DE GRUYTER OPEN
RIGA TECHNICAL UNIVERSITY 1862
Electrical, Control and Communication Engineering
ISSN 2255-9159 (online)
ISSN 2255-9140 (print)
2017, vol. 13, pp. 75–80
doi: 10.1515/ecce-2017-0011
https://www.degruyter.com/view/j/ecce

Computationally Efficient Chaotic Spreading Sequence Selection for Asynchronous DS-CDMA

Anna Litviņenko* (*lecturer, Riga Technical University*),
Artūrs Āboltiņš (*associate professor, Riga Technical University*)

***Abstract* – The choice of the spreading sequence for asynchronous direct-sequence code-division multiple-access (DS-CDMA) systems plays a crucial role for the mitigation of multiple-access interference. Considering the rich dynamics of chaotic sequences, their use for spreading allows overcoming the limitations of the classical spreading sequences. However, to ensure low cross-correlation between the sequences, careful selection must be performed. This paper presents a novel exhaustive search algorithm, which allows finding sets of chaotic spreading sequences of required length with a particularly low mutual cross-correlation. The efficiency of the search is verified by simulations, which show a significant advantage compared to non-selected chaotic sequences. Moreover, the impact of sequence length on the efficiency of the selection is studied.**

***Keywords* – Baseband; Binary sequences, Chaotic communication; Communication systems; Direct-sequence code-division multiple access; Minimization; Wireless sensor networks.**

I. Introduction

Digital communications play an important role for the well-being of modern society. Spread spectrum modulation schemes are employed in mobile networks, short-range communications and positioning systems. Moreover, code-division multiple access employs a spread spectrum approach for enabling access to the same frequency and time resources by multiple users. The choice of the spreading sequences in those systems is very important since it determines the level of multiple access interference (MAI) and, ultimately, the capacity of the communication system.

As has been noted in [1], for generation of spreading sequences in direct-sequence code-division multiple-access (DS-CDMA) systems, chaos phenomena can be used. As is shown in [2], chaotic spreading sequences have potential to increase the resistance to fading, as well as to increase the security level of wireless networks [3]. On other hand, chaotic synchronization [4] is suitable for providing time and frequency synchronization in the communication system.

Chaos allows generating a very large variety of sequences. However, not all sequences are well-suited for spread spectrum communication systems. Methods for the generation and selection of chaotic sequences for DS-CDMA have been extensively studied in scientific literature [5]–[6]. For example, in [5], a genetic programming approach for the generation and selection of sequences is presented, whereas in [7] over-sampled chaotic map binary sequences are discussed.

Instead of generating sequences with good cross-correlation properties, one can generate a large set of sequences and then select the most suitable ones. For example, papers [8], [9] and [10] present a possibility of generating short-length binary chaotic sequences with the same or better cross-correlation properties, compared to m-sequences, which are widely used in commercial DS-CDMA systems.

In this research, a computationally efficient exhaustive search and selection algorithm is used to obtain relatively short chaotic binary sequences with a length of 15–127 chips, which provide a much lower level of MAI compared to arbitrarily chosen ones. The proposed method allows building simple multi-user communication systems, especially suitable for low-traffic applications, such as sensor networks [3].

II. Description of the Research

A. Asynchronous DS-CDMA System

A block diagram of the DS-CDMA communication system is shown in Fig. 1. K transmitters and K correlation receivers are used, where the number of transmitter and receiver pairs K is equal to the number of selected chaotic spreading sequences, which is designated as N_{sel}.

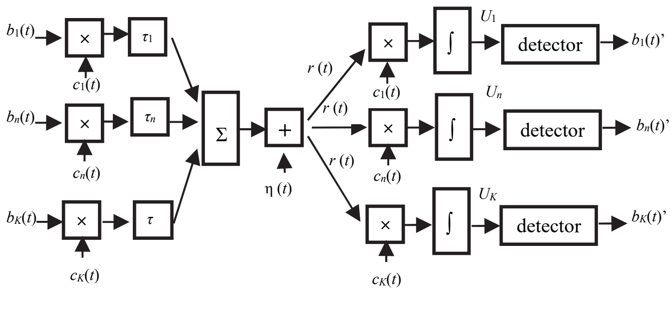

Fig. 1. Block diagram of asynchronous DS-CDMA baseband communication system model.

