-15	5.E-15	6.E-15	6.E-15	8.E-15	9.E-15	1.E-14	1.E-14	2.E-14	1	1	1
S_{e13}	2.E-14	2.E-14	2.E-14	2.E-14	2.E-14	3.E-14	3.E-14	3.E-14	4.E-14	5.E-14	7.E-14	8.E-14	1	1
S_{e14}	6.E-14	6.E-14	7.E-14	7.E-14	8.E-14	9.E-14	1.E-13	1.E-13	1.E-13	2.E-13	2.E-13	2.E-13	3.E-13	1
S_{e15}	2.E-13	2.E-13	2.E-13	2.E-13	2.E-13	3.E-13	3.E-13	3.E-13	4.E-13	4.E-13	5.E-13	7.E-13	8.E-13	9.E-13
S_{e16}	5.E-13	5.E-13	6.E-13	6.E-13	7.E-13	7.E-13	8.E-13	9.E-13	1.E-12	1.E-12	1.E-12	2.E-12	2.E-12	2.E-12
S_{e17}	1.E-12	1.E-12	1.E-12	2.E-12	2.E-12	2.E-12	2.E-12	2.E-12	2.E-12	3.E-12	3.E-12	4.E-12	4.E-12	5.E-12
S_{e18}	3.E-12	3.E-12	3.E-12	4.E-12	4.E-12	4.E-12	5.E-12	5.E-12	5.E-12	6.E-12	7.E-12	8.E-12	9.E-12	1.E-11
S_{e19}	7.E-12	7.E-12	8.E-12	8.E-12	9.E-12	9.E-12	1.E-11	1.E-11	1.E-11	1.E-11	1.E-11	2.E-11	2.E-11	2.E-11
S_{e20}	1.E-11	1.E-11	2.E-11	2.E-11	2.E-11	2.E-11	2.E-11	2.E-11	2.E-11	3.E-11	3.E-11	3.E-11	3.E-11	4.E-11
S_{e21}	3.E-11	3.E-11	3.E-11	3.E-11	3.E-11	4.E-11	4.E-11	4.E-11	4.E-11	5.E-11	5.E-11	6.E-11	6.E-11	7.E-11
S_{e22}	5.E-11	5.E-11	6.E-11	6.E-11	6.E-11	7.E-11	7.E-11	7.E-11	8.E-11	9.E-11	9.E-11	1.E-10	1.E-10	1.E-10
S_{e23}	9.E-11	1.E-10	1.E-10	1.E-10	1.E-10	1.E-10	1.E-10	1.E-10	1.E-10	1.E-10	2.E-10	2.E-10	2.E-10	2.E-10
S_{e24}	2.E-10	2.E-10	2.E-10	2.E-10	2.E-10	2.E-10	2.E-10	2.E-10	2.E-10	3.E-10	3.E-10	3.E-10	3.E-10	3.E-10
S_{e25}	3.E-10	3.E-10	3.E-10	3.E-10	3.E-10	3.E-10	3.E-10	4.E-10	4.E-10	4.E-10	4.E-10	5.E-10	5.E-10	5.E-10
S_{e26}	4.E-10	4.E-10	5.E-10	5.E-10	5.E-10	5.E-10	5.E-10	6.E-10	6.E-10	6.E-10	7.E-10	7.E-10	8.E-10	8.E-10
S_{e27}	6.E-10	7.E-10	7.E-10	7.E-10	8.E-10	8.E-10	8.E-10	9.E-10	9.E-10	1.E-09	1.E-09	1.E-09	1.E-09	1.E-09
S_{e28}	1.E-09	1.E-09	1.E-09	1.E-09	1.E-09	1.E-09	1.E-09	1.E-09	1.E-09	1.E-09	2.E-09	2.E-09	2.E-09	2.E-09
S_{e29}	1.E-09	2.E-09	2.E-09	2.E-09	2.E-09	2.E-09	2.E-09	2.E-09	2.E-09	2.E-09	2.E-09	2.E-09	2.E-09	3.E-09
S_{e30}	2.E-09	2.E-09	2.E-09	2.E-09	2.E-09	3.E-09	3.E-09	3.E-09	3.E-09	3.E-09	3.E-09	3.E-09	4.E-09	4.E-09
S_{e31}	3.E-09	3.E-09	3.E-09	3.E-09	3.E-09	4.E-09	4.E-09	4.E-09	4.E-09	4.E-09	4.E-09	5.E-09	5.E-09	5.E-09

