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NONLINEAR VIBRATIONS OF A PIPE CONTAINING FLOWING FLUID AND THEIR APPLICATIONS IN FLOW METERING

CAURULES AR CAURI PLŪSTOŠU ŠĶIDRUMU NELINEĀRAS SVĀRSTĪBAS UN TO IZMANTOŠANA ŠĶIDRUMA PATĒRIŅA NOTEIKŠANĀ

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1. Introduction

Problems, concerning the dynamics and stability of pipelines containing flowing fluid, in linear formulation have been solved by many authors [1-3]. The results of these investigations make a theoretical basis for the design of fundamentally new vibration method for the testing of the amount of flowing fluid (Coriolis method). Familiar designs of Coriolis flow meters [4] have a high reliability and are not in need of sensors inside the pipes. However these flow meters do not always fulfill practical requirements on accuracy of measurement. The last demerit may be mainly explained by the low flow rate sensitivity of linear vibration methods, which determine the principle of operation of such flow meters (the sensitivity of testing is not more than one arbitrary unit, i.e. any relative change in flow rate is accompanied by the same or even rather smaller relative change of vibration information parameters).
As was shown in [5, 6], considerable increase of sensitivity of control can be achieved by insertion of additional nonlinear elastic elements into the structure of the testing object.

Therefore it has been found advantageous to transform the dynamic model of the flow meter by insertion in it of additional nonlinear elastic support interacting with the pipe. Nonlinear formulation of the problem has the aim to develop new nonlinear methods of vibration flow metering in order to ensure the essential rise in the sensitivity of control.

2. Dynamic model

The flow meter model to be analyzed is a uniform viscoelastic fixed beam (span of a pipe) which is supported in the middle section by an additional nonlinear elastic element specially inserted into the structure of the system (Figure 1). Under the assumptions made by R.Stein and M.Tobriner [2] the differential equation of flexural vibrations of the pipe, excited by the harmonic force $P\sin\omega t$, can be represented as follows

$$EI(1+b_1\frac{\partial}{\partial t})\frac{\partial^4 y}{\partial x^4}+(\rho_f F_f V^2+pF_f)\frac{\partial^2 y}{\partial x^2}+(\rho_f F_f+\rho_p F_p)\frac{\partial^2 y}{\partial t^2}+2\rho_f F_f V\frac{\partial^2 y}{\partial t\cdot\partial x}+$$
$$+b_2\frac{\partial y}{\partial t}=P\sin\omega t\cdot\delta_1(x-x_P)-F_r(y)\cdot\delta_1(x-x_r)\ , \tag{1}$$

where EI is the flexural rigidity of a pipe, b_1 and b_2 are the coefficients of internal and external damping, ρ_p is a density of pipe material, ρ_f is a fluid density, F_p is the pipe cross-section area, F_f is a cross-section area of internal conduit of the pipe, P and ω are the amplitude and frequency of the external test harmonic excitation, $\delta_1(x-x_P)$ and $\delta_1(x-x_r)$ are Dirac delta functions, $F_r(y)$ is a nonlinear restoring force of the additional elastic support, V is a speed of flowing fluid and p is a fluid pressure on the pipe internal surface. The sense of other symbols is clear from Figure 1.

