

RIGA TECHNICAL UNIVERSITY
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**DETECTION OF THE MOST VULNERABLE
NETWORK ELEMENTS IN POWER SYSTEMS**

Summary of the Doctoral Thesis

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I hereby declare that the Doctoral Thesis submitted for the review to Riga Technical University for the promotion to the scientific degree of Doctor of Science (Ph. D.) is my own. I confirm that this Doctoral Thesis had not been submitted to any other university for the promotion to a scientific degree.

Aleksejs Soboļevskis (signature)

Date:

The doctoral thesis has been written in English and consists of Introduction; 3 chapters; Conclusions; 16 tables; 68 figures; the total number of pages is 140 including 2 appendices. The Bibliography contains 180 titles.

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INTRODUCTION

Topicality of the Subject of the Doctoral Thesis

With the development of market relations in the electric power industry, the calculation of regimes for their analysis and forecasting is a key element in ensuring the uninterrupted functioning of the electricity market. Ensuring regime reliability is one of the most priority tasks of operational dispatch management.

In connection with the expected reforms, as well as taking into account the need to address the priorities of the strategy of the European Union, the following aspects are important:

- increasing of energy efficiency;
- development of the electricity market;
- increasing of the reliability of supply;
- maximum use of renewable energy resources to increase the competitiveness of the price of such energy;
- preservation of environment during the production, transmission and distribution of different types of energy.

Electric power systems are complex objects of dispatching and management requiring compliance with the restrictions related to reliability, safety, quality and economy of their operating modes. These requirements must be fulfilled by all the subjects that make up the technological basis of the electricity market.

Stable operation of any power system depends on the ability of continuous provision of generated and consumed power balance, as well as the level required for the quality of electric power. In order to assess the stable operation of the power system, a whole range of different software package is designed, which can be used to model a variety of operating modes. In near future, precise methods of predicting and evaluating stability will be required [1] taking into account the prospects of growth of share of renewables.

Modern electric power systems are characterized by large sizes. Mathematical models that allow to forecast and minimize the number of violations of technical regulations are used for analysing power system modes. This also solves the tasks of increasing stability and quality of electric power system (EPS) operation modes, including determination of allowability of mode parameters in normal and emergency modes, increasing the transfer capability and satisfying the growing demand for electric power.

Building mathematical models of power systems provides an opportunity to control the EPS's operating mode, forecast the EPS's reaction to removal of certain parts of the EPS from operation for repairs as well as to identify the EPS's weak points. During the design phase, the obtained information allows to plan the building of an equally strong network and prevent dangerous and undesired situations. During the operation phase, it allows developing protection measures for minimizing damage to the power systems and economy caused by potential emergency situations.

Five power system model scenarios were analyzed and compared in this Thesis (Scenario 1, 2, 3, 4, and 5). The author of the present Thesis studies EPS models in conditions close to real-world. The said models partially reflect the situation of EPS in the Baltic States without claiming detailed similarity to the latter. The current one (according to the situation in Scenario 1) and the planned one according to the situation in Scenario 4 with the expected modernization and new interconnections of transmission lines, and another according to the situation in Scenario 5 with the expected modernization and new interconnections of transmission lines and construction of new large wind parks. The possibility and efficiency of the application of method of sensitivity indicators for solving problems of analysis and control of the modes of electrical networks shows that there is a wide range of applications of the developed method.

The Hypothesis, Aims and Objectives of the Thesis

The current stage in the development of world energy is characterized by the creation of powerful territorially unified energy systems. In the conditions of the free market at the moment, the energy markets of different countries are gradually unifying: the network configurations are changing, additional connections are emerging, new energy companies and unions are formed, and the question of finding and creating new methods and algorithms for calculating and analyzing complex EPS becomes more topical. In particular, this concerns the issue of solving the problems of static stability of EPS with detailed consideration of the main elements of systems.

Also, it should be noted that the active implementation of renewable energy is mainly justified by three reasons:

- environmental problems associated with the extraction and processing of energy resources of traditional energy;
- limited traditional energy resources and the unlimited resources of renewable energy;
- wind power is one of the most efficient alternatives.

The main aim of this dissertation is to propose a method for detecting network parameters that are most sensitive to external influences, determining their relation to EPS parameters. This information can be used to improve the behavioral characteristics of the EPS (reliability and controllability) by finding the most sensitive areas of EPS – nodes and lines. Singular Jacobian matrix and nodal conduction analysis methods are used to solve the issue and do not require statistical testing and a large number of variants of external exposure scenarios; thus, significantly simplifying the estimation process.

Hypothesis: The presence of weaknesses in the electric power system is determined by its structural properties (network topology, the presence and placement of generators and load nodes), as well as the parameters of the elements of the system (resistance of the connections, load parameters and generators, etc.). Timely and qualified determination of weaknesses in the electric power system allows developing and substantiating measures to strengthen them by changing the network scheme, the structure of generating capacities and location of generators, installation of compensating devices, the choice of means of emergency

automatics, etc. It is quite obvious that the determining factor of these activities is to provide an optimal structure of generating capacities, which includes the required range and required regulatory dynamics. In addition, like any complicated system, the EPS is heterogeneous and uneven. In particular, this is manifested by the fact that the perturbations applied in different places of the EPS cause noticeable reaction of the same EPS elements, i.e. localization of different perturbations, the voltage modules mostly change at the same nodes. Thus, the timely detection (localization) of the above-mentioned nodes and lines is necessary to increase the reliability of the ES and prevent the emergence of a number of emergency situations.

Methods and Tools of the research

The results of the Thesis were obtained by applying the following computer programs and methods:

- 1) calculations based on the method of using the inverse Jacobian matrix and matrix Z ;
- 2) EPS modeling in the environment of *AutoDESK AutoCAD*, *MathCAD*, and *Regus*;
- 3) EPS analysis using computer programs of *MathCAD*, *Regus*, and *Microsoft EXCEL*;
- 4) The result processing and graphical representation in *Microsoft EXCEL*, and *AutoDESK AutoCAD*.

Novelty of the Doctoral Thesis and Basic Results

The formation of the right directions for the development of power industry can be carried out on the basis of detailed studies of stationary and transitional modes of the electric power system, including static stability. At the same time, the complex of factors, structure and possible state of the electric power system must be taken into account. The starting point is the set requirements for the power system and, accordingly, for equipment and control systems.

This research is devoted to the detection of most vulnerable points of electricity networks, as well as an impact of integration of large-scale wind farm in the power system, in terms of sustainability.

Practical Importance of the Thesis

The purpose of the Thesis is to provide methods for the detection of network parameters that are most vulnerable (voltage drop) to external impacts, to determine their relationship with EPS parameters, and to try to use this information to improve behavioral properties of EPS. Thus, availability of information on the location of sensitive elements allows determining and controlling the nodes and lines in which the biggest oscillations of operational parameters are observed due to disturbances in the system.

Author's Personal Contribution to the Research Conducted

1. EPS models for a 330 kV network (Scenario 1, 2, 3, 4, and 5) were developed for the purpose of identifying the power system's most sensitive points.
2. Calculations for the studied EPS scenarios were made to find the most sensitive points.
3. The obtained results were analyzed and respective conclusions were made.

Approbation of the Doctoral Thesis

The results were reported and discussed at the following international conferences.

1. "Assessment of Static Stability of Power System of the Baltic States in View of the Planned Synchronization with Networks of Western Europe to 2030", 2018 IEEE International Conference on Environment and Electrical Engineering and 2018 IEEE Industrial and Commercial Power Systems Europe (EEEIC / I&CPS Europe), Italy, Palermo, 12–15 June 2018.
2. "Prediction of Latvian Electrical Power System for Reliability Evaluation Including Wind Energy", 2017 IEEE 17th International Conference on Environment and Electrical Engineering and 2017 IEEE Industrial and Commercial Power Systems Europe (EEEIC / I&CPS Europe), Italy, Milan, 6–9 June 2017.
3. "Wind Power Plant Influence on the Latvian EPS Stability", SEEP 2017: 10th International Conference on Sustainable Energy & Environmental Protection, Slovenia, Bled, 27–30 June 2017.
4. "Latvian Electrical Power System Stability's Analysis Taking into Account New Development Strategy until 2025", 11th International Conference on Electromechanical and Power Systems (SIELMEN 2017): Proceedings, Moldova, Chisinau, 12–13 October 2017.
5. "Analysis of Vulnerability of the Latvian Electrical Power System", 2016 IEEE 16th International Conference on Environment and Electrical Engineering (EEEIC), Italy, Florence, 7–10 June 2016.
6. "Assessing the Impact of Registering of Weak Points Calculating the Power System Operating Modes", 2016 57th International Scientific Conference on Power and Electrical Engineering of Riga Technical University (RTUCON 2016) Latvia, Riga, 13–14 October 2016.
7. "Vulnerability Assessment of Electric Power System for the Case of Latvian EPS", 2015 56th International Scientific Conference on Power and Electrical Engineering of Riga Technical University (RTUCON): Proceedings, Latvia, Riga, 14 October 2015.
8. "Localization of the Roots of the Characteristic Polynomial (CP) and Isolines to Analyze Static Stability", 2014 55th International Scientific Conference on Power and Electrical Engineering of Riga Technical University (RTUCON): Proceedings, Latvia, Riga, 14 October 2014.

Author's Publications

The results were reported and discussed at 8 international conferences.

1. I. Zicmane, K. Bērziņa, **A. Sobolevskis**, S. Kovaļenko. Assessment of Static Stability of Power System of the Baltic States in View of the Planned Synchronization with Networks of Western Europe to 2030. 2018 IEEE International Conference on Environment and Electrical Engineering and 2018 IEEE Industrial and Commercial Power Systems Europe (EEEIC / I&CPS Europe), Italy, Palermo, 12–15 June 2018.
2. **A. Sobolevskis**, I. Zicmane. Prediction of Latvian Electrical Power System for Reliability Evaluation Including Wind Energy. 2017 IEEE 17th International Conference on Environment and Electrical Engineering and 2017 IEEE Industrial and Commercial Power Systems Europe (EEEIC / I&CPS Europe), Italy, Milan, 6–9 June 2017.
3. K. Berzina, I. Zicmane, **A. Sobolevskis**, S. Kovalenko. Power system stability, wind energy integration, power system modelling. SEEP 2017: 10th International Conference on Sustainable Energy & Environmental Protection, Slovenia, Bled, 27–30 June 2017.
4. I. Zicmane, K. Berziņa, **A. Sobolevskis**, S. Kovalenko. Latvian Electrical Power System Stability's Analysis Taking into Account New Development Strategy until 2025. 11th International Conference on Electromechanical and Power Systems (SIELMEN 2017): Proceedings, Moldova, Chisinau, 12–13 October 2017.
5. **A. Sobolevskis**, I. Zicmane. Analysis of Vulnerability of the Latvian Electrical Power System. 2016 IEEE 16th International Conference on Environment and Electrical Engineering (EEEIC), Italy, Florence, 7–10 June 2016.
6. **A. Sobolevskis**, I. Zicmane. Assessing the Impact of Registering of Weak Points Calculating the Power System Operating Modes. 2016 57th International Scientific Conference on Power and Electrical Engineering of Riga Technical University (RTUCON 2016), Latvia, Riga, 13–14 October 2016.
7. **A. Sobolevskis**, V. Murac, I. Zicmane. Vulnerability Assessment of Electric Power System for the Case of Latvian EPS. 2015 56th International Scientific Conference on Power and Electrical Engineering of Riga Technical University (RTUCON 2015), Latvia, Riga, 14 October 2015.
8. I. Zicmane, S. Kovalenko, **A. Sobolevskis**, A. Sauhats. Localization of the Roots of the Characteristic Polynomial (CP) and Isolines to Analyze Static Stability. 2014 55th International Scientific Conference on Power and Electrical Engineering of Riga Technical University (RTUCON 2014), Latvia, Riga, 14 October 2014.

Structure and Volume of the Doctoral Thesis

The Doctoral Thesis consists of an introduction, 3 chapters, conclusions, and bibliography with 180 reference sources. The volume of the Thesis is 140 pages; it contains 16 tables and 68 figures.

The introduction contains substantiation of the topicality of the subject, defines the aim and tasks of the Thesis, as well as formulates scientific novelty. The personal contribution of the author is described and conferences and publications are listed.

- Chapter 1 describes the current methods and approaches to the analysis of EPS.
- Chapter 2 describes the theoretical basis and the proposed method.
- Chapter 3 analyzes the EPS using the approach described in the theoretical chapter;
- Conclusions are about the proposed method.

1. OVERVIEW OF APPROACHES TO THE INVESTIGATION OF SENSITIVITY AND HETEROGENEITIES OF EPS

Electric power systems are complex objects of dispatching and management, requiring compliance with restrictions related to the reliability, safety, quality and efficiency of their operating regimes. These requirements must be fulfilled by all subjects that make up the technological basis of the electricity market.

The problem of the sensitivity of EPS in steady-state regimes was first studied in connection with the analysis of the influence of errors in the initial data. From the point of view of EES modelling the influence of errors of the initial data on the result for a good model is quite adequate to the effect of real disturbances on the values of the regime parameters.

That is why the results of the study of errors are also interesting for the problem of weak points. Thus, in [4], attention was drawn to the role of a condition number of the Jacobian matrix in estimating the error of calculation results and it was shown that the main reason for poor conditionality is the difference in resistance of branches of the electric network.

In general, the list of the weakest points in the power system is received in terms of simulating single or simultaneous failures of the full or mathematical model of EPS on the basis of expert assessments and heuristic rules.

Traditionally, the list of the most “dangerous” points of the system is obtained by simulation of single or joint failures using a complete or mathematical model of EPS based on expert judgements and heuristics.

Thus, for example, EPS vulnerability may be assessed on the basis of general theory of complex systems [5]. As a result, only one weak point (branch) of the network (according to overload level by comparison with other lines) loss of which leads to a relative decrease in operability and overall efficiency of the entire network is determined for a 34-node diagram.

The second approach [6] provides a formalized technique of analysis of cascade-developing emergency processes, which allows obtaining a fairly adequate reflection of the actual events, states and processes in the EPS occurred that appear in cascade development of the accident, but does not allow analysing emergency situations before they occur.

The third approach [7] for identifying the weak points offers to find the critical (weak) sections (by static stability) on the basis of models. The disadvantage of this approach is the difficulty of analysis and systematization of the results of numerous calculations.

The disadvantage of most approaches is the need for numerous calculations. At the same time, the main difficulty is related not so much to the modelling of the situation and the execution of calculations, but rather to the analysis and systematization of the results obtained. Also, it should be noted that the calculation of the roots is very simple for the characteristic equation of the first and second degree. There are general expressions for the roots of equations of the third and fourth degree, but these expressions are cumbersome and of little use. As for the equation of higher orders, it is generally not possible to write a general expression for them. Therefore, a direct solution of the characteristic equation has not been widely used so far in the calculations of stability.