$$\lambda_f = \frac{1}{L_f} = \frac{\pi_{S_f}}{\Delta}$$

Fig. 15. $\lambda_{f(b,\,e)}$ **set.**

$$g_{b,e} = \sum_{i=b}^{e} \pi_i q_i$$

	S_{b1}	S_{b2}	S_{b3}	S_{b4}	S_{b5}	S_{b6}	S_{b7}	S_{b8}	S_{b9}	S_{b10}	S_{b11}	S_{b12}	S_{b13}	S_{b14}
S_{e1}	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf
S_{e2}	-507	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.	[illegible]	nf
S_{e3}	-269	-467	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.	[illegible]	nf
S_{e4}	-146	-236	-429	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf
S_{e5}	-68.1	-117	-205	-393	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf
S_{e6}	-13.8	-41.8	-89.6	-176	-360	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf
S_{e7}	27.1	10.6	-17.2	-64.1	-149	-328	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf
S_{e8}	59.5	50.0	33.4	6.0	-40.1	-123	-298	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf
S_{e9}	86.2	81.1	71.4	54.8	27.7	-17.6	-98.9	-269	.-inf	.-inf	.-inf	.-inf	.-inf	.-inf
S_{e10}	108.6	106.6	101.3	91.4	74.9	48.0	3.5	-76.1	-243	.-inf	.-inf	.-inf	.-inf	.-inf
S_{e11}	127.9	128.1	125.7	120.1	110.1	93.6	67.1	23.3	-54.7	-217	.-inf	.-inf	.-inf	.-inf
S_{e12}	144.9	146.6	146.3	143.6	137.7	127.5	111.1	84.9	41.8	-34.6	-194	.-inf	.-inf	.-inf
S_{e13}	159.9	162.7	163.9	163.2	160.2	154.1	143.8	127.4	101.5	59.1	-15.7	-171	.-inf	.-inf
S_{e14}	173.4	177.0	179.3	180.1	179.0	175.7	169.4	159.0	142.6	117.0	75.3	2.0	-150	.-inf
S_{e15}	185.5	189.7	192.9	194.7	195.1	193.7	190.1	183.6	173.1	156.8	131.4	90.4	18.5	-130
S_{e16}	196.6	201.2	205.0	207.6	209.0	209.0	207.3	203.4	196.7	186.1	169.9	144.8	104.5	34.0
S_{e17}	206.7	211.7	215.9	219.1	221.3	222.3	221.9	219.9	215.8	208.9	198.2	182.1	157.3	117.6
S_{e18}	216.0	221.2	225.7	229.4	232.2	233.9	234.6	233.9	231.6	227.2	220.2	209.4	193.3	168.8
S_{e19}	224.5	229.9	234.7	238.7	241.9	244.2	245.6	245.9	244.9	242.3	237.7	230.5	219.7	203.7
S_{e20}	232.4	237.9	242.9	247.1	250.7	253.4	255.4	256.4	256.3	255.0	252.2	247.4	240.0	229.2
S_{e21}	239.7	245.3	250.4	254.8	258.6	261.7	264.1	265.6	266.3	265.9	264.3	261.2	256.2	248.7
S_{e22}	246.4	252.1	257.2	261.8	265.8	269.1	271.8	273.8	274.9	275.3	274.6	272.7	269.4	264.2
S_{e23}	252.6	258.4	263.5	268.2	272.3	275.8	278.7	281.0	282.6	283.4	283.4	282.4	280.3	276.7
S_{e24}	258.3	264.1	269.3	274.0	278.2	281.8	284.9	287.4	289.3	290.5	291.0	290.7	289.4	287.0
S_{e25}	263.6	269.3	274.5	279.2	283.5	287.2	290.4	293.1	295.2	296.7	297.6	297.8	297.1	295.6
S_{e26}	268.4	274.0	279.2	283.9	288.2	292.0	295.3	298.0	300.3	302.1	303.2	303.7	303.6	302.6
S_{e27}	272.6	278.2	283.4	288.1	292.3	296.1	299.5	302.3	304.7	306.6	307.9	308.7	308.9	308.4
S_{e28}	276.3	281.9	287.0	291.6	295.9	299.6	303.0	305.9	308.3	310.3	311.8	312.7	313.1	313.0
S_{e29}	279.5	284.9	290.0	294.6	298.7	302.5	305.8	308.7	311.1	313.1	314.7	315.8	316.3	316.4
S_{e30}	282.0	287.3	292.3	296.8	300.9	304.5	307.8	310.7	313.1	315.1	316.7	317.8	318.5	318.6
S_{e31}	283.8	289.0	293.8	298.2	302.2	305.8	309.0	311.8	314.1	316.1	317.7	318.8	319.4	319.6