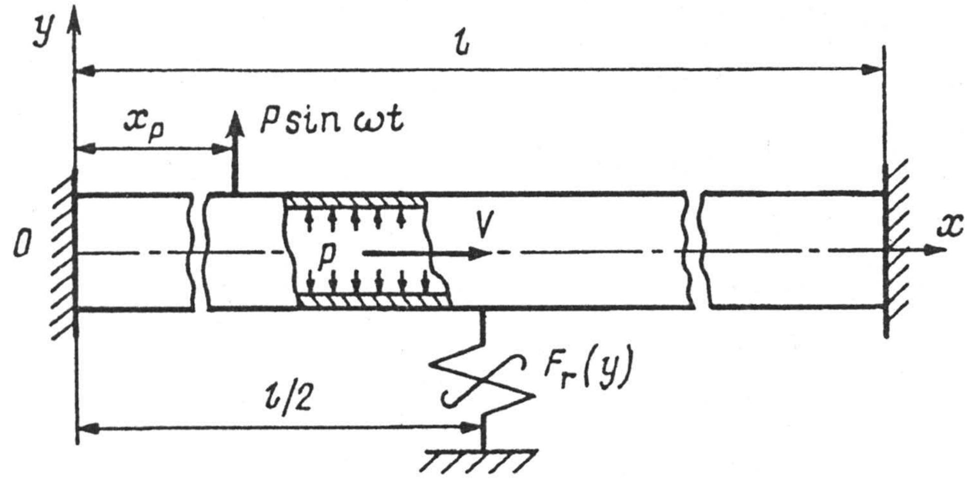

Figure 1. Dynamic model of the flow meter

The variant of nonlinear elastic support with characteristic $F_r(y)$ of preload type is considered:

$$F_r(y) = ky + F_0 \operatorname{sign} y. \tag{2}$$

By changing the variables to $u = y/l$, $z = x/l$ and $\tau = \omega_1 t$ equations (1) and (2) can be transformed into dimensionless forms

$$\frac{\partial^4 u}{\partial z^4} + \beta_1 \frac{\partial^5 u}{\partial\tau \cdot \partial z^4} + \varepsilon_1^4 \frac{\partial^2 y}{\partial\tau^2} + \beta_V \frac{\partial^2 u}{\partial\tau \cdot \partial z} + \beta_2 \frac{\partial u}{\partial\tau} + \gamma \frac{\partial^2 u}{\partial z^2} = q \sin\nu\tau \cdot \delta_1(z - z_P) - \\ - f_r(u) \cdot \delta_1(z - z_r)\ ; \tag{3}$$

$$f_r(u) = k_r u + f_0 \operatorname{sign} u\ , \tag{4}$$

where the following notation is taken: $\beta_1 = b_1\omega_1$; $\beta_V = 2\rho_f F_f V \omega_1 l^3 / (EI)$; $\nu = \omega/\omega_1$; $\beta_2 = b_2\omega_1 l^4 / (EI)$; $\gamma = (\rho_f F_f V^2 + pF_f) l^2 / (EI)$; $q = Pl^2/(EI)$; $f_0 = F_0 l^2/(EI)$; $k_r = kl^3/(EI)$; $z_P = x_P/l$; $z_r = x_r/l$; ω_1 is the first natural frequency of the linear system (pipe interacting with the linear elastic support k) ; ε_1 is a dimensionless coefficient, which is dependent on the boundary conditions at the pipe ends [7].

In the case studied the end boundary conditions are the following:

$$u(0,\tau) = 0\ ,\ u(1,\tau) = 0\ ,\ \frac{\partial u}{\partial z}(0,\tau) = 0\ ,\ \frac{\partial u}{\partial z}(1,\tau) = 0\ . \tag{5}$$

Equation (3), subject to the conditions (4) - (5), was solved on an analogue-digital computer system predominantly set up for the solution of complex nonlinear dynamic problems [8, 9]. The principle of operation of this computer system is described in more detail in references [10, 11].

The problem considered in this paper was solved assuming the parameters of equations (3) - (4) to be the following: $\beta_1 = 0.00785$; $\beta_2 = 10^{-6}$; $q = 1.5$; $f_0 = 1$; $k_r = 20$; $z_P = 0.3$; $z_r = 0.5$; $\varepsilon_1 = 7.1$. Parameters β_V and γ connected with the speed V of flowing fluid have been varied within the limits of $\beta_V = 0$ - 12 and $\gamma = 0$ - 0.25, but frequency ν of external harmonic excitation - over the range of $\nu = 0.2$ - 3.0.

3. Analysis of vibrations

On the whole the dynamic behaviour of the system under study may be illustrated by the amplitude-frequency characteristic (AFC), which graphically represents mutual connections between the driving frequency ν and the half-swing of oscillations u_0 in the pipe cross-section with co-ordinate $z = 0.4$ (Figure 2).