Each transmitter and receiver pair uses a unique chaotic spreading sequence $c(t)$ with chip duration T_c. In the transmitter, randomly generated information bits $b(t)$ are spread by the corresponding unique spreading sequence $c(t)$ and then transferred to the communication channel. Each transmitter has its own transmission start moment τ, the duration of which, T_b, is less than one bit and is equal to the integer number of chips. In the common communication channel with AWGN noise $\eta(t)$, all transmitted signals with

* Corresponding author.
E-mail: anna.litvinenko@rtu.lv

different starting moment delays τ are summed together causing MAI to one another. In the proposed scenario with high MAI distortion, the input signal at each receiver (1) is simply the sum of the transmitted signals.

$$r(t)=\sum_{i=1}^{K} b_i(t)c_i(t)+\eta(t). \tag{1}$$

At time moment $jT_b = jLT_c$, the following signal values U_n at the output of correlation receiver n and at the input of threshold detector n are obtained:

$$U_n = A\int_{jT_b-T_b}^{jT_b}\left[\sum_{i=1}^{K} b_i(t)c_i(t)+\eta(t)\right]c_n(t)\mathrm{d}t, \tag{2}$$

where A is a parameter that characterizes correlation receiver design, $T_b = LT_c$ is the duration of one received spread bit or the duration of a chaotic spreading sequence, and j is the index of the received information bit. For converting the received signal into information signal $b_n(t)$', the threshold detector compares obtained values U_n to zero. The channel noise and MAI have a direct impact on the correctness of detector decisions.

B. Generation of Chaotic Spreading Sequences

For the spreading and despreading of the information bits in DS-CDMA systems, spreading sequences are employed. In this study, binary spreading sequences are used and they are obtained from chaotic sequences. Chaotic sequence x is generated by means of a one-dimensional (1D) map. In this study, the chaotic sequence generation algorithm employs the well-known logistic 1D map (3). The bifurcation parameters of the logistic 1D map have been adjusted to obtain chaotic behaviour of the generated sequences. A logistic map with $\lambda = 4$ and $x \in (0; 1)$ is defined as follows:

$$x_{k+1} = \lambda x_k (1 - x_k). \tag{3}$$

The initial condition x_0 is randomly chosen from the x domain.

Since chaotic sequences coming from the logistic equation have an infinite alphabet, conversion into binary form is necessary. For converting chaotic sequence x into binary non-return-to-zero (NRZ) sequence c, the threshold rule (4) is used.

$$c_k = \begin{cases} 1, & x_k \ge \Theta; \\ -1, & x_k < \Theta. \end{cases} \tag{4}$$

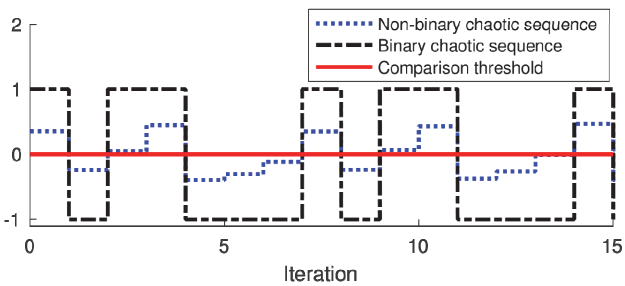

Fig. 2. Conversion of chaotic sequences into binary form.

The value of the threshold $\Theta = 0.5$ is equal to the mid-point of the x domain to provide balance between "–1" and "1" elements within the generated chaotic sequences. As an example, Fig. 2 illustrates conversion of a 15-chip-long chaotic sequence x into a binary chaotic sequence c.

As was demonstrated in [5], division of the state space of a chaotic 1D map into parts and using different initial conditions leads to the generation of completely different binary sequences. This property allows generating a large set of binary chaotic sequences with the aim of selecting the required number of sequences with the desired cross-correlation properties.

C. Selection of Chaotic Spreading Sequences

To minimize MAI within the DS-CDMA system, binary sequences with a low mutual correlation must be used. Note that in the case of asynchronous DS-CDMA, the peak value of the cross-correlation function plays the key role. Taking into account these considerations, the following hypothesis is formed: minimization of the sum of periodic cross-correlation (PCC) maximum absolute values for selected sequences from all the possible pair combinations will lead to the minimization of MAI within the DS-CDMA communication system. A selection algorithm that performs an exhaustive search for the smallest sum of the absolute values of PCC peaks has been created. A version of this algorithm for selecting four sequences is reflected in Algorithm 1. It consists of multiple nested loops, which provide combination of all the possible PCC absolute peak values Φ. Practical implementations of the algorithm in MATLAB and C++ are based on the use of a recursive function, which allows selecting an arbitrary number of sequences from the given set. Please note that the proposed algorithm can be used with any sets of sequences where minimization of cross-correlation peaks is required. The largest drawback of the proposed algorithm lies in the necessity to check a very large number of combinations. The number of possible combinations can be found by using the well-known combination formula (5). For example, in the case of 8 out of 100 sequences there are more than $18 \cdot 10^{10}$ combinations.