Fig. 16. $g_{(b,e)}$ **set.**

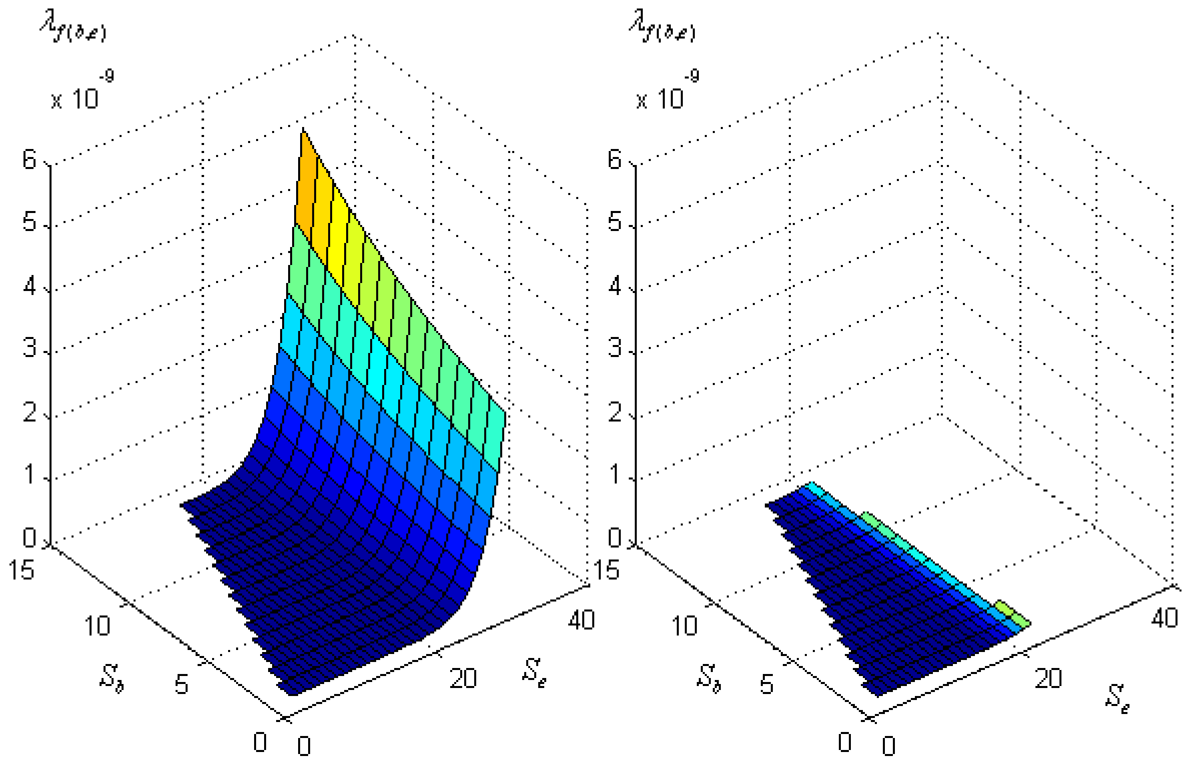

Fig. 17. $\lambda_{f(b,\,e)}$ **full and limited sets.**

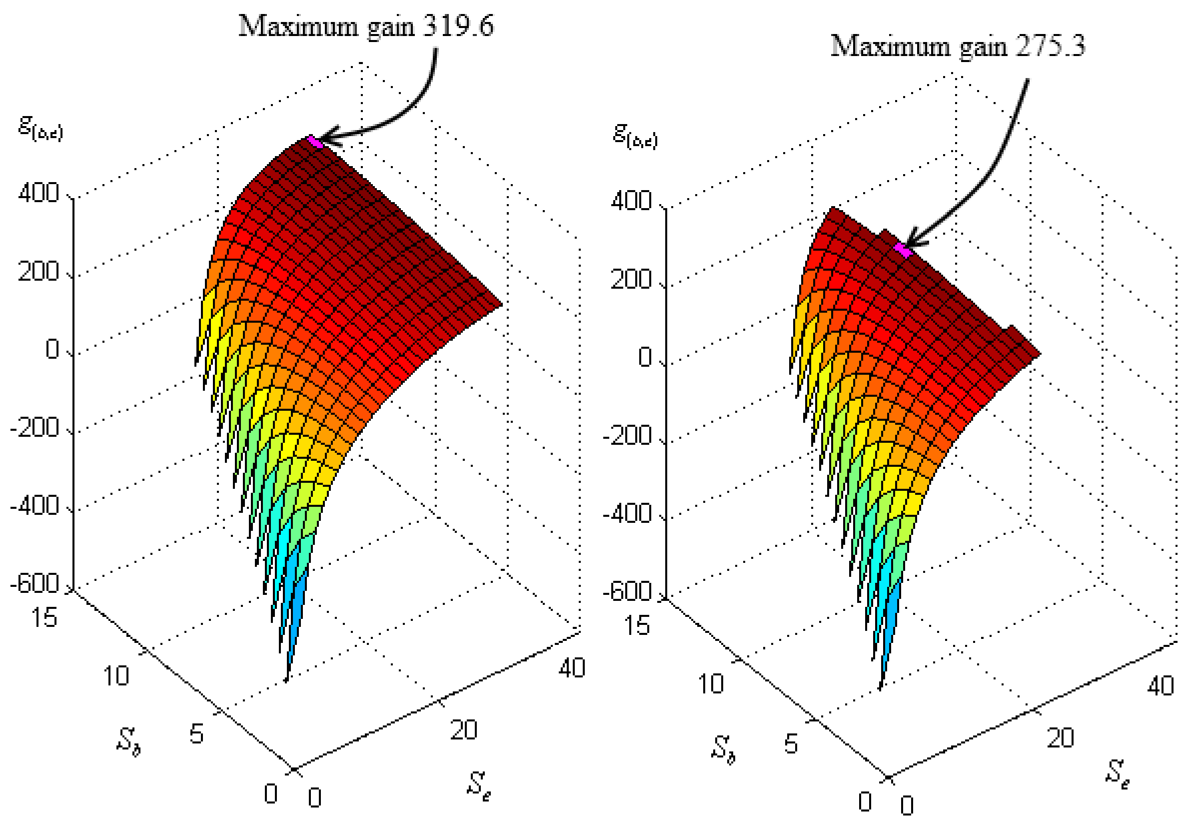

Fig. 18. $g_{(b,\,e)}$ **full and limited sets.**

Appendices

In appendices, a specific case is considered when cumulative distribution function of durability has standard normal distribution. This kind of approximation can make sense when final failure is a result of accumulation of small defects and the time to failure is the sum of lager number of small intervals between specific defects. We suppose that random variables T_d and T_c have the same type of cumulative distribution function with the same scale parameter, but with the different location parameter. For this case, the total number of inspections made before the first failure in fleet R_k for the selected inspection program can be found as a function of scale parameter θ_1, factor $\delta = \frac{\theta_{c0} - \theta_{d0}}{\theta_1}$, cumulative distribution function of $\overset{\circ}{T}_d$, $\overset{\circ}{T}_c$, $\overset{\circ}{\theta}_{0c}$ and number of aircraft in fleet n_f, the rate of beginning of operation of new aircraft (it is defined by the value $\frac{1}{a\theta_1}$) and interval between inspections, $b\theta_1$, where a and b are some variables.