The calculation based on the bus admittance matrix and Jacobian matrix provides a more accurate calculation of small eigenvalues in comparison with competing algorithms. To determine the sensitivity based on the Jacobian matrix and the bus admittance matrix, a smaller number of operations is required, while sensitivity analysis under normal modes requires less resources, providing the accuracy of the results for estimates. The advantage of using the Jacobian inverse matrix is that it is not necessary to solve a system of linear equations in this case. However, the use of the Jacobian matrix itself has its advantages, which will be considered further.

2. USE OF THE INVERSE JACOBIAN MATRIX AND Z MATRIX

The bus admittance matrix Y and the Jacobian matrix J connect the primary parameters of the regime and the secondary ones. With the help of these matrices, defined for a random power system, a complete characteristic of its current modes and network properties can be obtained. In this Doctoral Thesis we will consider the sensitivity indicators of high-voltage network points.

In terms of the heterogeneity of the electricity system two concepts are important – sensitivity and coherence.

Sensitivity is the degree of response of the regime parameter to a unit disturbance, which can be found either by numerical experiment or by some indirect indicators, in particular by the indicators proposed below, related to the singular and eigenvalues of the sensitivity matrices. Those parameters of the regime and the corresponding elements of EPS, whose sensitivity is noticeably higher than the others, are called sensitive.

The most interesting in the context of EPS vulnerability research is localization of diagram branches, change in parameters of which affects sensitivity most of all. As is known, arithmetic values of square roots of the common eigenvalues λ of real matrices $A^T A$ and AA^T are called matrix singular values A .

$$\sigma_i(A) = \sqrt{\lambda_i(A^T A)} = \sqrt{\lambda_i(AA^T)}, \quad (2.1)$$

if $i = 1, \dots, k$, and indicator of the sensitivity increase is reduction of the minimum singular value σ_1 of the Jacobian matrix [8], [9].

Therefore, to assess the impact of the diagram parameters and operational parameters on the sensitivity of its elements, the matrix derived from the Jacobian matrix by conductance of branches y_{ij} of the analysed diagram has been studied.

Availability and location of sensitivity and weak points has been determined by numerical and analytical methods for studying EPS diagram and its parameters [8], [9]: singular analysis of the Jacobian matrix (Eq. 2.2), and nodal conductance matrix (Eq. 2.3), as well as generalized weakness indicators (Eq. 2.4) and (Eq. 2.5).

$$J = W \sum V^T = \sum_{i=1}^k w_i \sigma_i v_i^T, \quad (2.2)$$

$$Y = W_Y \sum_Y V_Y^T = \sum_{i=1}^k w_{Yi} \sigma_{Yi} v_{Yi}^T, \quad (2.3)$$

where $\sum = \text{diag}(\sigma_1, \dots, \sigma_k)$ and $\sum_Y = \text{diag}(\sigma_{Y1}, \dots, \sigma_{Yk})$ are diagonal matrices of singular values; $W = (w_1, \dots, w_k)$, $V = (v_1, \dots, v_k)$, are orthogonal matrices of $k \times k$, size; their i -th columns are respectively the i -th left and the i -th right singular vectors of corresponding matrices and these conditions are true for them: $w_i^T w_i = v_i^T v_i = 1$, $w_i^T w_j = v_i^T v_j = 0$, when $i \neq j$.

Indicators $\chi_{J\sigma}$ [8], [9] and $\chi_{Y\sigma}$ have been used as generalized indicators to highlight weak branches based on singular analysis:

$$\chi_{J\sigma} = \frac{\partial \sigma_1}{\partial y_{ij}} = w_{1\delta}^T \left(\frac{\partial^2 P}{\partial \delta \partial y_{ij}} \right) v_{1\delta} + w_{1\delta}^T \left(\frac{\partial^2 P}{\partial U \partial y_{ij}} \right) v_{1U} + w_{1U}^T \left(\frac{\partial^2 Q}{\partial \delta \partial y_{ij}} \right) v_{1\delta} + w_{1U}^T \left(\frac{\partial^2 Q}{\partial U \partial y_{ij}} \right) v_{1U}, \quad (2.4)$$

$$\chi_{Y\sigma} = (w_{y_{p1}} - w_{y_{q1}})(v_{y_{p1}} - v_{y_{q1}}), \quad (2.5)$$

where $(\partial^2 P)/(\partial \delta \partial y_{ij})$, $(\partial^2 P)/(\partial U \partial y_{ij})$, $(\partial^2 Q)/(\partial \delta \partial y_{ij})$, $(\partial^2 Q)/(\partial U \partial y_{ij})$ are derivatives of Jacobian matrix by conductance of branches y_{ij} ; w_1 , v_1 are the 1st left and the 1st right singular vectors of the Jacobian matrix; and w_{Y1} , v_{Y1} are the 1st left and the 1st right singular vectors of the nodal conductance matrix.

Using the approach offered, it is possible to analyze the power system: identify weaknesses, predict the balance/imbalance of the system under deficit and surplus of electricity to take further measures to achieve a normal balanced mode.

Comparison of estimates of sensitivity obtained via the singular analysis of the Jacobian matrix and the singular analysis of the nodal conductance matrix shows that practically the same components and branches are sensitive for deviation of voltage modules. The difference of results is due to the fact that sensitivity of some EPS elements depends on the diagram topology parameters, while sensitivity of others – on operational parameters too.

3. FINDING LOCATION OF SENSITIVE ELEMENTS FOR THE EPS MODEL

3.1. Finding Location of Sensitive Elements for the EPS Model (for Scenario 1)

The most important phase in the vulnerability analysis of EPS is the generation of impact scenarios leading to the development of the EPS crash. For networks with a relatively small number of elements such scenarios can be created by manually listing different variants, but their number increases rapidly as network scheme configuration becomes more complex. As a result, the method proposed in this dissertation allows discovering the most vulnerable parts of the power system, the impact of which can cause crashes of the EPS by searching for so-called sensitive points (nodes, lines) without performing statistical tests and counting a huge number of external exposure scenarios. The following parameters are used for the calculations: EPS stationary mode parameters (node voltage, power, load and generation, angles and modules), network parameters (internal and external communication set for power transmission, branch resistance). In order to ensure convergence (balance between generation and consumption) of the operational mode, the balance node and reactive power were set arbitrarily under the model considered, which is closer to the real conditions of the EPS's models. The calculation of the respective modes is approximate as well and does not allow the estimation of active and reactive power flows with high reliability.

The task of analyzing an EPS's heterogeneities consists of using matrices of generalized values of EPS elements and information about the points of application of perturbations for searching and localizing the EPS's weak points. For the steady-state modes, the generalized values are expressed via parameters of the nodal conductance matrix and Jacobi matrix of equations of the steady-state mode, which are the main sources of information about the sensitivity and heterogeneity of EPS in steady-state modes. Sensitive places and their location will be identified with the help of numerical and analytical methods of studying the EPS's scheme and parameters. An example of using the method (with nodal conductance matrices and Jacobi matrices) and the process of identifying sensitive points are shown in appendix 1. The algorithm for the calculation of steady-state parameters of the EPS is shown in the flowchart (Fig. 3.1).

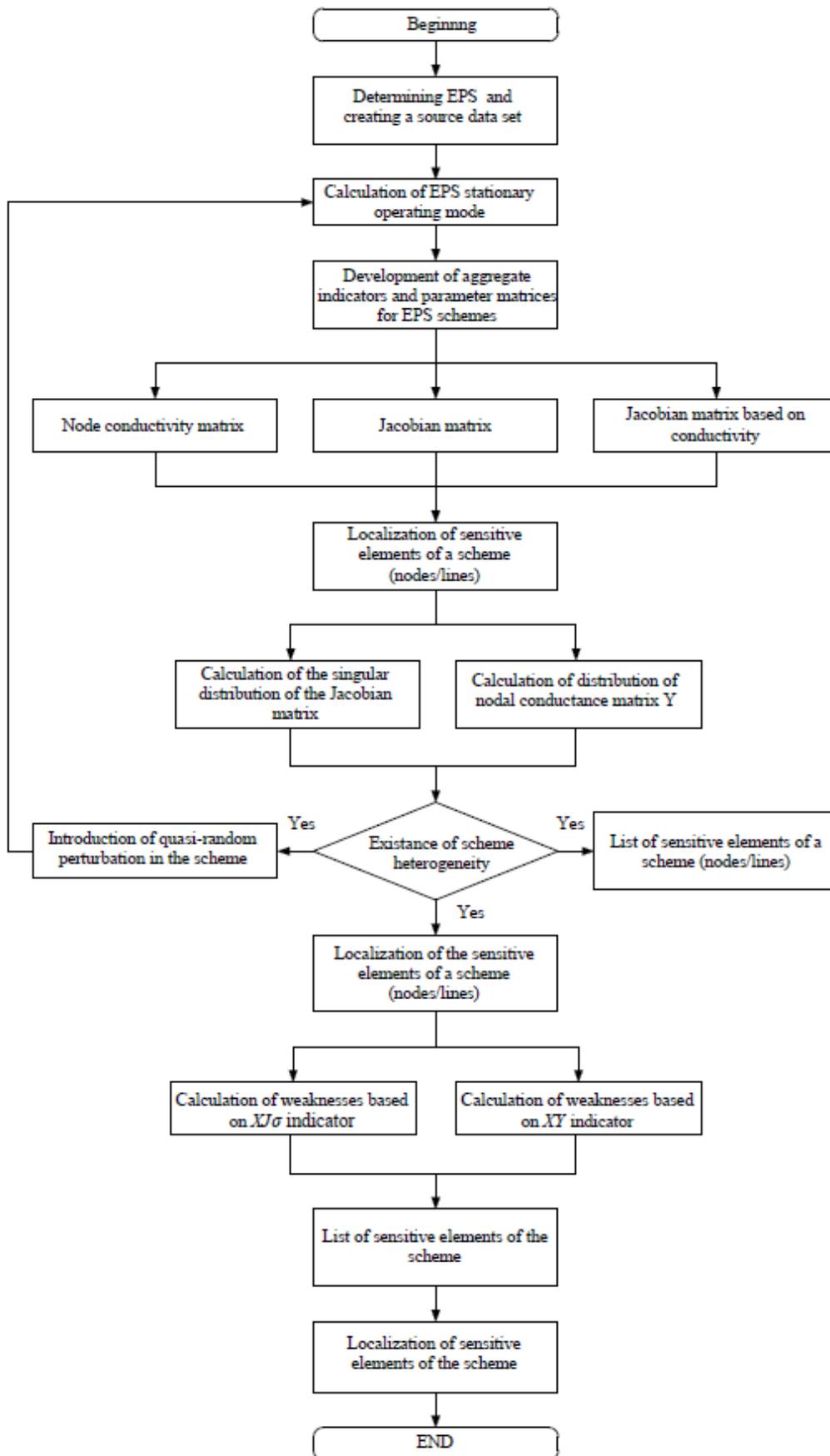


Fig. 3.1. Flowchart of the algorithm for the calculation of steady-state parameters of the EPS.

Five EPS model scenarios were analyzed and compared in this work. The EPS models in this work are close to real-world conditions and partially reflect the situation in the Baltic States' EPS without claiming detailed similarity to the latter.

The obtained results of the localization of the EPS vulnerabilities allow providing for measures related to the construction of an equally stable network, thus preventing dangerous or undesirable situations, as well as developing protective measures aimed at minimizing damage to the power system during its operational phase from possible external exposures.

To study the chosen approach/method, a model of Scenario 1 power system has been considered. The scheme shown in Fig. 3.2 contains 18 nodes and 26 branches (lines). Nodes 1–16 are power plant and power station. Node 17' belongs to EPS-1, node 18' belongs to EPS-3 [34].

Scenario 1

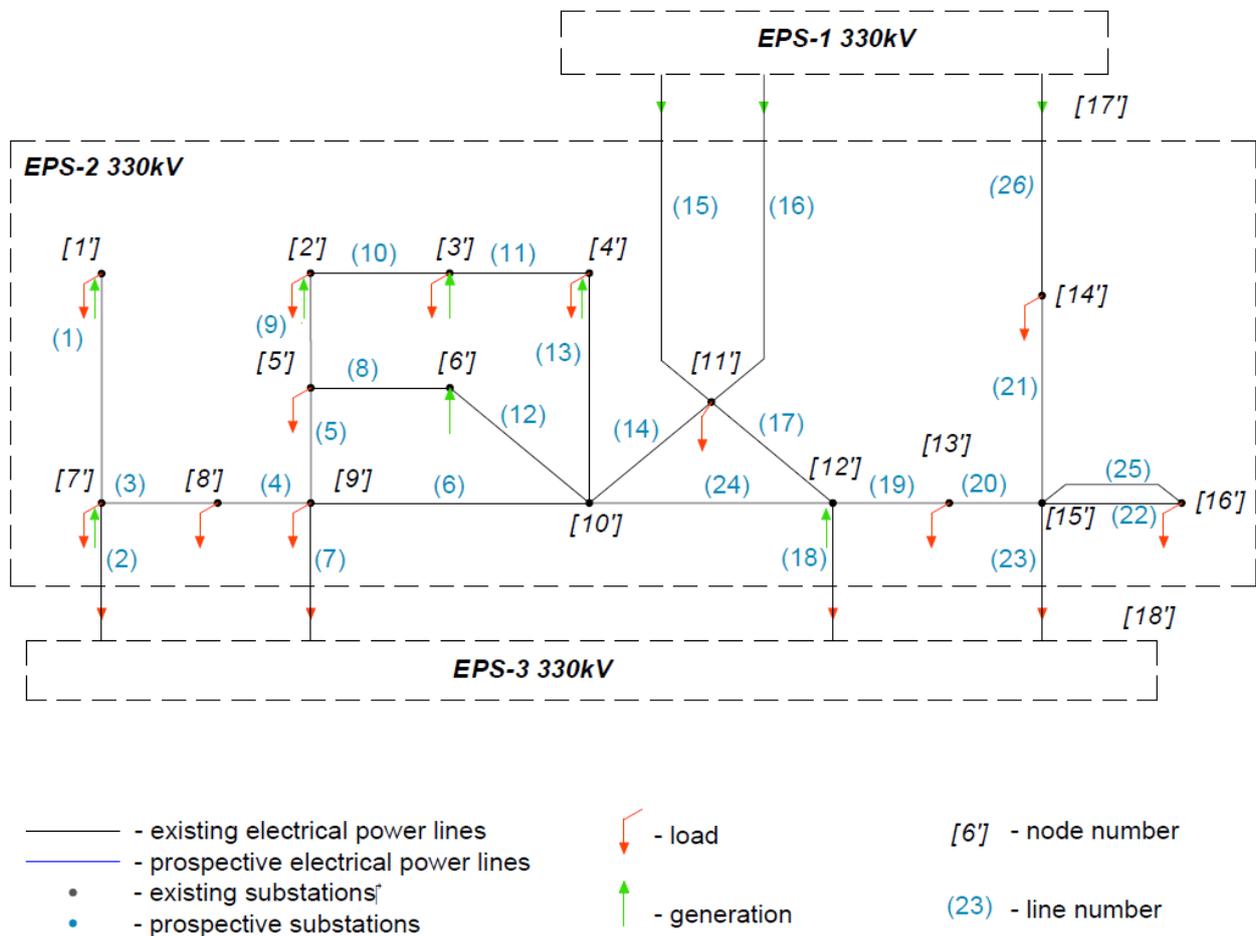


Fig. 3.2. The studied EPS 330 kV model (for Scenario 1).