Data: Periodic cross-correlation functions for all possible generated chaotic sequences
    $[R_{1,2}, R_{1,3}, \ldots, R_{1,N_{all}}, \ldots, R_{N_{all}-1,N_{all}}]$
Data: Number of generated sequences N_{all}
Result: Indexes of selected seq-ces N_{1final}, N_{2final}, N_{3final}, N_{4final}
initialize the value of selection criterion $\Phi_{sel} = \infty$;
for $N_1 = 1 : N_{all} - 3$ do
    for $N_2 = N_1 + 1 : N_{all} - 2$ do
        for $N_3 = N_2 + 1 : N_{all} - 1$ do
            for $N_4 = N_3 + 1 : N_{all}$ do
                $\Phi = \max|R_{N1,N2}| + \max|R_{N1,N3}| + \max|R_{N1,N4}| + \max|R_{N2,N3}| + \max|R_{N2,N4}| + \max|R_{N3,N4}|$;
                if $\Phi_{sel} > \Phi$ then
                    $N_{1final} = N1$;
                    $N_{2final} = N2$;
                    $N_{3final} = N3$;
                    $N_{4final} = N4$;
                end
            end
        end
    end
end

Algorithm 1. Algorithm for the selection of four sequences.

$$N_{all}N_{sel} = \frac{N_{all}!}{N_{sel}!(N_{all}-N_{sel})!}. \quad (5)$$

To reduce the number of possible combinations, we propose a second algorithm: only sequence pairs with cross-correlation peaks less than defined threshold value ξ are sent to the selection algorithm. Since the maximum absolute PCC values depend on the length of sequences, L, threshold ξ also changes according to it. Moreover, since the set size N_{all} is fixed, the increasing of the number of selected sequences N_{sel}, i.e. users of the DS-CDMA system, decreases the possibility of selecting sequences with a low cross-correlation level, and threshold value ξ must be increased. Considering these reasons, the estimation of the threshold value is made iterative and adaptive.

Data: accuracy level Δ
Result: threshold ξ_{final} for periodic cross-correlation level
initialize interval of search from $\xi_{min} = 0$ to $\xi_{max} = 1$
while $|(\xi_{max} - \xi_{min})| > \Delta$ **do**
    set threshold at the middle $\xi = (\xi_{max} + \xi_{min})/2$
    search for a set of sequences that have all periodic cross-correlation values lower than threshold ξ
    if success **then**
        $\xi_{max} = \xi$;
        $\xi_{final} = \xi$;
    else
        $\xi_{min} = \xi$;
    end
end

Algorithm 2. Algorithm of PCC threshold adaptation.

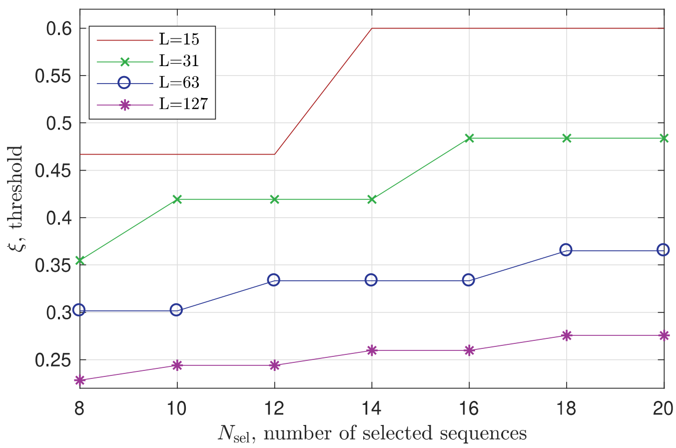

Fig. 3. Threshold levels for 15 to 127 chip-long chaotic spreading sequences.