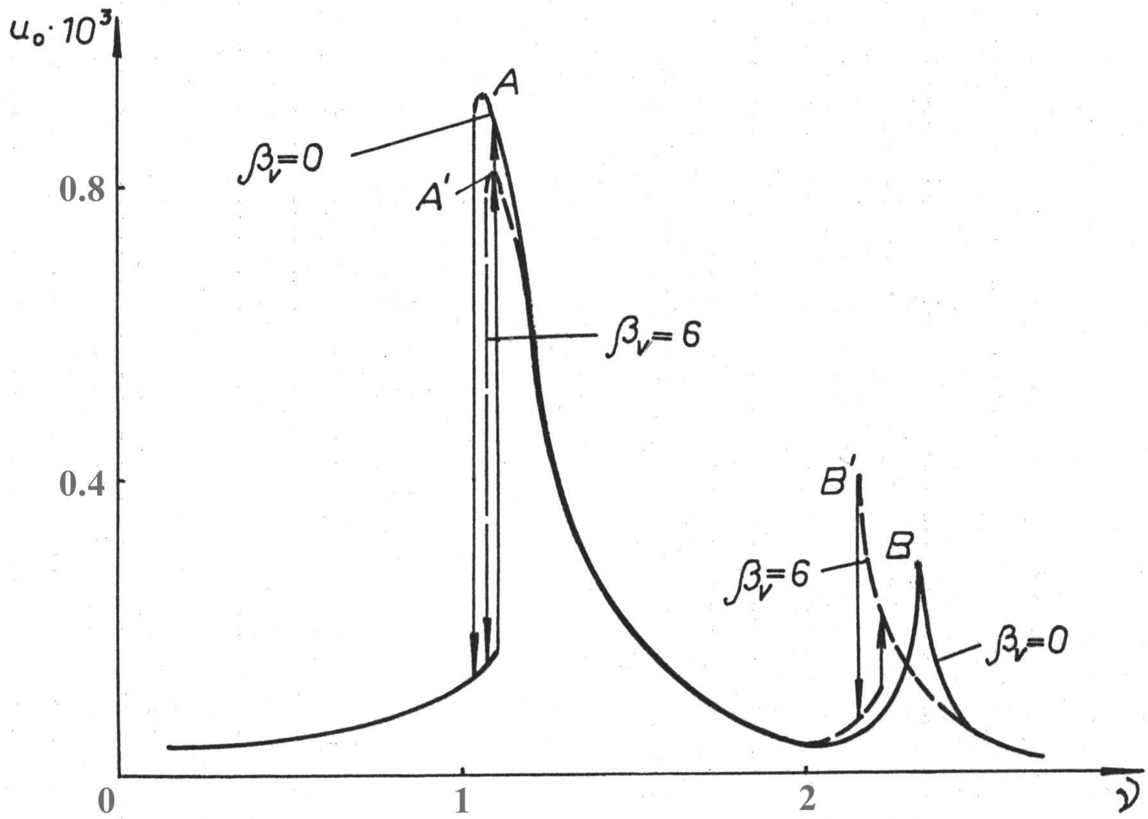

Figure 2. Amplitude-frequency characteristic of the pipe flexural vibrations

Continuous lines show the resonant curves for the case $\beta_V = 0$ and $\gamma = 0$ (the flow rate $Q = \rho_f F_f V$ is equal to zero), dotted lines correspond to the case $\beta_V = 6$ and $\gamma = 0.15$ (the flow rate Q is other than zero).

Peak points of the AFC correspond to the first (points A, A') and the second (points B, B') modes of pipe resonant flexural vibrations. Graphs of the first and the second modes of pipe vibrations corresponding to the two different values of speed V are presented in Figure 3. As additional information the time responses $u = f(\tau)$ and the spectrograms for points B and B' are presented.

At $Q = 0$ the crest of the first mode and the nodal point of the second mode coincide with the co-ordinate $z_r = 0.5$ of location of additional nonlinear elastic support. Under given conditions, nonlinearity of elastic support has an effect only on the first resonant regime and practically does not influence the resonant oscillations of the second mode. Therefore at $Q = 0$ the zone of ambiguity peculiar to nonlinear systems is realized only on the first mode of pipe resonant flexural vibrations (the vicinity of A point of the AFC, see Figure 2).