Mathematical model includes 330 kV transmitting line lengths, electrical generation, and consumer (Table 3.1). A normal balanced operation mode of the studied EPS model is considered as initial. The following parameters are used for the calculations: EPS stationary mode parameters (nodes voltage, power, load and generation, angles and voltage modules),

network parameters (internal and external communication set for power transmission, branch resistance).

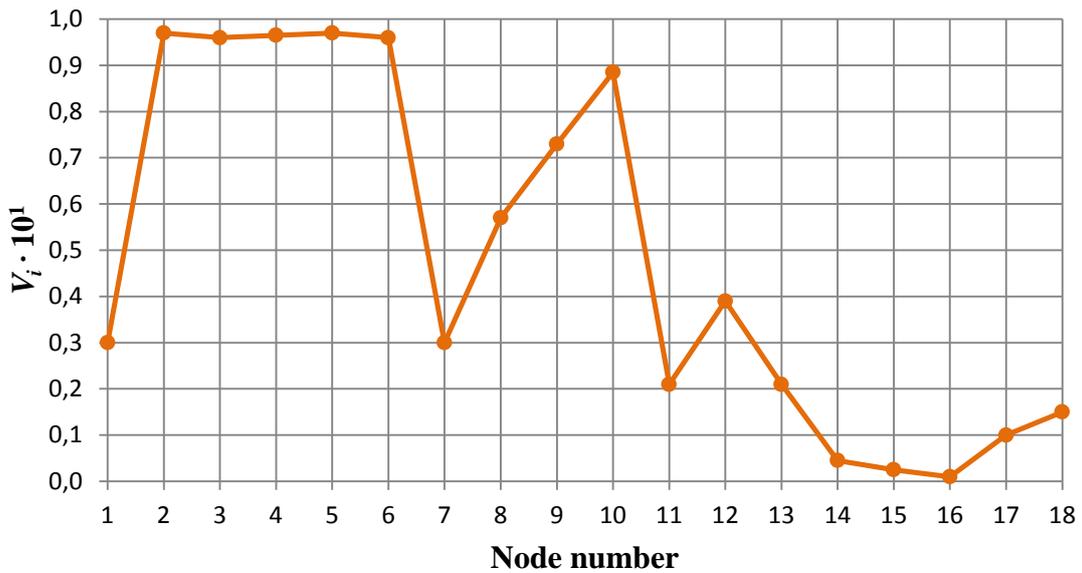
Table 3.1

Consumer and Generation Nodes of the Model (for Scenario 1)

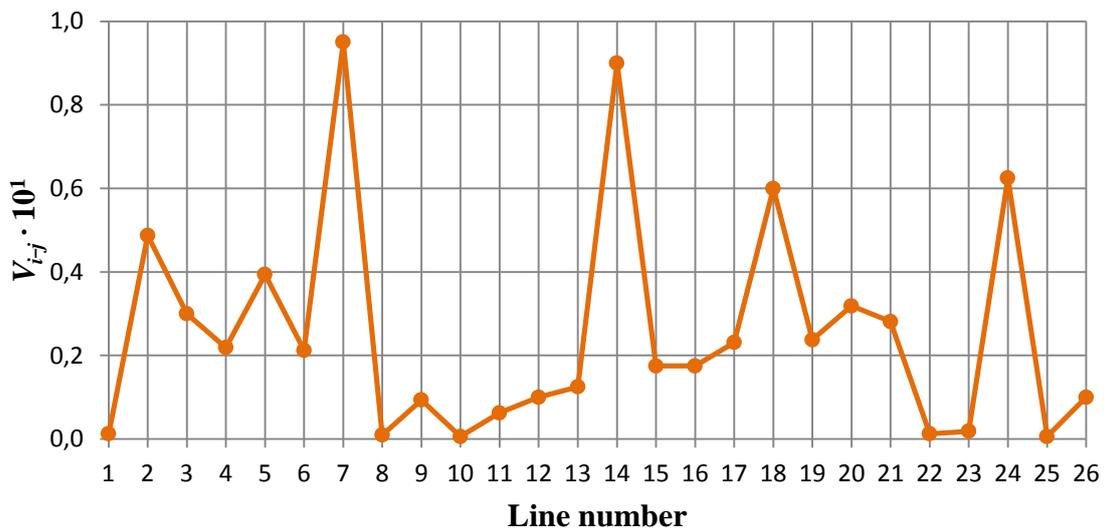
Node No.	Consumer <i>P</i> , MW	Generation <i>P</i> , MW
[1']	34	4
[2']	143	13
[3']	143	46
[4']	143	270
[5']	143	0
[6']	0	96
[7']	59	8
[8']	17	0
[9']	42	0
[10']	0	0
[11']	17	0
[12']	0	192
[13']	17	0
[14']	25	0
[15']	0	0
[16']	59	0
[17']	0	600
[18']	387	0
Balance:		1229 1229

For the calculated network consisting of 18 nodes and 26 branches (Fig. 3.2), the number of possible impact scenarios when disconnecting one branch or a combination of two to five branches is 83 681 [34].

Using the methods of singular analysis of the Jacobian matrix and the nodal conductance matrix for the diagram shown in Fig. 3.2 nodes 2', 3', 4', 5', 6', 10' (Fig. 3.3a) have been identified as sensitive by maximum components of the right singular vector V_1 , corresponding to the minimum singular value σ_{\min} of the Jacobian matrix. Branches 2, 7, 14, 18, 24 (Fig. 3.3b) are sensitive by voltage loss.



a)

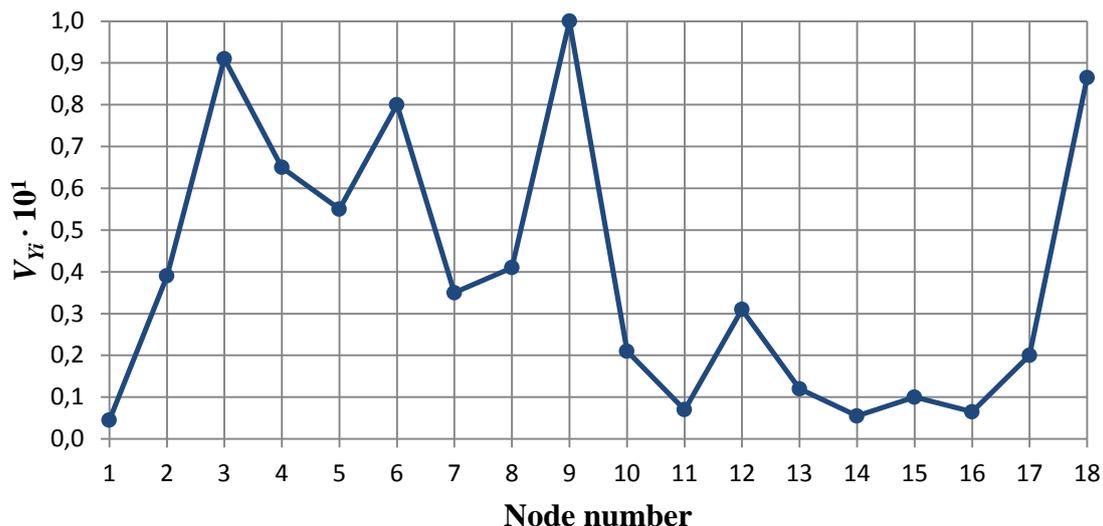


b)

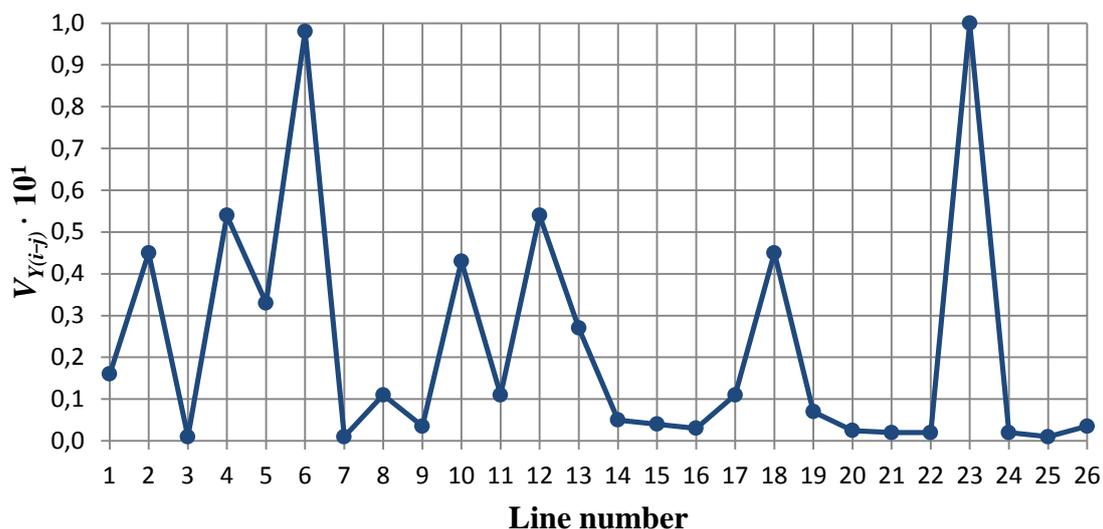
Fig. 3.3. Calculation results of sensitivity of nodes and branches of the studied network diagram based on singular analysis of the Jacobian matrix: a) node sensitivity by voltage; b) line (branch) sensitivity by voltage loss.

The nodes and branches with maximum calculated values of components of the first right singular vector V_1 of matrix Y identify nodes 3', 4', 5', 6', 9', 18' as sensitive by voltage (Fig. 3.4a).

Evaluation of the differences corresponding to the nodes at the ends of the branches of the components of the first right singular vector of matrix Y makes it possible to identify branches sensitive by voltage loss – 2, 4, 6, 10, 12, 18, 23 (Fig. 3.4b).



a)



b)

Fig. 3.4. Calculation results of sensitivity of nodes and branches of the studied network diagram based on singular analysis of the nodal conductance matrix Y : a) node sensitivity by voltage; b) line (branch) sensitivity by voltage loss.

Comparison of estimates of sensitivity obtained via the singular analysis of the Jacobian matrix (Fig. 3.3) and the singular analysis of the nodal conductance matrix (Fig. 3.4) shows that practically the same components and branches are sensitive for deviation of voltage modules. The difference of results is due to the fact that sensitivity of some EPS elements depends on the diagram topology parameters, while sensitivity of others – on operational parameters too [34].

Using expression

$$\chi_{J\sigma} = \frac{\partial \sigma_1}{\partial y_{ij}} = w_{1s}^T \left(\frac{\partial^2 P}{\partial \delta \partial y_{ij}} \right) v_{1s} + w_{1s}^T \left(\frac{\partial^2 P}{\partial U \partial y_{ij}} \right) v_{1U} + w_{1U}^T \left(\frac{\partial^2 Q}{\partial \delta \partial y_{ij}} \right) v_{1s} + w_{1U}^T \left(\frac{\partial^2 Q}{\partial U \partial y_{ij}} \right) v_{1U} \quad (3.1.)$$

for the considered diagram branches 2, 6, 12, 13, 14, 20, 23 have been selected as weak (Fig. 3.5).

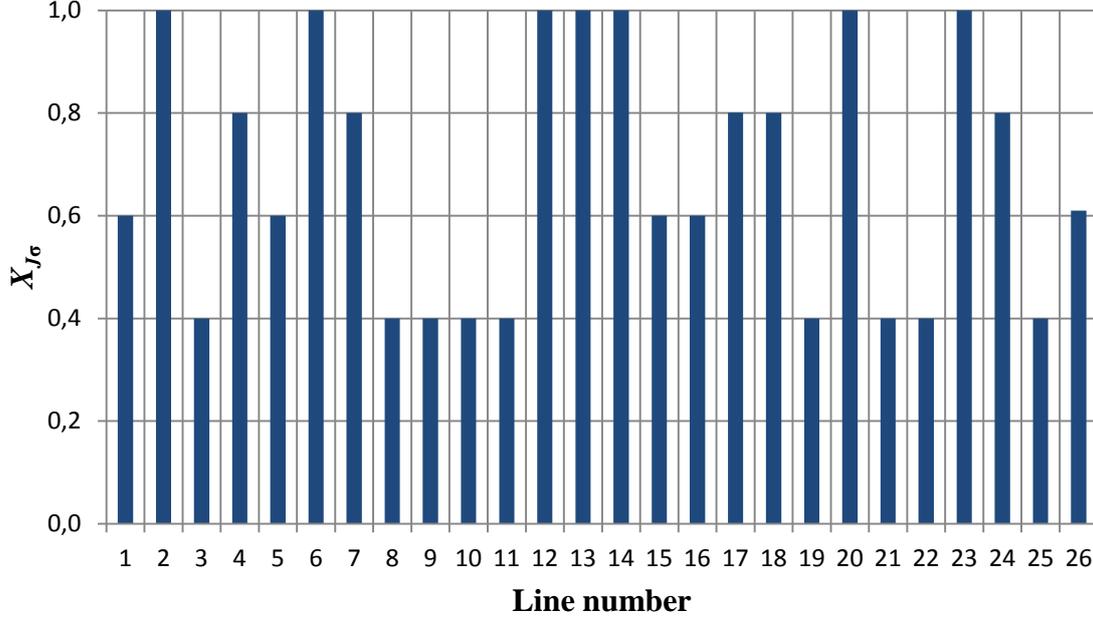


Fig. 3.5. Ranking of branches of diagram by weakness indicator $\chi_{J\sigma}$ based on the singular analysis of the Jacobian matrix.