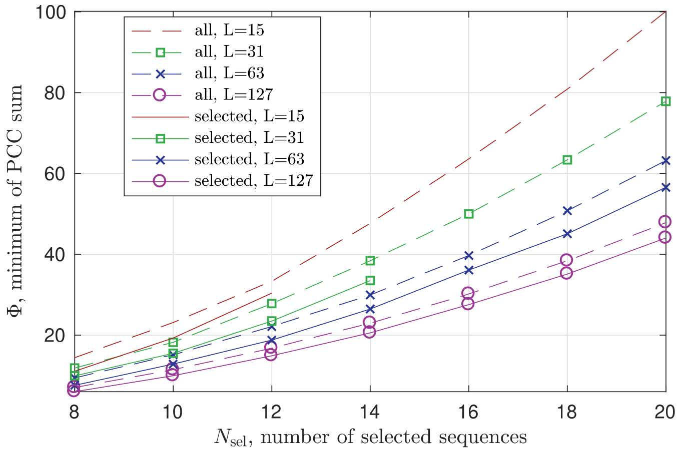

Fig. 4. Dependence of PCC peak sum on the number of selected or all generated chaotic sequences.

The bisection-based threshold estimation method is reflected in Algorithm 2.

If the required accuracy is $\Delta = 10^{-12}$, then the number of required iterations for threshold estimation is more than 20. Fig. 3 presents the dependence of estimated threshold levels on the number and length of the selected sequences. In most cases, the proposed method for selecting chaotic spreading sequences reduces the number of combinations more than 10^6 times, resulting in the possibility of sequence selection in reasonable time. The selection of most of the sequences was completed in less than 24 hours if the sequence search was performed by a C++ program running on a single computer core. The use of a supercomputer allowed performing selection of various sets of sequences simultaneously.

Fig. 4 shows the dependence of the selection criteria on the number and length of the sequences. From this figure, it can be noticed that selection considerably reduces the sum of absolute maximum PCC values. The absence of some values is caused by the impossibility of selecting the desired number of sequences in reasonable time.

III. Performance Evaluation

A. Communication System Simulation

For the verification of the proposed hypotheses and algorithms MATLAB- and C++ based simulations of baseband asynchronous DS-CDMA systems, built in accordance with in Section II.A, have been created. Data transmission is carried out in AWGN channel and at the conditions of perfect chip synchronization. The parameters for the simulation of asynchronous DS-CDMA communication systems are presented in Table I.

TABLE I
PARAMETERS OF THE MODEL

Parameter	Value
Information bits *b*	Random NRZ sequences
Chaotic spreading sequence c	NRZ sequences generated by 1D maps and comparison rule, and selected according to Algorithm 3.1, Algorithms 3.1 and 3.2, or without selection
Length of spreading sequences	L_c = 15; 31; 63; 127
Initial parameters for generation of chaotic sequences x_0	Randomly generated from *x* domain corresponding to map
Number of transmitter-receiver pairs *K*	*K* = [4; 8] or *K* = [8; 20]
Transfer mode	Synchronous or asynchronous
Starting moment τ	Synchronous mode: τ = 0; Asynchronous mode: $\tau \in [0; L_c - 1]$
Message duration	128 bits
Number of generated sequences N_{all}	Variable, depending on selection algorithm

B. Analysis of Simulation Results

This section presents the validation of chaotic sequence selection using the performance analysis of an asynchronous DS-CDMA system based on chaotic spreading sequences.

Fig. 5 shows the average bit error ratio (BER) for an asynchronous DS-CDMA system based on chaotic spreading sequences with diverse numbers of users in case of E_b/N_0 = 20 dB. It can be noted that a DS-CDMA system with longer chaotic spreading sequences enables satisfactory communication amongst a larger number of users. However, it must be remembered that the increasing of spreading sequence length reduces the data transmission rate.

Figs. 6–9 show the dependence of performance on the AWGN noise for an asynchronous DS-CDMA system based on chaotic spreading sequences of different length.

According to Fig. 6, in case of 15-chip-long sequences, only a 8-user communication system shows a satisfactory BER level below 10^{-3}. Thus, to implement data transfer in an asynchronous DS-CDMA system based on chaotic spreading sequences with the number of users equal to or higher than 8, either the length of the chaotic spreading sequences should be larger than 15 chips, or the selection of the chaotic sequences should be performed from a set larger than 100 sequences.