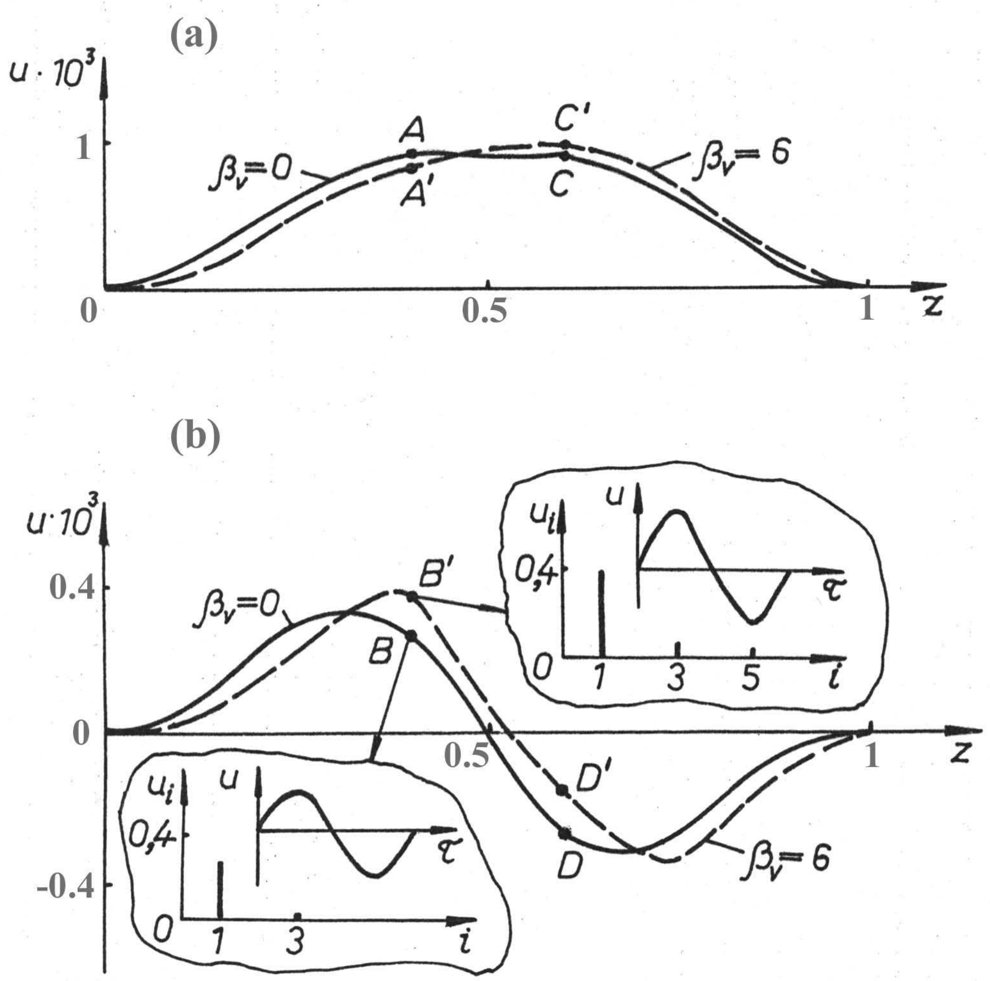

Figure 3. Modes of pipe resonant flexural vibrations: (a) the first resonant mode; (b) the second resonant mode

As the flow rate Q goes up, the original symmetry of pipe resonant modes is gradually disturbed by the action of inertial Coriolis forces. The antinode of the first mode and the node of the second mode are moved in the direction of fluid motion. And the value of this displacement is increased with the rise of fluid speed V. Therefore nonlinear elastic support, placed in section $z_r = 0.5$, begins active interactions with the vibrating pipe not only at the first resonant mode, but also at the second one. In consequence of these nonlinear properties of the system on the second resonant regime are sufficiently intensified, and as the result the corresponding change of the AFC occurs. Thus, at $\beta_V = 6$ the AFC takes on the ambiguity zone in the vicinity of B' point (Figure 2). But on the first resonant regime the fluid motion doesn't influence substantially system's properties and only favors a little contraction of the ambiguity zone (compare position of points A and A' on the AFC).