Figure 3.6 shows the calculation of the weakness indicator $\chi_{Y\sigma}$ of branches using relation:

$$\chi_{Y\sigma} = (w_{Yp1} - w_{Yq1})(v_{Yp1} - v_{q1}). \quad (3.2.)$$

The analysis of received results reveals that branches 2, 7, 14, 23, components of which are maximal, are weak. In doing so, branches 2, 4, 6, 7, 10, 12, 14, 18, 23, 24, were previously determined as sensitive (Figs. 3.3b and 3.4b), and branches 2, 6, 12, 13, 14, 20, 23 – as weak by indicator $\chi_{J\sigma}$ (Fig. 3.5). Differences between the results of ranking the weak branches are due to the fact that the singular analysis of the matrix does not take into account the operational parameters but only by locating a structural heterogeneity of EPS.

Summarizing the above information about sensitive elements of the studied diagram, it can be stated that nodes 3', 4', 5', 6' and lines 2, 6, 7, 12, 14, 18, 23 are most sensitive by voltage. For further analysis of the situation the sensitive nodes and lines were highlighted in red on the model's schemes of the power system for Scenario 1 (see Fig. 3.7 for clarity).

Thus, totality of localized weak points defines a set of scenarios of disturbances that are most dangerous in terms of the EPS vulnerability assessment. The number of such scenarios, when disconnecting one branch or a combination of two to five branches, is 1585, which is 50 times less than the number of scenarios of impacts drawn up without taking into account information about the weak points.

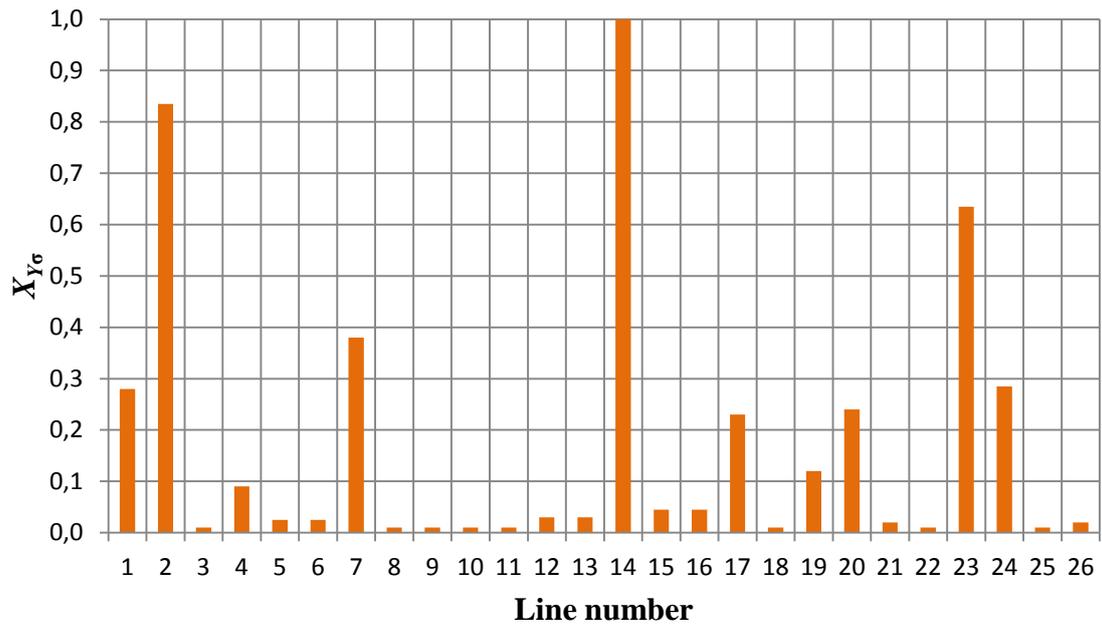


Fig. 3.6. Ranking of the branches of the diagram by χ_{Y_G} based on the singular analysis of the nodal conductance matrix.

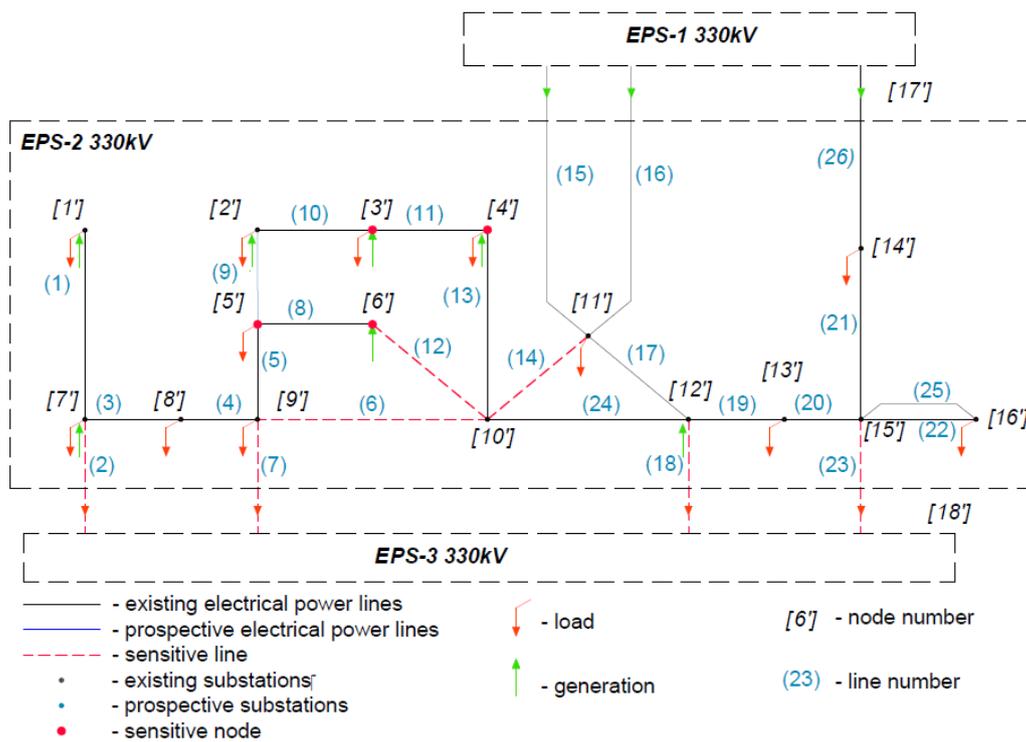


Fig. 3.7. The studied 330 kV network diagram with identified weak nodes and branches (for Scenario 1).

3.2. Sensitivity Calculation of EPS Model for Scenario 5

The current stage in the development of world energy is characterized by the creation of powerful territorially-unified energy systems. Under the conditions of the free market, the energy markets of different countries currently are gradually unifying: the network configurations change, additional connections emerge, and new energy companies and unions are formed. Thus, a merged EPS is presented in Scenario 5 where EPS-1 and EPS-3 are connected to EPS-2, which was discussed above, and therefore affect each other. This Thesis studies EPS models that approximate real-world conditions and are to a certain extent similar to the existing EPSs in the Baltic States.

The initial data for Scenario 5 is shown in Table 3.2. The scheme (Fig. 3.8) contains 51 node and 79 branches (lines).

Table 3.2

The Initial Data of the Expected Scenario 5 for the EPS Model

Node No.	EPS	Consumer P, MW	Generation P, MW
[1']	EPS-3.1	0	130
[2']	EPS-1.1	55	1218
[3']	EPS-3.1	39	148
[4']	EPS-2	172	22
[5']		172	63
[6']		172	387
[7']		0	177
[8']		9	393
[9']		76	9
[10']		39	26
[11']		13	0
[12']		62	0
[13']		172	0
[14']		0	0
[15']		25	0
[16']		25	0
[17']		0	0
[18']		93	0
[19']		34	0
[20']		1	0
[21']		0	0
[22']		EPS-1.1	55
[23']	290		0

The Initial Data of the Expected Scenario 5 for the EPS Model

Node No.	EPS:	Consumer <i>P</i> , MW	Generation <i>P</i> , MW
[24']	EPS-1.1	290	148
[25']		31	0
[26']		2	148
[27']		290	0
[28']		16	0
[29']		3	0
[30']		8	0
[31']		4	0
[32']		1	0
[33']		181	0
[34']		EPS-3.1	0
[35']	315		27
[36']	171		201
[37']	103		0
[38']	59		148
[39']	197		45
[40']	32		0
[41']	0		0
[42']	31		0
[43']	37		0
[44']	197		0
[45']	197		0
[46']	117		0
[47']	0		0
[48']	0		0
[49']	14		0
[50']	0		0
[51']	21	0	

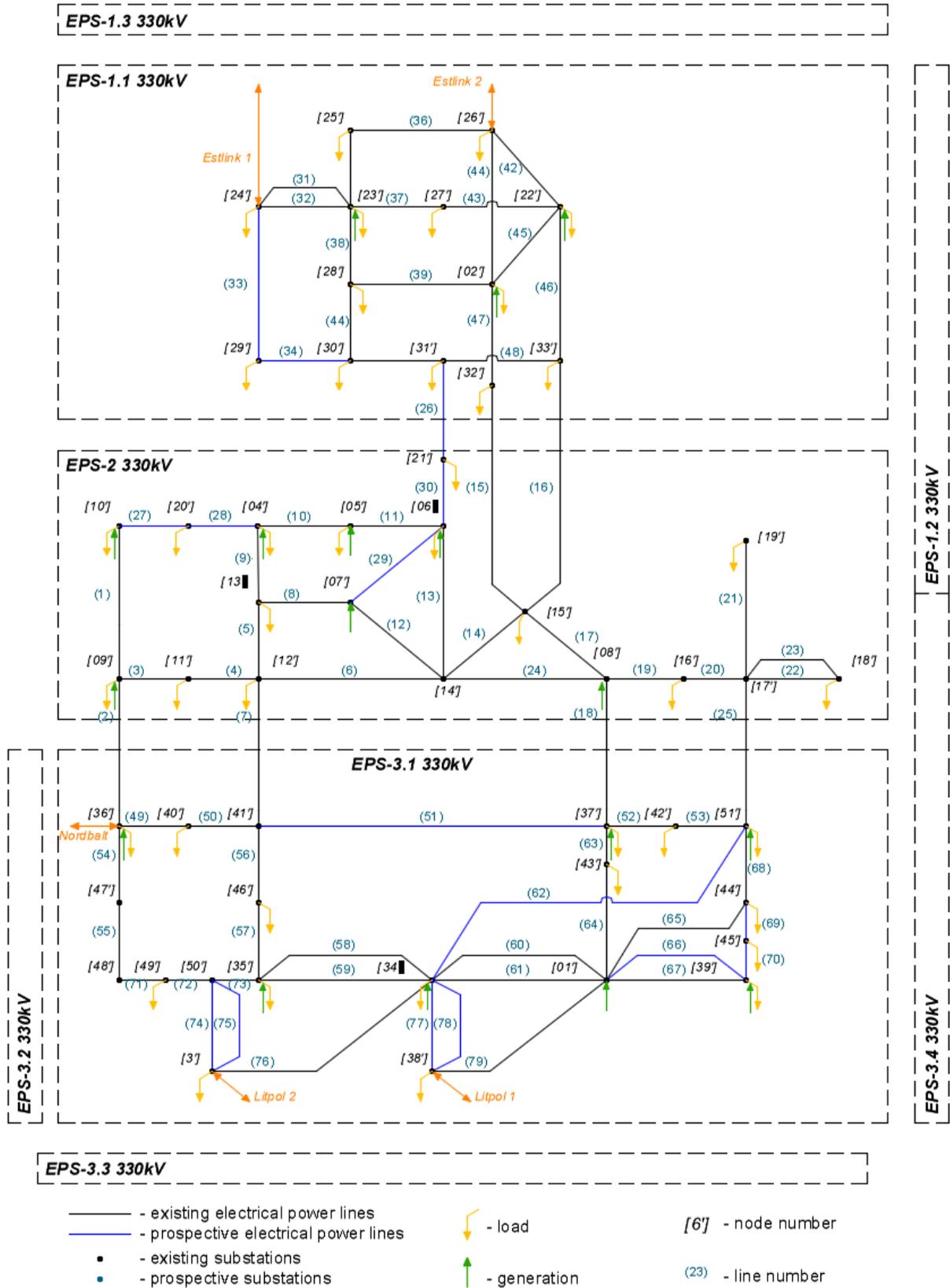


Fig. 3.8. The studied EPS 330 kV model for Scenario 5.

The following parameters are used for calculations: EPS stationary mode parameters (nodes voltage, power, load and generation, angles and voltage modules), network parameters (internal and external communication set for power transmission, branch resistance). A normally balanced operation mode of the studied EPS model Fig. 3.8 is considered as initial.

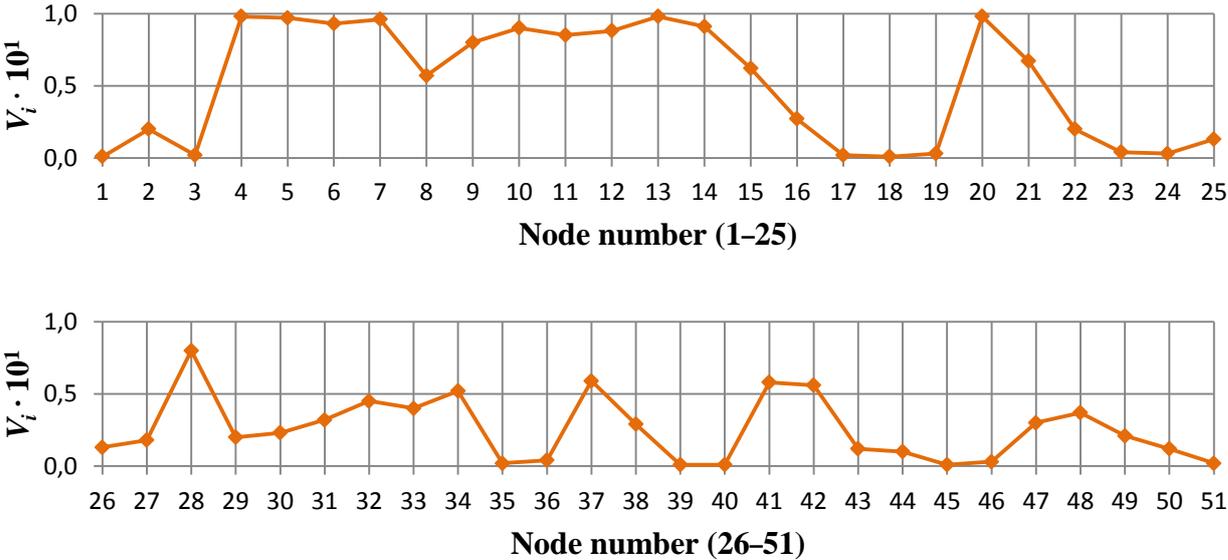
Figure 3.9 shows the results of calculations with the method of singular analysis of the Jacobian matrix and the bus admittance matrix for Scenario 5. Thus, the maximal components of the right singular vector corresponding to the minimal singular value of the Jacobian matrix are identified nodes 4', 5', 6', 7', 8', 9', 10', 11', 12', 13', 14', 15', 20', 21', 36', 40', 41' (Fig. 3.9a). Branches 2, 7, 14, 18, 19, 20, 24, 30, 46, 47, 51, 57 are sensitive in terms of the loss of voltage (Fig. 3.9b).