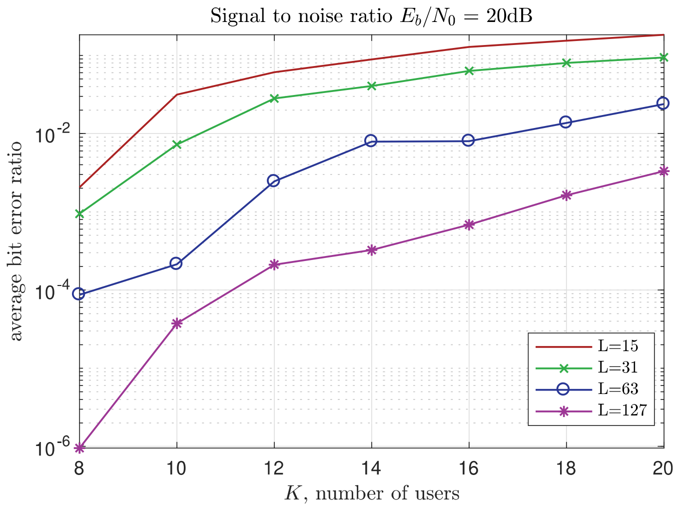

Fig. 5. Asynchronous DS-CDMA: BER dependence on the number of users for 15 to 127 chip-long chaotic sequences generated using logistic, Gauss, cubic maps.

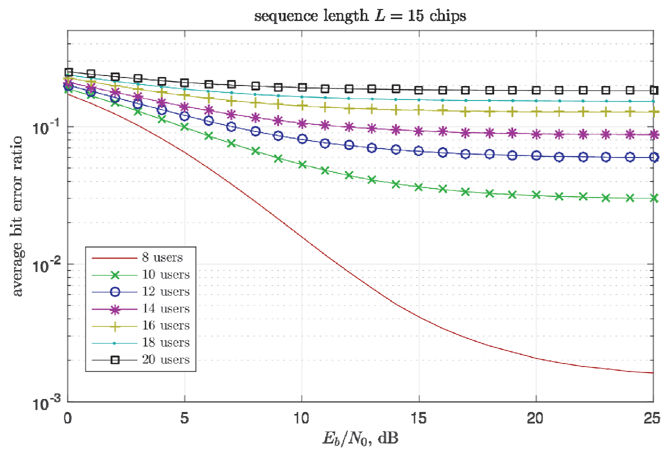

Fig. 6. Asynchronous DS-CDMA: BER performance in case of 15-chip-long chaotic spreading sequence.

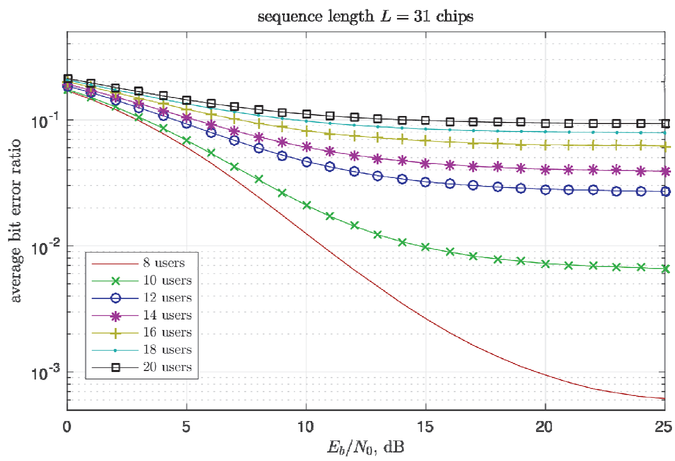

Fig. 7. Asynchronous DS-CDMA: BER performance in case of 31-chip-long chaotic spreading sequence.

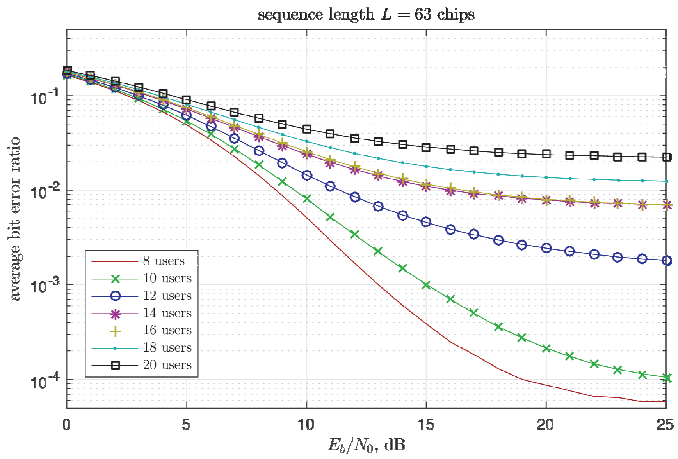

Fig. 8. Asynchronous DS-CDMA: BER performance in case of 63-chip-long chaotic spreading sequence.