With the increasing of flow rate Q Fourier spectrum of pipe resonant vibrations is also varied (especially at the second resonant regime). As follows from the spectrograms presented in Figure 3, at $\beta_V = 0$ spectrum of the pipe second resonant regime of flexural vibrations is close to the monoharmonic one (spectral ratio u_3/u_1 makes up only 0.007). But at $\beta_V = 6$ the influence of nonlinear elastic support is reinforced and due to this spectral ratio u_3/u_1 is increased more than ten times ($u_3/u_1 = 0.08$). Besides, fifth harmonic component u_5 additionally emerges in vibration spectrum ($u_5/u_1 = 0.01$).

4. Flow metering

Specified distortions of modes and spectrum of pipe flexural vibrations due to flowing fluid can be used in flow metering as information signs. Quantitative test of flow rate Q may be executed by the recording of a difference in amplitude Δu between two points of the pipe equally distant from its middle (e.g. $\Delta u_1 = u_A - u_C$; $\Delta u_2 = u_B - u_D$) or by the measuring of spectral ratio u_3/u_1 .For the case studied in this paper calibration curves $\Delta u = f(\beta_V)$ and $u_3/u_1 = f(\beta_V)$ as well as sensitivity functions $\eta = f(\beta_V)$ are shown in Figure 4. The sensitivity η of flow metering is estimated in arbitrary units indicating the ratio between the relative change of vibration information parameter (Δu or u_3/u_1) and the corresponding relative change in flow rate Q. Numerals *1* and *2* indicate the order of resonant regime (the first or the second), for which the corresponding graph is plotted. For comparison, the same graphs are presented for the system with linear elastic support k (dotted lines).

From the analysis of graphs presented it can be concluded, that sensitivity of flow metering is sufficiently higher when the system is tuned on the second order resonant regime. In this case the flow rate sensitivity is about 1.4 - 1.5 arbitrary units (in case of measuring of spectral ratio u_3/u_1) and 2.0 - 2.1 units, if the difference in amplitude Δu is recorded. Such level of sensitivity is basically unattainable for linear Coriolis vibration methods [4], which sensitivity can not exceed the value of one arbitrary unit.

Fluid flow metering by the proposed nonlinear vibration method can be carried out in the following operational procedure. At first, the rectilinear part of a pipe to be tested is chosen, then the ends of this part of the pipe have to be fixed, but its middle section must be connected with an additional nonlinear elastic support. After that the second mode of pipe resonant flexural vibrations is excited and parameters Δu or u_3/u_1 of these vibrations are recorded.