Those nodes and branches which have the maximal calculated values of the components of the first right singular vector V_1 of the matrix Y are identified as sensitive for voltage. Those are nodes 8', 9', 15', 16', 30', 33', 36', 37', 48', 49', 51' (Fig. 3.10a).

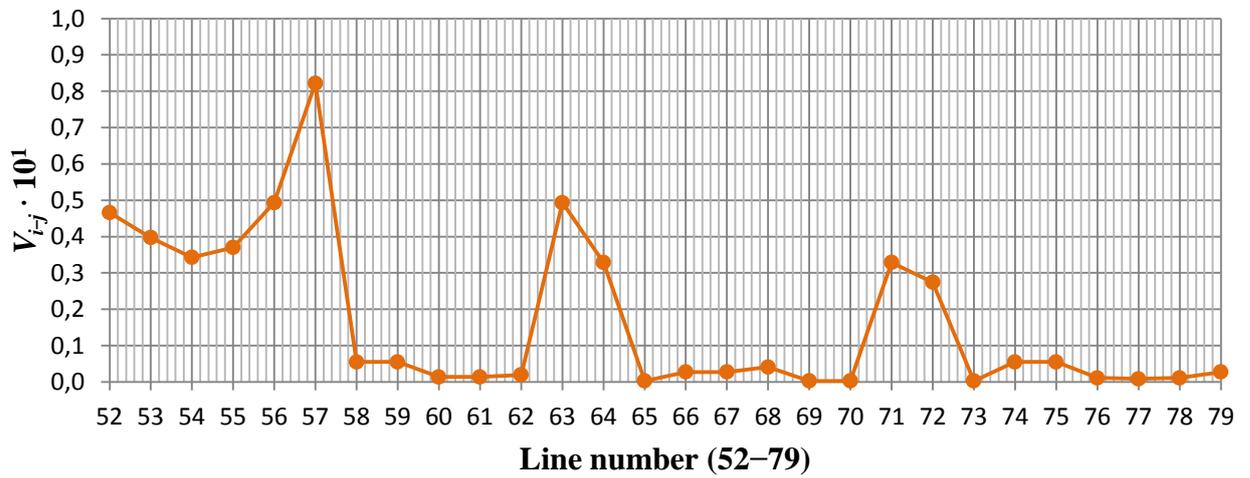
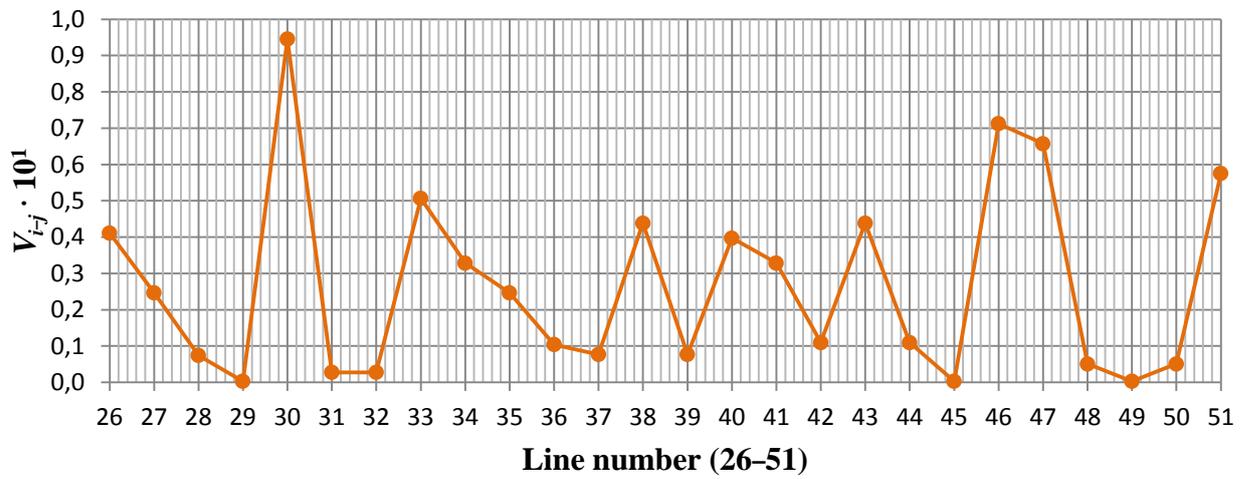
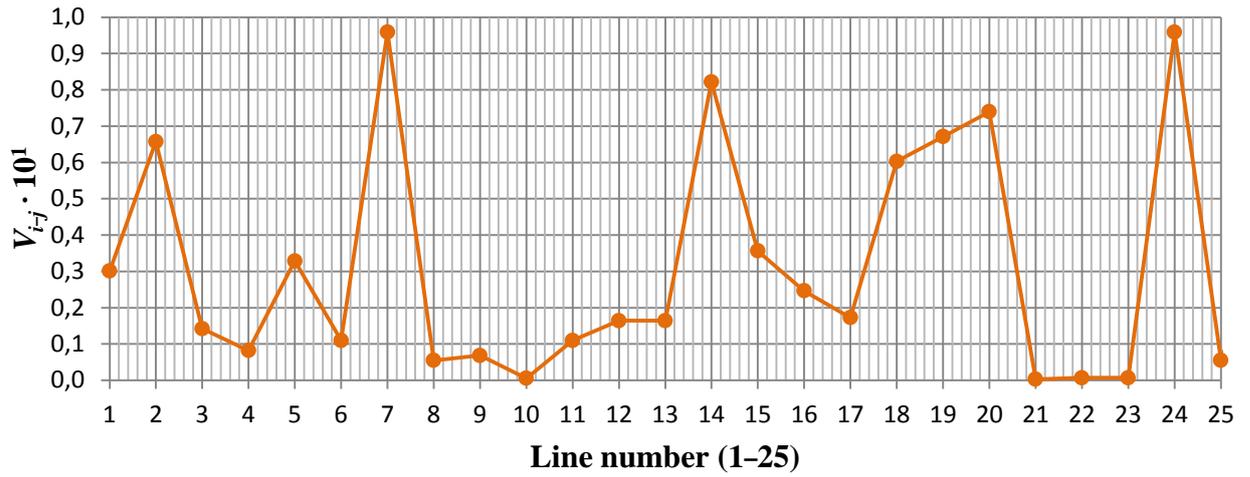
Estimating the difference to the nodes at the ends of the branches of the components of the first right singular vector of the matrix makes it possible to identify the branches sensitive to the loss of voltage, namely: 14, 20, 24, 46, 48, 49, 54, 63 (Fig. 3.10b).

From Figures 3.9 and 3.10 it is apparent that the comparison of the estimates of sensitivity obtained by two different methods, i.e. the singular analysis of the Jacobian matrix (Fig. 3.9) and the bus admittance matrix (Fig. 3.10) in terms of deviation of voltage modules, practically the same nodes and branches are sensitive. So, the analysis of the obtained results allows establishing that the next branches, whose components are maximal, are weak, namely: 14, 20, 24, 46.

The difference in results, using both methods, can be explained by the fact that the sensitivity of some elements of the EPS depends only on the topological parameters of the network, while others on the regime parameters as well.



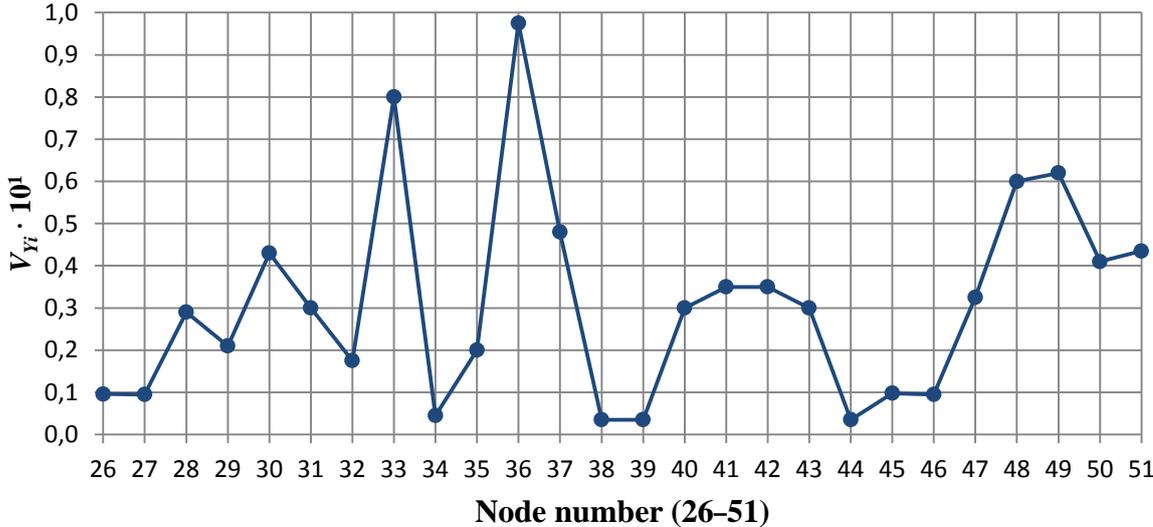
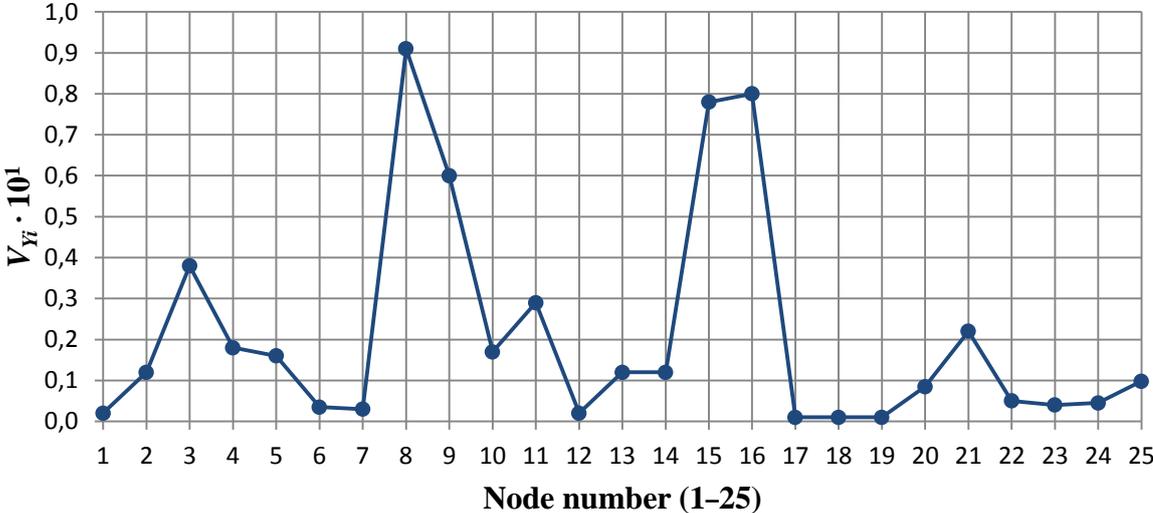
a)



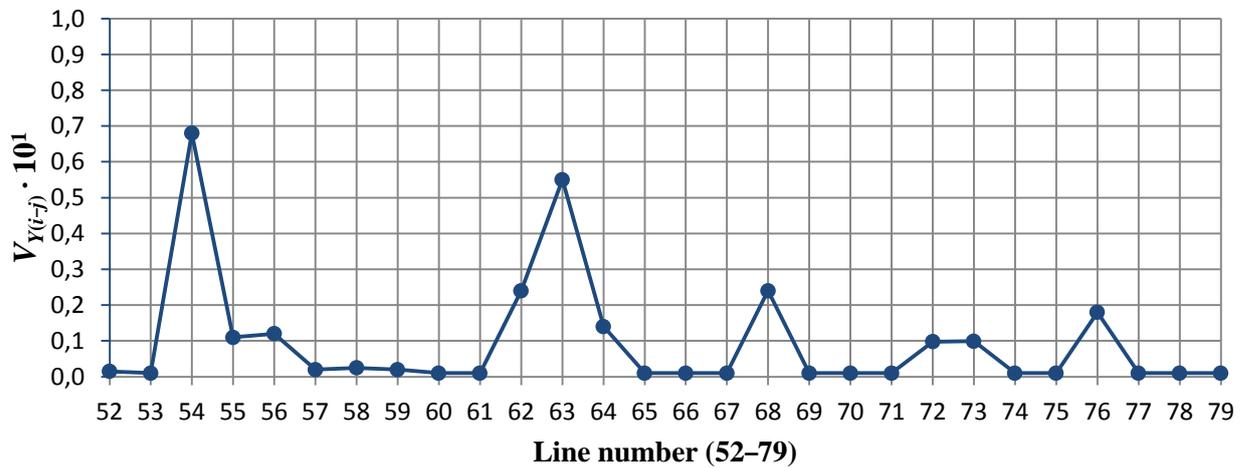
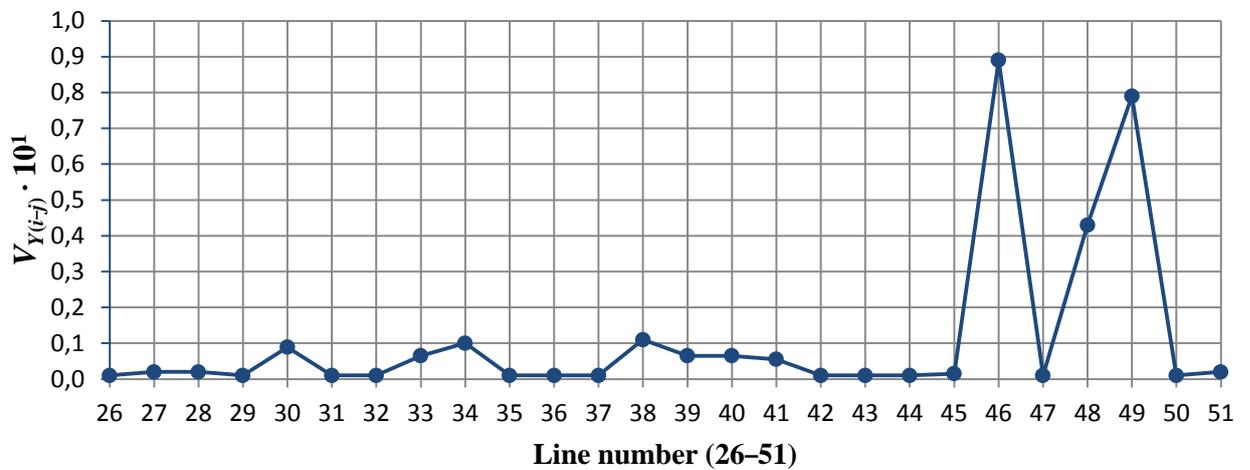
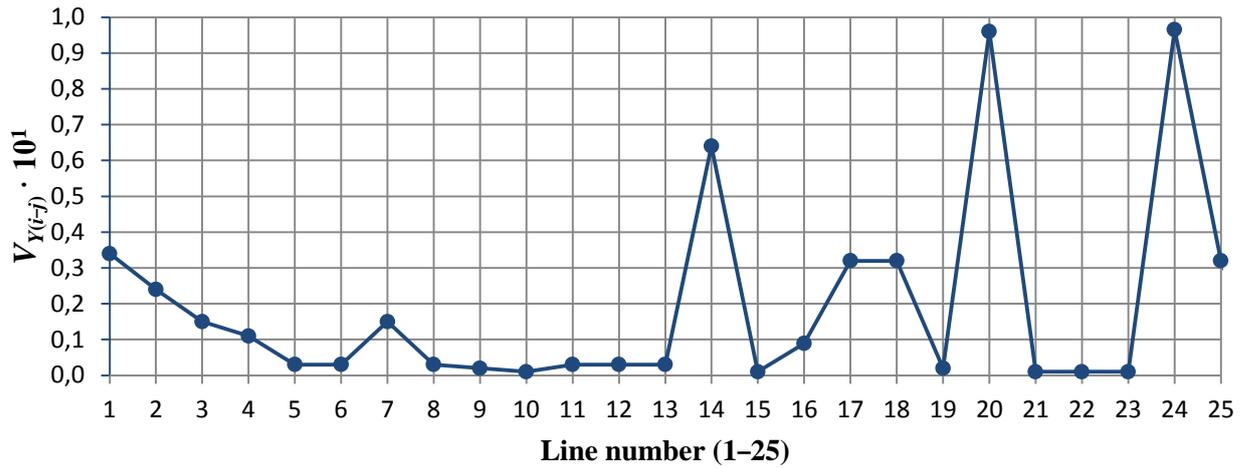
b)