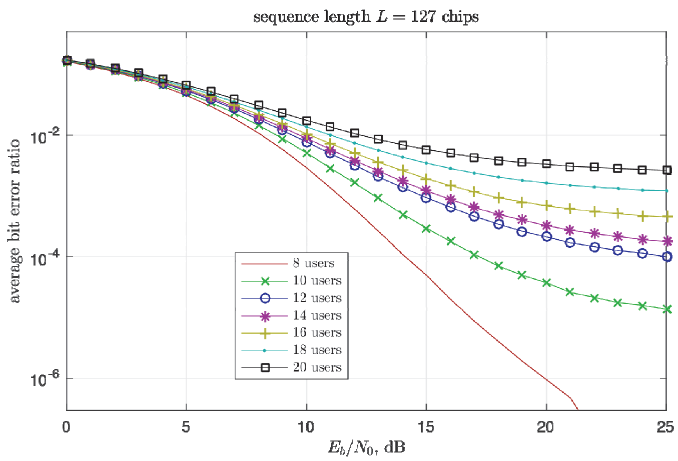

Fig. 9. Asynchronous DS-CDMA: BER performance in case of 127-chip-long chaotic spreading sequence.

Fig. 7 shows the results for 31-chip-long sequences. These sequences could be used in a DS-CDMA system with up to eight users. A further increase of the number of users up to 10 degrades system performance about tenfold in the case of SNR per bit at a level of 20 dB.

Fig. 8 shows that, in the case of 63-chip sequences, a system with ten users can provide a satisfactory BER level if E_b/N_0 exceeds 15 dB.

Fig. 9 presents the BER performance results for a communication system employing 127-chip-long sequences. In this case, the number of users may be increased to up to 16–18, with a BER level of around 10^{-3} at SNR per bit at a level of 20 dB. Moreover, by using those sequences, the BER level can be decreased to 10^{-6} for up to eight simultaneous users. This mode is especially suitable for simple communication systems that do not employ forward error correction (FEC).

Finally, the selection effectiveness is depicted in Figs. 11–13, which present a comparison of BER in communication systems employing selected and

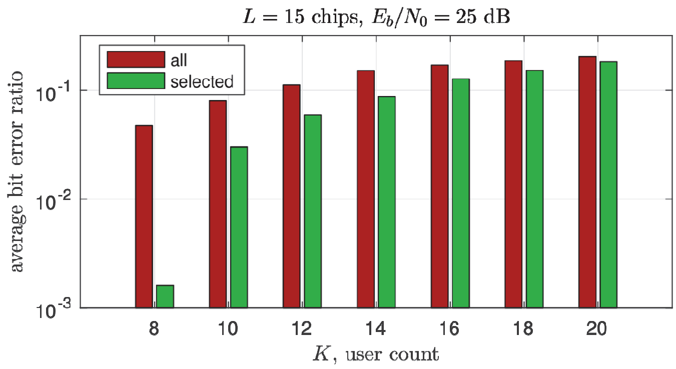

Fig. 10. Asynchronous DS-CDMA: BER levels for selected and all 15-chip-long sequences generated by logistic map at SNR per bit at a level of 25 dB.

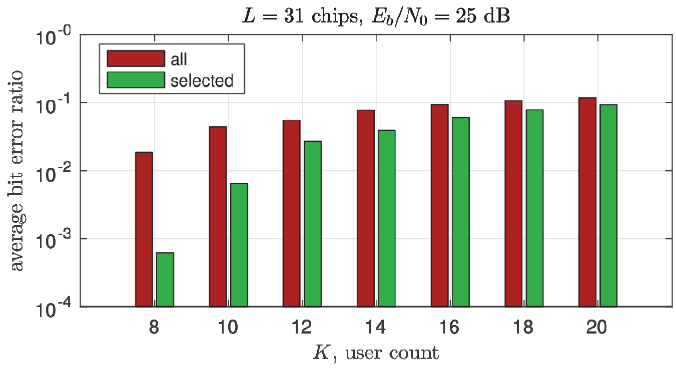

Fig. 11. Asynchronous DS-CDMA: BER levels for selected and all 31-chip-long sequences generated by logistic map at SNR per bit at a level of 25 dB.