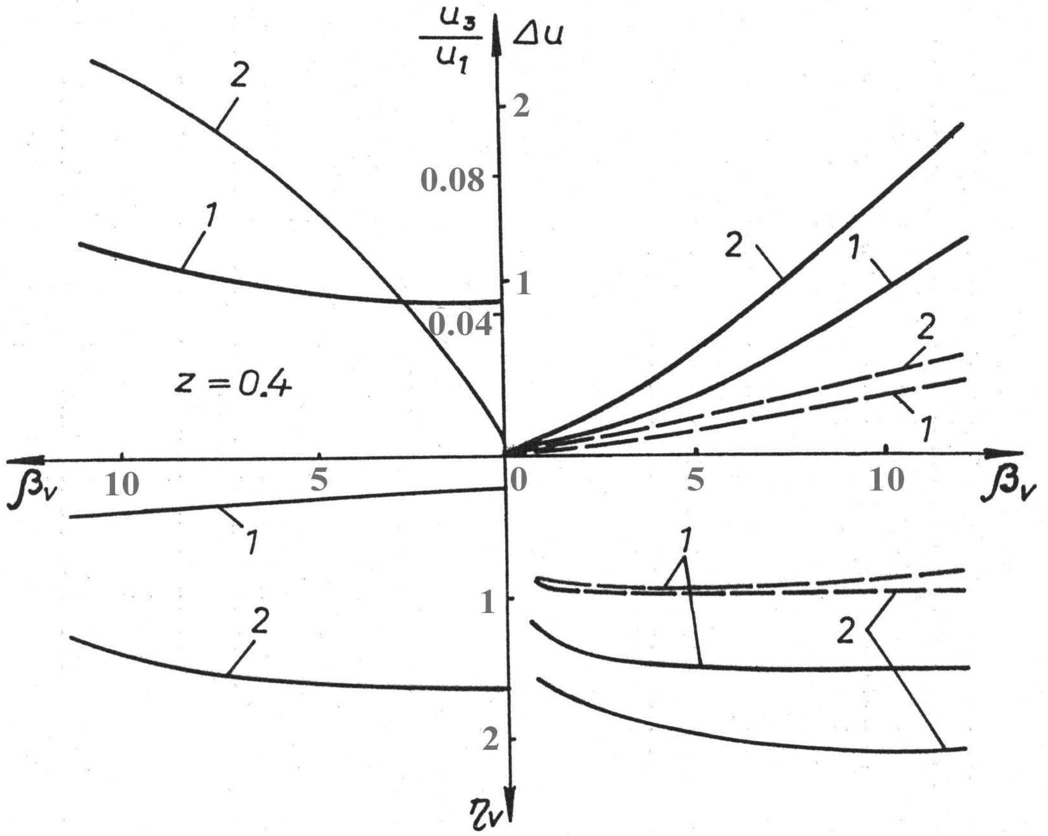

Figure 4. Calibration curves and sensitivity functions

The flow rate Q is evaluated by the measured value of vibration parameter (Δu or u_3/u_1) with the aid of calibration curve preliminary constructed (similar curves are presented in Figure 4).

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Cifanskis S., Beresņevičs V. Caurules ar cauri plūstošu šķidrumu nelineāras svārstības un to izmantošana šķidruma patēriņa noteikšanā
Izpētīta izplūstošā šķidruma plūsmas ietekme uz caurules iecirkņa, kurš savstarpēji iedarbojas ar nelineāru elastīgu balstu, uzspiestajām lieces svārstībām. Atklātas caurules ar šķidruma plūsmu tajā rezonanses lieces svārstību formu un spektrālo raksturlielumu specifiskas izmaiņas. Izmantojot atklātās nelineāro svārstību īpatnības, piedāvātas jaunas pa cauruli izplūstošā šķidruma daudzuma noteikšanas metodes, kuru jutīgums 1,5 – 2 reizes augstāks, salīdzinot ar tradicionālajām lineārajām kontroles metodēm.

Tsyfansky S., Beresnevich V. Nonlinear Vibrations of a Pipe Containing Flowing Fluid and Their Applications in Flowmetering
Forced flexural vibrations of a pipe conveying fluid and interacting with a nonlinear elastic support are investigated. Specific distortions of flexural modes and spectrum of pipe's resonant vibrations due to flowing fluid are studied. It is shown, that the rise of flow rate leads to the pronounced amplification of the third and the fifth harmonic components in vibration spectrum. The utilization of nonlinear phenomena has made it possible to develop new vibration methods of flow-metering. The sensitivity of proposed nonlinear vibration procedures is about 1.5 - 2 times higher in comparison with familiar linear methods.

Цыфанский С., Бересневич В. Нелинейные колебания трубы с протекающей жидкостью и их использование для определения расхода
Изучено влияние потока протекающей жидкости на вынужденные изгибные колебания участка трубы, взаимодействующего с нелинейной упругой опорой. Вскрыты специфические искажения спектральных характеристик и форм резонансных изгибных колебаний трубы под воздействием потока жидкости. На основе использования обнаруженных нелинейных свойств предложены новые вибрационные методы определения расхода, чувствительность которых в 1,5 – 2 раза выше по сравнению с чувствительностью традиционных линейных методов контроля.