Fig. 3.9. Calculation results of sensitivity of nodes and branches of the studied network diagram (for Scenario 5) based on the singular analysis of the Jacobian matrix: a) node sensitivity by voltage (1–25 and 26–51); b) line (branch) sensitivity by voltage loss.

Using the weakness indicator of branches $\chi_{J\sigma}$ based on the singular analysis of the Jacobian matrix, for the circuit shown on Fig. 3.8(for Scenario 5), branches 20, 21, 58, 59, 60, 61, 64, 76, 79 have been selected as weak (Fig. 3.11).



a)



b)

Fig. 3.10. Calculation results of sensitivity of nodes and branches of the studied network diagram (for Scenario 5) based on the singular analysis of the nodal conductance matrix Y : a) node sensitivity by voltage; b) line (branch) sensitivity by voltage loss.

Figure 3.12 shows calculation of the weakness indicator of branches $\chi_{Y\sigma}$ based on the singular analysis of the nodal conductance matrix. The analysis of received results reveals that branches 7, 19, 20, 24, 33, 35, 43, 47, 57, components of which are maximal, are weak.

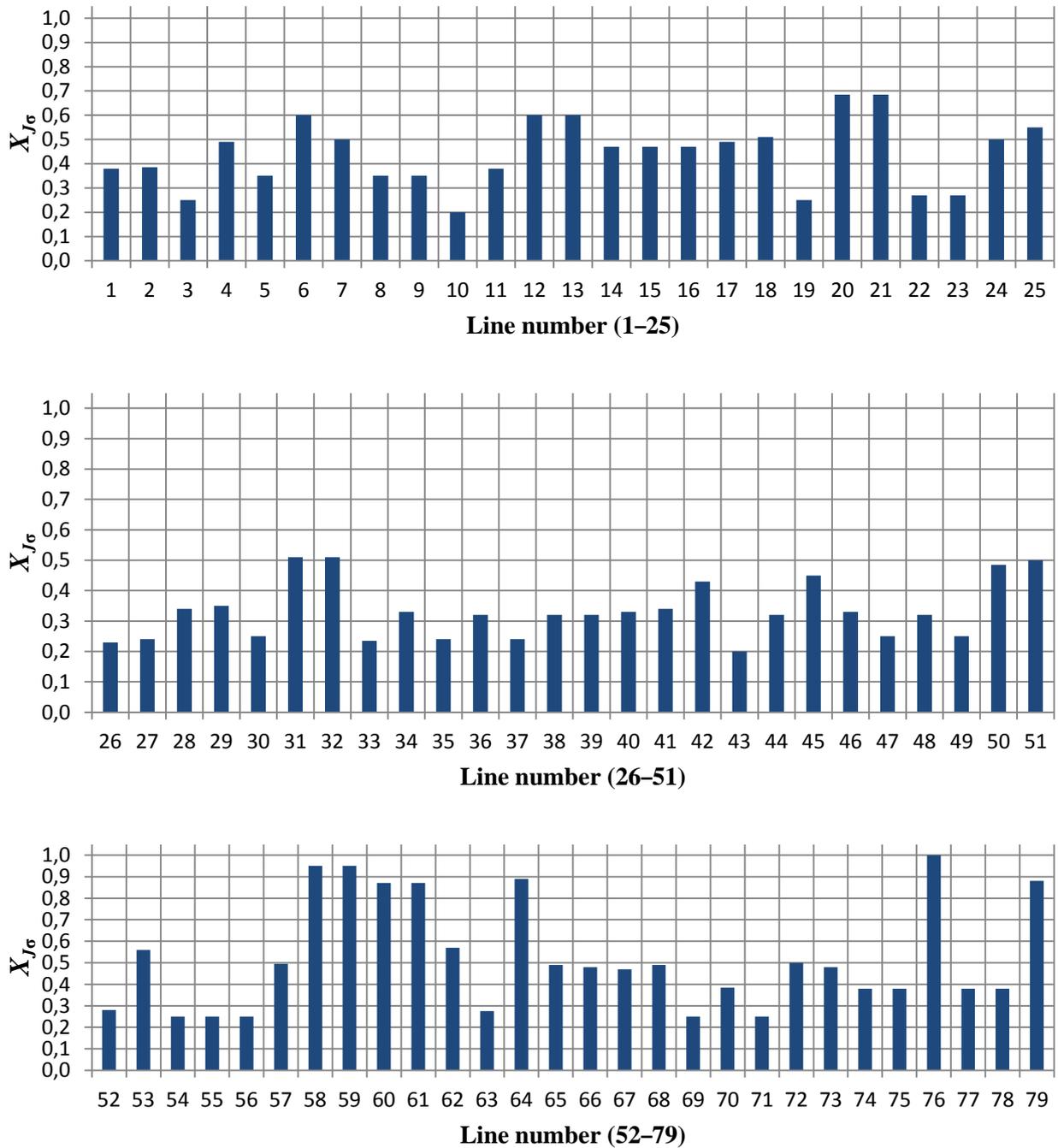


Fig. 3.11. Ranking the branches of diagram (for Scenario 5) by weakness indicator $\chi_{Y\sigma}$ of the branches of diagram based on the singular analysis of the Jacobian matrix.

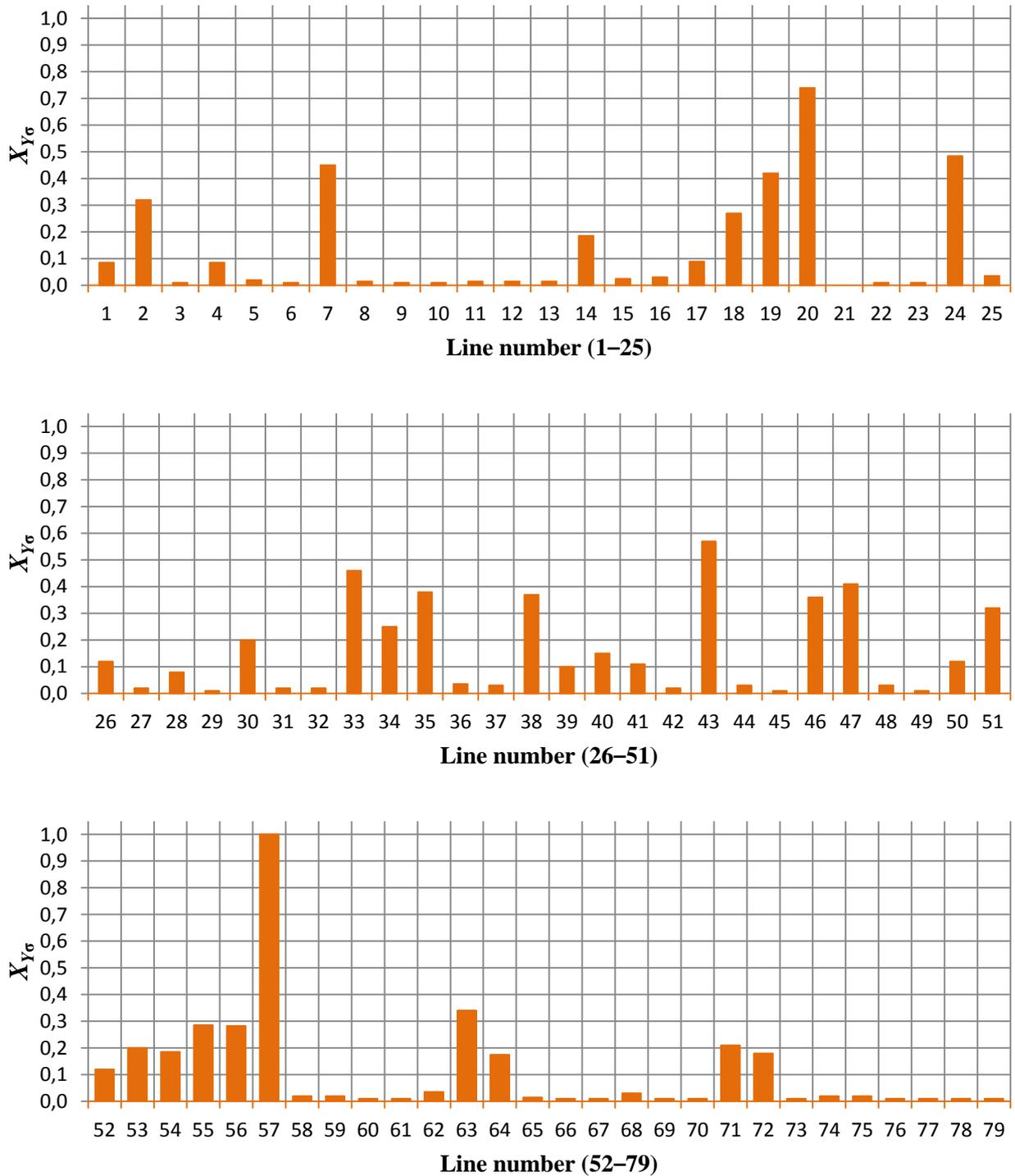


Fig. 3.12. Ranking the branches of diagram (for Scenario 5) by weakness indicator χ_{Y_G} of the branches of diagram based on the singular analysis of the nodal conductance matrix.

Summarizing the above information about sensitive points of the studied diagram, it can be stated that nodes 8', 9', 15', and 36' and lines 7, 14, 19, 20, 24, 46, 47, and 57 are most sensitive by voltage losses.

In Fig. 3.13 the results of the work, i.e. the effect on the degree of sensitivity of the nodes of the weighting scheme on the reactive load power in the sensitive line and nodes, identified weaknesses, was presented.

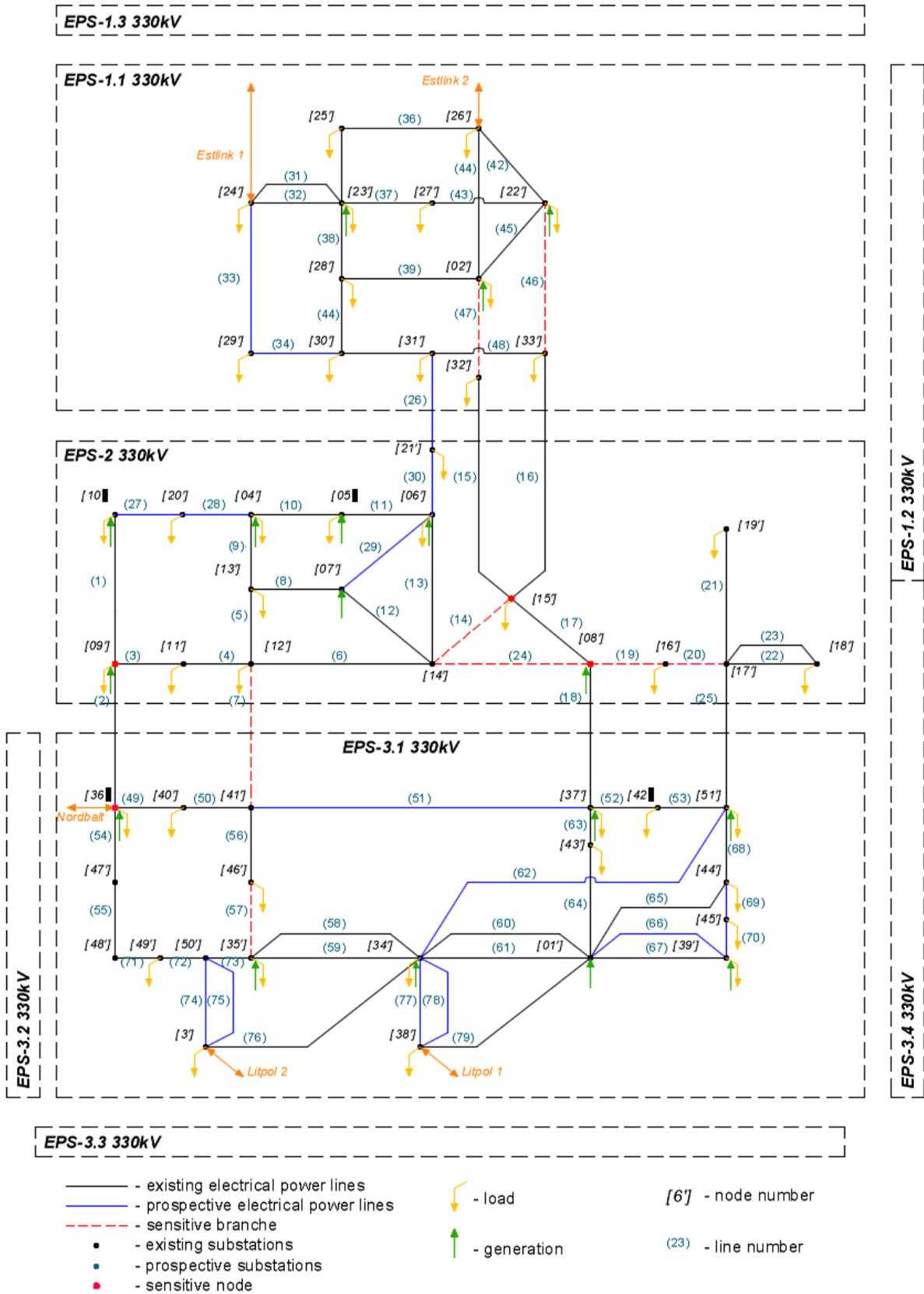


Fig. 3.13. The studied 330 kV network diagram with identified sensitive nodes and lines (for Scenario 5).