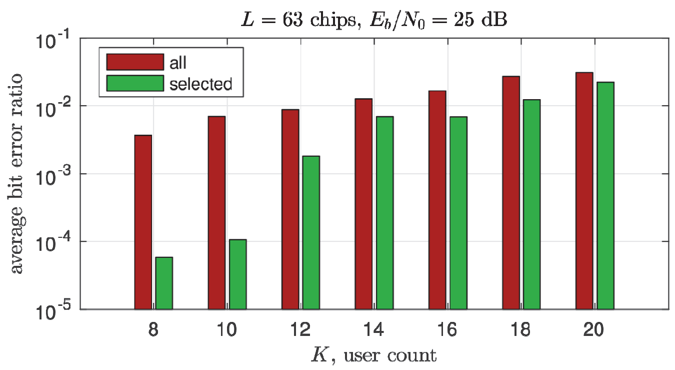

Fig. 12. Asynchronous DS-CDMA: BER levels for selected and all 63-chip-long sequences generated by logistic map at SNR per bit at a level of 25 dB.

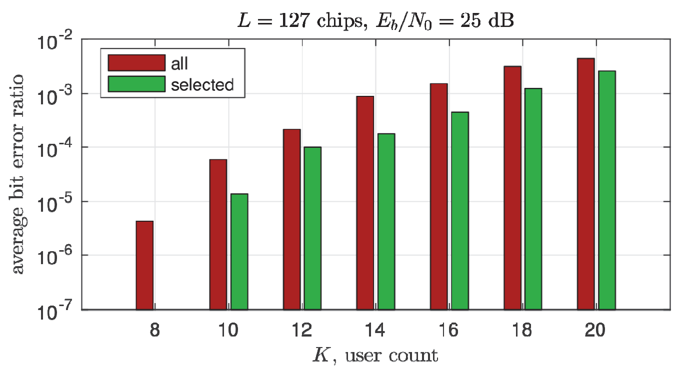

Fig. 13. Asynchronous DS-CDMA: BER levels for selected and all 127-chip-long sequences generated by logistic map at SNR per bit at a level of 25 dB.

arbitrarily chosen sequences for different numbers of users with E_b/N_0 at a level of 25 dB. All the plots show significantly lower BER levels in systems employing selected chaotic spreading sequences for a number of users below 10 and this evidences the effectiveness of the proposed methods.

A small difference between the BER levels of selected and arbitrarily chosen sequences points to an insufficient number of sequences in the set. Therefore, to achieve a higher quality of data transmission for twelve or more users, the selection of chaotic spreading sequences should be performed within a larger set.

IV. Discussion

In this study, we have presented a novel approach to the selection of chaotic spreading sequences for asynchronous DS-CDMA communication systems. It is based on the minimization of the periodic cross-correlation (PCC) peak sum among the selected sequences. Relatively short (15 to 127 chip-long) spreading sequences were analysed since in this case the selection provides the most noticeable gain. Due to computational complexity, the proposed algorithm can ensure the selection of up to 20 sequences out of 100 if the length of the sequence is 15 chips. The performed Monte-Carlo simulations of DS-CDMA systems by employing chaotic spreading sequences have confirmed the validity of the proposed approach. Using the selected chaotic spreading sequences has led to a more than tenfold reduction of the bit error ratio (BER). Moreover, it has been noticed that the highest gain can be obtained if the number of simultaneous users is small, i.e. less than 12. DS-CDMA communication systems employing 15 to 31 chip-long spreading sequences can efficiently accommodate up to eight users, whereas systems with longer sequences can serve up to 18 users with satisfactory $\mathrm{BER} < 10^{-3}$. These BERs are sufficient for employing forward error correction (FEC) in the communication system to improve the transmission quality.

Another advantage of the proposed method lies in the employment of a PCC threshold for the reduction of sequences to be analysed before the selection. In the case of sequences used in this study, the threshold-based selection algorithm reduces the number of checked combinations of sequences on an average of more than 10^6 times. In some cases, the algorithm provides even an up to 10^{19}-fold reduction in the number of combinations and allows working with sets that require the testing of more than 10^{21} combinations.