In the second part of appendices, the developed Matlab scripts used for described methodology implementation are given. The list of those scripts is presented in Table 2.

Table 2

Matlab Scripts

No.	Module name	Type	Short description
1.	Fleet_Reliability.m	Program	This program allows developing the inspection program for the fleet of aircraft. Different curves could be obtained.
2.	Fleet_Reliability.fig	Interface	This is interface of the program that makes the use of the program convenient.
3.	f_Fleet_Pf	Function	This function calculates fleet failure probability for the selected inspection program.
4.	f_PfInt	Function	Function that makes curve extrapolation.
5.	Safe_Life_Approach.m	Program	This program allows calculating the most profitable safe-life under the fatigue failure rate limitation.
6.	Safe_Life_Approach.fig	Interface	This is interface of the program that makes the use of the program convenient.
7.	f_AC_Step	Function	This function calculates the most profitable policies using Howard's algorithm.

CONCLUSIONS

The main conclusions of the research are the following:

- Two types of tasks have been considered: planning of fatigue-prone aircraft fleet inspection program; determination of aircraft replacement time under condition of airline fatigue failure rate limitation taking into account its economic effectiveness.
- In the first task, in contradistinction to existing solutions to the analogous task, the information exchange about the discovery of any fatigue crack in fleet is considered to ensure the required reliability for the whole fleet. Influence of aircraft service beginning rate has also been considered. It is especially important for the service beginning of a new aircraft type (difference of the present research from previous studies is demonstrated graphically in Doctoral Thesis Fig. 3.8 distinction from Fig. 3.6 of the Doctoral Thesis). Human factor has also been considered (the probability of planned inspection has been taken into account).
- Definition of p-set function for the complete set of fatigue life of the whole aircraft fleet has been presented for the first time. Using minimax approach solution allows providing the aircraft fleet reliability for the unknown parameters of durability distribution. Parameters have been estimated using acceptance test results. This means that aircraft that does not satisfy acceptance requirements is not allowed to be used in service and is sent to redesign. In this case, the aircraft reliability in service is guaranteed for all unknown crack growth parameters because only aircraft that satisfies acceptance requirements is used in service.
- The main solution to the problem has been provided for the lognormal distribution that is usually used to describe fatigue durability. The case of normal distribution has also been considered. Normal distribution estimation of durability can be accepted for the cases when final failure is a result of a particular failure sum. This model allows simplifying item inspection program development by reducing a number of parameters to consider.
- In the second task, the determination of replacement time and reliability of a system with unlimited service time have been considered. In the case of failure, a new item that is analogical to the previous one is brought to service. As opposed to the solution proposed, for example by Howard, the optimisation of economic effectiveness of the system (conditionally called airline) has been considered under the limitation of system failure rate (number of failures per time unit).

- The modular software package has been created to implement the model specified above. This software package is a convenient, flexible and functional working tool of the researcher, the specialized test bench, allowing for the calculation of parameters of inspection or for safe life determination. The results of modelling can be stored for further use.

The Areas of Further Research

The described model is a simplified reflection of real processes. It is possible to improve a model by including additional variables that can offer more reliable calculations and, as a result, more rare inspection programs with the same level of reliability. The first step could be determination of the first inspection using the inspection interval that can decrease in time.

It is also possible to develop an individual adaptive inspection program for each aircraft in the fleet using calendar time of aircraft service initiation and service data of existing aircraft in the fleet. This could result in possible dynamic inspection programs of all the aircraft in fleet that could change with an increasing number of inspected airframes.

It is necessary to find a solution to safe life and replacement time determination with unknown crack growth parameters. Minmax approach can be used for such a methodology.

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