CONCLUSIONS AND FINDINGS

1. The main disadvantage of most approaches used to analyze the static stability of EPS is the need for extensional calculations. At the same time, the main difficulty is not so much with the modelling of the situation and carrying out of calculations, but rather with the analysis and systematization of the results obtained. The purpose of this Thesis is to provide methods for detection of network parameters that are most sensitive to external impacts and to determine their relationship with EPS parameters. This method does not require statistical tests and busting of a huge number of scenarios.
2. To determine the sensitivity based on the Jacobian matrix and the bus admittance matrix, fewer operations are required, while sensitivity analysis in normal modes requires less resources. A minimum set of data is required to identify potential in stability problems. Usually these data can be given by the model of the system under consideration. Any power system can be represented as a model, including a number of generators (power plants) and a number of loads (power distribution centers) that are connected through a network of power transmission lines. Thus, the network model has nodes (buses), branches (lines), generators and loads.
3. Using the offered approach, it is possible to analyze the power system: to identify weaknesses of the system and to take further measures to achieve a normal balanced mode.
4. Calculation of the scenarios of the EPS model provides an opportunity to visually see changes in the sensitive elements of EPS. Considering several scenarios for the development of an EPS model, it is evident that the sensitivity in the corresponding places has decreased. Thereby, enabling the transit flow to be increased and new power plants to be integrated, as well as to increase the stability of EPS as a whole.
5. According to the planned big changes in the configuration of EPS model, connected with the modernization and new interconnections of transmission lines as well as with construction of new large wind parks, the static stability of power system model has to be estimated based on the detection methods of weakest network parameters in relation to external impacts, taking into account the need to identify such nodes of the electrical network, the increased sensitivity of which contributes to the occurrence in them of significant violations in the deviation.
6. It should be noted that the existence of weak spots is inherent in any power supply system; however, their sensitivity can be reduced. So, for example, the results obtained from localization of weak points and vulnerability valuation in the design stage allow providing the next steps for increasing the static stability of EPS model, namely, it is necessary to work out in more detail the identified weak nodes and lines.
7. Analyzing the obtained results, it is possible to observe the following dependence: the more electric transmission lines there are between nodes of the power system, the less sensible the EPS nodes are.
8. According to the obtained results for the model of the scheme of Scenario 5 the most sensitive are the hubs in the center of the EPS (in EPS-2). Thus, energy system EPS-2

becomes the most vulnerable sector from the point of view of maintaining the stability of the EPS model, which is shown by the results of modelling in calculating nodes and branches. It means that there is a need for placing of voltage and frequency regulators in EPS-2 that allow providing increasing of EPS static stability on the whole.

9. The following source data were used to perform the calculations: EPS stationary mode parameters (power, load, node voltage and angle), network parameters (internal and external connection set for power transmission, branch resistance).

In order to ensure convergence (balance between generation and consumption) of the operational mode, the balance node and reactive power were set arbitrarily according to the model under consideration, which are closer to the real conditions of the EPS models. The calculation of the respective modes is approximate as well, and does not allow the estimation of active and reactive power flows with high reliability.

BIBLIOGRAPHY

- [1] Ordinance of the Cabinet of Ministers No.129 Energy Development guidelines 2016-2020 (Latvian);
- [2] <http://www.inforse.org/europe/VisionLA.htm>.
- [3] Zicmane, I., Berzina, K., Sobolevskis, A., Kovalenko, S. Evaluation of Latvian Electrical Power System Static Stability according the New Development Strategy until 2025, *Journal Energy*.
- [4] Идельчик В.к. К вопросу о влиянии погрешностей исходных данных на результат расчета стационарного режима энергосистем. *Энергетика и транспорт*. 1968, pp. 9–15.
- [5] Sobolevskis, A., Zicmane, I., Murach, V. “Vulnerability assessment of electric power system for the case of Latvian EPS”, 2015 56th International Scientific Conference on Power and Electrical Engineering of Riga Technical University, Riga, 2015.
- [6] Воропай, Н.И. Анализ механизмов развития системных аварий в ЭЭС / Н.И. Воропай, Д.Н. Ефимов, В.И. Решетов // *Электричество*. 2008. № 10. С. 12–24.
- [7] Wasley, R.G. Identification and ranking of critical contingencies in dependent variable scale / R.G. Wasley, M. Danesdoost // *IEEE Trans. Power Appar. and Syst.* 1983. Vol. 102, pp. 881–892.
- [8] Анализ неоднородностей электроэнергетических систем/О.Н. Войтов [и др.]. – Новосибирск: Наука; Сиб. отд-ние РАН, 1999. 256 с.
- [9] Гамм, А. 3. Сенсоры и слабые места в электроэнергетических системах / А. 3. Гамм, И. И. Голуб. – Иркутск: Наука; Сиб. отд-ние РАН, 1996. 99 с.
- [10] Elektroenerģijas pārvades sistēmas attīstības plāns, AST, Rīga. 2016, 21–30 lpp.
- [11] Ordinance of the Cabinet of Ministers No.129 Energy Development guidelines 2016-2020 (Latvian).
- [12] <http://www.litgrid.eu/index.php/news-events/news/baltic-tsos-launch-a-common-baltic-balancing-market-from-2018/3760>.
- [13] <https://cyberleninka.ru/article/n/matematiceskaya-model-energositemy-kaliningradskoy-oblasti-i-otsenka-na-ney-elementov-glavnoy-shemy-tes-pregolskaya>
- [14] Criteria and Countermeasures for Voltage Collapse. CIGRE TF 38.02.12. Final report // *Electra*. 1991. No. 124, pp. 118–132.
- [15] Approaches to the Security Analysis of Power Systems: Defence Strategies Against Malicious Threats / E. Bompard [et al.] // Office for Official Publications of the European Communities. 2007. 51 p.
- [16] Latvijas tīkla kodekss: <http://likumi.lv/doc.php?id=257943>
- [17] Kundur, P., Paserba, J., Ajarapu, V., Andersson, G., Bose, A., Canizares, C., Hatziargyriou, N., Hill, D., Stankovic, A., Taylor, C., Van Cutsen, T., Vittal, V. Definition and Classification of Power System Stability. – *IEEE Transactions on power systems*, Vol. 19, No. 2, May 2004, pp.1387–1401.
- [18] Nepomnyaschij, V., Gerhards, J., Mahņitko, A., Lomane, T. Reliability of Latvian Power System's 330 kV Substations. *Latvian Journal of Physics and Technical Sciences*, 2014, Vol.51, Iss.3, pp. 15–23. ISSN 0868-8257. Available from: doi:10.2478/lpts-2014-0016.
- [19] Utans, A.; Sauhats, A., Bieļa-Dailidoviča, E. Wide-Area Measurements-Based Out-Of-Step Protection System. In 2015 56th International Scientific Conference on Power and Electrical Engineering of Riga Technical University (RTUCON): Proceedings, Latvia, Riga, 14–14 October 2015. Riga: Riga Technical University, 2015, pp. 11–15.
- [20] Sauhats, A.; Utans, A.; Svalova, I., Svalovs, A., Antonovs, D., Bochkarjova, G. Two-Terminal Out-of-Step Protection for Multi-Machine Grids Using Synchronised Measurements. *PowerTech*, Netherlands, Eindhoven, 2015.
- [21] Sauhats, A., Utans, A., Antonovs, D., Svalovs, A. Multi-Terminal Out-of-Step Protection System. In 2016 IEEE 16th International Conference on Environment and Electrical Engineering (EEEIC), Florence, 2016.
- [22] Sauhats, A., Utans, A., Antonovs, D. and Svalovs A. Angle Control-Based Multi-Terminal Out-of-Step Protection System. *Energies*, Vol. 10, 2017, 308 p.
- [23] “Electromechanical transition processes in electric systems” under edition of I. Zicmane. Published/Created Riga: RTU Izd., 2012. 402 p.

- [24] B. Papkovs, I. Zicmane “Electromagnetic transition processes in electric systems”, Published/Created Riga: RTU Izd., 2007. 306 p. Library of Congress.
- [25] Ersen Akdeniz, Mustafa Bagriyanik “A knowledge based decision support algorithm for power transmission system vulnerability impact reduction” *Electrical Power and Energy Systems* 78 (2016), pp. 436–444, <http://dx.doi.org/10.1016/j.ijepes.2015.11.041>.
- [26] Nima Amjadya, Masoud Esmailib. “Improving voltage security assessment and ranking vulnerable buses with consideration of power system limits” *Electrical Power and Energy Systems* 25 (2003) pp. 705–715. doi: 10.1016/S0142-0615(03)00020-6.
- [27] Cepeda, J. C., Colome, D. G. Vulnerability assessment of electric power systems through identification and ranking of vulnerable areas. *Int J Emerg Electr Power Syst*, 2011.
- [28] Jun Yang, Kai Jiang “The sensitive line identification in resilient power system based on fault chain model” *Electrical Power and Energy Systems* 92 (2017), pp. 212–220, <http://dx.doi.org/10.1016/j.ijepes.2017.05.004>.
- [29] Sobolevskis, A., Zicmane, I., Murach V. “Vulnerability assessment of electric power system for the case of Latvian EPS”, 2015 56th International Scientific Conference on Power and Electrical Engineering of Riga Technical University, Riga, 2015.
- [30] Sobolevskis, A., Zicmane, I., “Analysis of vulnerability of the Latvian electrical power system” 2016 16 IEEE International Conference on Environment and Electrical Engineering, La Palazzina de' Servi, Florence, Italy, 2016.
- [31] Sobolevskis, A., Zicmane, I., “Assessing the Impact of Registering of Weak Points Calculating the Power System Operating Modes”, 2016 57th International Scientific Conference on Power and Electrical Engineering of Riga Technical University, Riga, 2016.
- [32] Sobolevskis, A., Zicmane, I., “Prediction Of Latvian Electrical Power System For Reliability Evaluation Including Wind Energy” 17th IEEE International Conference on Environment and Electrical Engineering, Milan, Italy, 2017.
- [33] Zicmane, I., Berzina, K., Sobolevskis, A., Kovalenko, S. Latvian Electrical Power System Stability’s Analysis Taking into Account New Development Strategy until 2025. In: 11th International Conference on Electromechanical and Power Systems (SIELMEN 2017): Proceedings, Moldova, Chisinau, 12–13 October 2017. Chisinau: 2017, pp. 1–6.
- [34] Zicmane, I., Sobolevskis, A., Murac, V. Vulnerability Assessment of Electric Power System for the Case of Latvian EPS // 2015 56th International Scientific Conference on Power and Electrical Engineering of Riga Technical University (RTUCON 2015), Latvija, Riga, October 14–14, 2015.
- [35] <http://www.ast.lv>.
- [36] <http://www.let.lv>.

APPENDICES

Example of Identifying Sensitive Elements of EPS (Sensitivity Calculation of EPS Model for Scenario 2)

The task of analyzing heterogeneities of an EPS consists of using matrices of generalized values of EPS elements and information about the points of application of perturbations for searching and localizing the weak points of EPS. For the steady-state modes, the generalized values are expressed via parameters of the nodal conductance matrix and Jacobi matrix of equations of the steady-state mode, which are the main sources of information about the sensitivity and heterogeneity of EPS in steady-state modes. Sensitive places and their location will be identified with the help of numerical and analytical methods of studying the EPS' scheme and parameters.

The following parameters are used for the calculations: EPS stationary mode parameters (node voltage, power, load and generation, angles and modules), network parameters (internal and external communication set for power transmission, branch resistance). In order to ensure convergence (balance between generation and consumption) of the operational mode, the balance node and reactive power were set arbitrarily under the model considered, which is closer to the real conditions of the EPS's models. The calculation of the respective modes is approximate as well and does not allow the estimation of active and reactive power flows with high reliability.

An example of using the method (with nodal conductance matrices and Jacobi matrices) and the process of identifying sensitive points is shown further.

CALCULATION OF THE MAIN VALUES FOR ANALYSING THE EPS's SENSITIVE ELEMENTS

Units of measurement

ORIGIN := 1
 kvolt ≡ 1000 · volt k amp ≡ 1000 · amp grad ≡ °
 MVA ≡ kvolt · k amp Mvar ≡ MVA Mwatt ≡ MVA

INITIAL DATA

The network's steadystate 330 kV.