Selection of the chaotic spreading sequences from larger pools of available sequences would allow finding even better spreading sequences in terms of PCC level and MAI within the DS-CDMA communication system. However, it is a computationally demanding task and requires the implementation of highly efficient algorithms.

References

[1] A. Abel and W. Schwarz, "Chaos communications - Principles, schemes, and system analysis," *Proceedings of the IEEE*, vol. 90, no. 5. pp. 691–710, May-2002. https://doi.org/10.1109/jproc.2002.1015002

[2] S. M. Berber and F. Shu, "Chaos-Based Physical Layer Design for WSN Applications," in *Proceedings of the 17th International Conference on Communications (part of CSCC'13)*, 2013, vol. 1, no. 2, pp. 157–162.

[3] D. P. Kong, J. Da Zhu, T. He, and Y. F. Zhou, "The study of chaos encryption algorithm for wireless sensor networks based on the reconfigure technology of FPGA," *J. Chem. Pharm. Res.*, vol. 6, no. 4, pp. 359–364, 2014.

[4] G. Setti, R. Rovatti, and G. Mazzini, "Synchronization Mechanism and Optimization of Spreading Sequences in Chaos-Based DS-CDMA Systems," *IEICE Trans. Fundam. Electron. Commun. Comput. Sci.*, vol. E82–A, no. 9, pp. 1737–1746, 1999.

[5] V. Varadan and H. Leung, "Design of piecewise maps for chaotic spread-spectrum communications using genetic programming," *IEEE Trans. Circuits Syst. I Fundam. Theory Appl.*, vol. 49, no. 11, pp. 1543–1553, Nov. 2002. https://doi.org/10.1109/tcsi.2002.804545

[6] A. Litvinenko and A. Aboltins, "Use of cross-correlation minimization for performance enhancement of chaotic spreading sequence based asynchronous DS-CDMA system," in *2016 IEEE 4th Workshop on Advances in Information, Electronic and Electrical Engineering (AIEEE)*, 2016, pp. 1–6. https://doi.org/10.1109/aieee.2016.7821812

[7] J. Jie Yang, M. H. Moon Ho Lee, and M. Mingqi Jiang, "Optimal over sampled chaotic map binary sequences for CDMA," in *2002 IEEE International Symposium on Circuits and Systems. Proceedings (Cat. No.02CH37353)*, 2002, vol. 2, p. II-289-II-291. https://doi.org/10.1109/iscas.2002.1010981

[8] A. Litvinenko and E. Bekeris, "Correlation properties of binary spreading sequences generated by chaotic logistic map," *Sci. J. RTU, Telecommun. Electron.*, vol. 9, pp. 39–43, 2009.

[9] A. Litvinenko and E. Bekeris, "Statistical Analysis of Multiple Access Interference in Chaotic Spreading Sequence Based DS-CDMA Systems," *Electronics*, vol. 21, no. 1, pp. 34–37, 2017.

[10] A. Litvinenko and A. Aboltins, "Selection and performance analysis of chaotic spreading sequences for DS-CDMA systems," in *2016 Advances in Wireless and Optical Communications (RTUWO)*, 2016, pp. 38–45. https://doi.org/10.1109/rtuwo.2016.7821852

Anna Litviņenko received her B. sc. and M. sc. in electronics and telecommunications engineering from Riga Technical University (RTU), Latvia, in 2008 and 2010, respectively. In 2017 she has received a Dr. sc. ing. degree at RTU.

She is a researcher at the Institute of Radioelectronics and a lecturer at the Department of Electronics Fundamentals, RTU, Latvia. Her research interests include spread spectrum and multicarrier communications. She is a member of the IEEE and an active volunteer at the IEEE Latvia Section. She received the Best Paper Presentation Award at the 2016 IEEE 4th Workshop on Advances in Information, Electronic and Electrical Engineering (AIEEE).

ORCID iD: http://orcid.org/0000-0003-4734-2182

Artūrs Āboltiņš holds a Dr. sc. ing. degree in electronics and telecommunications engineering from Riga Technical University (RTU), Latvia (2013).

He is a Senior Researcher at the Institute of Radioelectronics and an associate professor at the Department of Electronics Fundamentals, RTU, Latvia. His research interests are related to the use of unitary transformations and software-defined radio. He is an organizer and TPC chair of the 2017 scientific conference "Advances in Wireless and Optical Communications" (RTUWO). He is a member of the IEEE and an officer at the IEEE Latvia Section.

ORCID iD: http://orcid.org/0000-0001-6901-9787