Number of nodes in the scheme: Number of nodes without the basic node: Number of branches in the scheme:

n := 20 i := 1.. n k := 1.. (n-1) nv := 29
 ny := 1.. (n-1) · 2

Initial continuous form of the "nodes-branches" incidence matrix:

$$I_{\text{WB}} = \begin{pmatrix} 1 & 7 & 7 & 8 & 9 & 9 & 9 & 5 & 5 & 2 & 3 & 6 & 4 & 10 & 11 & 11 & 11 & 12 & 12 & 13 & 15 & 15 & 15 & 10 & 15 & 14 & 1 & 18 & 19 \\ 19 & 20 & 8 & 9 & 5 & 10 & 20 & 6 & 2 & 3 & 4 & 10 & 10 & 11 & 17 & 17 & 12 & 20 & 13 & 15 & 14 & 16 & 20 & 12 & 16 & 17 & 18 & 2 & 7 \end{pmatrix}$$



Incidence matrix "nodes-branches":

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Multitude of indexes of the branches adjacent to node i:

		nodes																		
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
$S_i =$	1	19	5	2	3	9	5	20	7	8	9	10	11	12	15	13	15	11	1	1
	2	18	3	4	10	6	10	8	9	5	6	17	20	15	17	14	15	11	2	7
	3	0	18	0	0	2	0	19	0	10	4	17	13	0	0	16	0	14	0	0
	4	0	0	0	0	0	0	0	0	20	11	12	10	0	0	20	0	0	0	0
	5	0	0	0	0	0	0	0	0	0	12	0	0	0	0	16	0	0	0	0

numbers of nodes adjacent to them

Continuation of Appendix 1

Voltage module and phase in the steady state:

$$\begin{matrix}
 \begin{matrix}
 346.83 \\
 344.27 \\
 344.26 \\
 344.6 \\
 344.57 \\
 345.09 \\
 347.63 \\
 346.35 \\
 345.41 \\
 345.3 \\
 349.3 \\
 347.71 \\
 347.22 \\
 348.04 \\
 346.95 \\
 346.87 \\
 352.58 \\
 344.94 \\
 347.48 \\
 347
 \end{matrix} \\
 xU_y :=
 \end{matrix}
 \cdot \text{kvolt}
 \qquad
 \begin{matrix}
 \begin{matrix}
 -1.38 \\
 -0.02 \\
 -0.08 \\
 -0.4 \\
 -0.23 \\
 -0.73 \\
 -1.55 \\
 -0.89 \\
 -0.43 \\
 -0.99 \\
 -5.37 \\
 -2.71 \\
 -1.98 \\
 -3.48 \\
 -1.3 \\
 -1.24 \\
 -8.16 \\
 -0.35 \\
 -1.67 \\
 0
 \end{matrix} \\
 x\delta_y :=
 \end{matrix}
 \text{grad}$$

$\text{last}(xU_y) = 20$
 $\text{last}(x\delta_y) = 20$

BUILDING THE MATRIX OF OWN AND MUTUAL NODAL CONDUCTANCES

$$Y(R_v, X_v) = \begin{matrix}
 \begin{matrix}
 1 & 2 & 3 & 4 & 5 \\
 1 & 0.0155+0.0732j & 0 & 0 & 0 \\
 2 & 0 & 0.0944+0.4451j & -0.0439-0.2071j & 0 \\
 3 & 0 & -0.0439-0.2071j & 0.0952+0.4488j & -0.0513-0.2417j \\
 4 & 0 & 0 & -0.0513-0.2417j & 0.0952+0.4488j \\
 5 & 0 & -0.041-0.1933j & 0 & 0.0954+0.4497j \\
 6 & 0 & 0 & 0 & 0 \\
 7 & 0 & 0 & 0 & 0 \\
 8 & 0 & 0 & 0 & 0 \\
 9 & 0 & 0 & 0 & 0 \\
 \dots & & & &
 \end{matrix} \\
 \text{siemens}
 \end{matrix}$$

LOCALIZATION OF THE EPS'S SENSITIVE POINTS USING SINGULAR VALUE DECOMPOSITION OF THE JACOBIAN MATRIX

Rectangular system of coordinates:

$$J := \begin{pmatrix} \frac{dW_p}{dU_a} & \frac{dW_p}{dU_r} \\ \frac{dW_q}{dU_a} & \frac{dW_q}{dU_r} \end{pmatrix}$$

Polar system of coordinates:

$$J := \begin{pmatrix} \frac{dW_p}{dU} & \frac{dW_p}{d\delta} \\ \frac{dW_q}{dU} & \frac{dW_q}{d\delta} \end{pmatrix}$$

Analysis Jacobian matrices

$$MJ^{-1} = \begin{matrix}
 \begin{matrix}
 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
 1 & -0.0231 & -0.0061 & -0.0056 & -0.0051 & -0.0054 & -0.005 & -0.0098 & -0.007 & -0.0045 & -0.0045 \\
 2 & -0.0061 & -0.01 & -0.009 & -0.008 & -0.0085 & -0.0077 & -0.0038 & -0.0049 & -0.0058 & -0.0069 \\
 3 & -0.0055 & -0.009 & -0.0102 & -0.0088 & -0.0079 & -0.0075 & -0.0035 & -0.0047 & -0.0057 & -0.0071 \\
 4 & -0.005 & -0.008 & -0.0088 & -0.0094 & -0.0074 & -0.0073 & -0.0033 & -0.0045 & -0.0055 & -0.0073 \\
 5 & -0.0054 & -0.0085 & -0.0079 & -0.0074 & -0.0093 & -0.0081 & -0.0036 & -0.0049 & -0.0061 & \dots
 \end{matrix}
 \end{matrix}$$

$$MJ = \begin{matrix}
 \begin{matrix}
 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
 1 & -5.4063 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 2 & 0 & -32.4925 & 15.1239 & 0 & 14.1126 & 0 & 0 & 0 & 0 & 0 \\
 3 & 0 & 15.1235 & -32.7687 & 17.6344 & 0 & 0 & 0 & 0 & 0 & 0 \\
 4 & 0 & 0 & 17.6519 & -32.8337 & 0 & 0 & 0 & 0 & 0 & 15.1104 \\
 5 & 0 & 14.1249 & 0 & 0 & -32.8763 & 14.1105 & 0 & 0 & 4.6032 & \dots
 \end{matrix}
 \end{matrix}$$

Continuation of Appendix 1

ANALYSIS OF SINGULAR VALUE DECOMPOSITION OF THE FULL JAKOBIAN MATRIX

Singular value decomposition of the Jacobian matrix:

$$J_{svd} (MJ) =$$

	1	2	3	4	5
1	3.2222·10 ⁻¹⁰	-0.0006	0.0015	-0.0007	4.6544·10 ⁻⁶
2	6.4013·10 ⁻⁷	-0.3782	0.508	-0.2313	0.001
3	-1.9779·10 ⁻⁶	0.4715	-0.0177	0.5259	-0.0005
4	7.9568·10 ⁻⁶	-0.4874	-0.4191	-0.2049	-0.0007
5	-1.3311·10 ⁻⁶	0.3111	-0.5501	-0.3412	0.0001
6	5.9479·10 ⁻⁶	-0.2666	0.0954	0.5204	-0.001
7	1.0636·10 ⁻⁹	-0.0001	0.0001	0.0004	-9.4145·10 ⁻⁷
8	-4.4332·10 ⁻⁸	0.0031	-0.0013	-0.0069	0
9	1.6527·10 ⁻⁶	-0.0639	0.0206	0.1022	-0.0002
10	-0	0.4252	0.4552	-0.4299	0.0009
11	-0	-0.0172	-0.026	0.0262	-0.0012
12	0.0008	-0.0257	-0.0378	0.0379	0.0073
13	-0.0214	0.0016	0.0033	-0.0035	...

i-th left singular vectors of the Jacobian matrix

$$W (MJ) =$$

	1	2	3	4	5
1	3.2222·10 ⁻¹⁰	-0.0006	0.0015	-0.0007	4.6544·10 ⁻⁶
2	6.4013·10 ⁻⁷	-0.3782	0.508	-0.2313	0.001
3	-1.9779·10 ⁻⁶	0.4715	-0.0177	0.5259	-0.0005
4	7.9568·10 ⁻⁶	-0.4874	-0.4191	-0.2049	-0.0007
5	-1.3311·10 ⁻⁶	0.3111	-0.5501	-0.3412	0.0001
6	5.9479·10 ⁻⁶	-0.2666	0.0954	0.5204	-0.001
7	1.0636·10 ⁻⁹	-0.0001	0.0001	0.0004	-9.4145·10 ⁻⁷
8	-4.4332·10 ⁻⁸	0.0031	-0.0013	-0.0069	0
9	1.6527·10 ⁻⁶	-0.0639	0.0206	0.1022	-0.0002
10	-0	0.4252	0.4552	-0.4299	0.0009
11	-0	-0.0172	-0.026	0.0262	-0.0012
12	0.0008	-0.0257	-0.0378	0.0379	0.0073
13	-0.0214	0.0016	0.0033	-0.0035	...

Diagonal matrix of singular values of the Jakobian matrix:

$$\Sigma (MJ) =$$

	1	2	3
1	180407.8135	0	0
2	0	104825.3293	0
3	0	0	80623.3279
4	0	0	0
5	0	0	0
6	0	0	0
7	0	0	0
8	0	0	0
9	0	0	0
10	0	0	0
11	0	0	0
12	0	0	0
13	0	0	...

Vector of singular values:

$$\sigma (MJ) \cdot 10^0 =$$

	1
1	180407.8135
2	104825.3293
3	80623.3279
4	76854.1288
5	65089.148
6	34300.4128
7	31638.4599
8	28425.1142
9	21787.4159
10	20603.8003
11	20399.0776
12	13244.0726
13	...

i-th right singular vectors of the Jakobian matrix:

$$V (MJ) =$$

	1	2	3	4	5
1	0	-4.4679·10 ⁻¹⁰	8.5783·10 ⁻¹⁰	-4.1454·10 ⁻¹⁰	2.0221·10 ⁻¹²
2	-2.2481·10 ⁻¹³	1.0776·10 ⁻⁷	-1.4922·10 ⁻⁷	1.7447·10 ⁻⁸	-1.6914·10 ⁻¹⁰
3	1.2389·10 ⁻¹²	-1.9399·10 ⁻⁷	-9.5384·10 ⁻⁸	-1.807·10 ⁻⁷	-5.8428·10 ⁻¹¹
4	-1.2·10 ⁻¹¹	4.274·10 ⁻⁷	5.8454·10 ⁻⁷	2.8104·10 ⁻⁸	1.0258·10 ⁻⁹
5	9.9637·10 ⁻¹³	-1.9201·10 ⁻⁷	3.5968·10 ⁻⁷	3.8769·10 ⁻⁷	-4.6403·10 ⁻¹⁰
6	-8.665·10 ⁻¹²	2.5294·10 ⁻⁷	2.6058·10 ⁻⁹	-7.3689·10 ⁻⁷	1.5024·10 ⁻⁹
7	0	-3.6213·10 ⁻¹¹	-3.6033·10 ⁻¹²	1.1475·10 ⁻¹¹	1.082·10 ⁻¹³
8	7.0471·10 ⁻¹⁵	-4.9158·10 ⁻¹¹	-1.6616·10 ⁻⁹	6.7346·10 ⁻¹⁰	-3.8356·10 ⁻¹²
9	-2.3736·10 ⁻¹²	3.2974·10 ⁻⁸	1.8759·10 ⁻⁸	-8.0541·10 ⁻⁸	8.8893·10 ⁻¹¹
10	7.5198·10 ⁻¹¹	-7.7712·10 ⁻⁷	-1.1993·10 ⁻⁶	1.2959·10 ⁻⁶	-1.7047·10 ⁻⁹
11	2.3482·10 ⁻¹¹	1.5575·10 ⁻⁸	6.6642·10 ⁻⁸	-7.9144·10 ⁻⁸	7.682·10 ⁻⁹
12	2.2166·10 ⁻¹⁰	-4.5863·10 ⁻⁹	1.1905·10 ⁻⁸	-1.9743·10 ⁻⁸	-1.0815·10 ⁻⁸
13	-4.3212·10 ⁻⁹	-1.2192·10 ⁻¹⁰	-1.1641·10 ⁻⁹	1.4449·10 ⁻⁹	...

Values of the components of the first right V singular vector corresponding to nodal voltage modules (maximal components correspond to nodes sensitive to voltage loss):

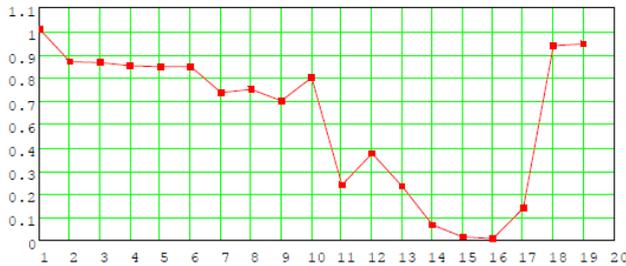


Fig.2a

Values of differences related to nodal voltage modules of the ith and jth component of the right singular vector of the full Jakobian matrix (branches sensitive to voltage loss):

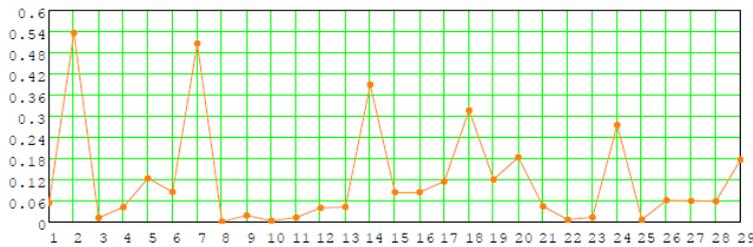


Fig.2b

LOCALIZATION OF THE EPS's SENSITIVE POINTS USING SINGULAR DECOMPOSITION OF THE MATRIX Y

Evaluation of sensitivity of links by the difference of the corresponding nodes at the ends of the branches of the first rightsingular vector of the matrix Ygb (are sensitive to voltage loss)

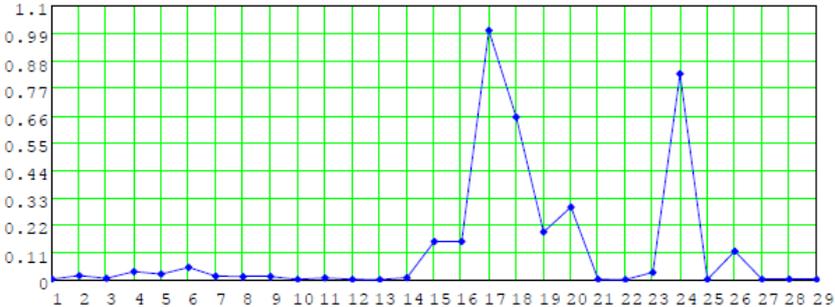


Fig.3b
 $f_{V_{Yij}_{\sigma a}}$

Estimation of sensitivity of branches by components of the first right singular vector of the matrix Ygb:

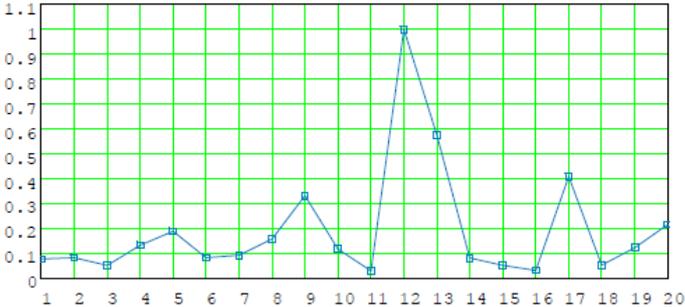


Fig.3a
 $f_{V_{Yv}_{\sigma a}}$