

MECHANICS  
MEHĀNIKAMETHODS OF OPIMAL SYNTHESIS OF STRONGLY  
NON-LINEAR (IMPACT) SYSTEMSSTIPRI NELINEĀRU (IMPULSU) SISTĒMU  
OPTIMĀLĀS SINTĒZES METODE

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## Introduction

Algorithm of optimal synthesis of strongly non-linear (impact) systems includes five tasks of formal order to invent a principally new adaptive dynamical system [1]. They are:

Preliminary analysis of main goal given to scientist to solve the problem.  
Optimization of main fundamental system. Analysis of ideal control actions.  
Synthesis a new structural schemes. Calculation of optimal parameters [2].

### 1. Preliminary analysis of main goal given to scientist to solve the problem

Preliminary analysis includes following 5 steps:

1. Analysis of technological processes.
2. Choice of base system.
3. Choice of control actions.
4. Clarification of criteria of optimization.
5. Choice of limits.

Example of material crush processes by hammer is shown in Fig. 1. The main goal is to pulp the material by impact interaction with hard solid body in very short time.

## 2. Optimization of main fundamental system

After the first task is given, the next task of optimization can be formulated in such way (Fig. 3.1.).

To find from all allowable (force  $u$  or impulse  $S$ ) control actions (1),

$$u \in u_0, \quad S_j \in S_0, \quad (1)$$

which moves the base system (2)

$$\dot{x} = f(t, x, u) + \frac{S_j}{m} \delta(t - t_j) \quad (2)$$

from initial position  $x_0$  to defined end position  $x_1$ , and ensures minimal (or maximal) value of criteria of optimization

$$K = \int_{t_0}^{t_1} f_0(t, x, u, S_j, t_j) dt \quad (3)$$

if additional limits on phase co-ordinates and control impulses exist (4):

$$|\dot{x}| \leq v_0; \quad |x| \leq A_0; \quad z \leq z_0; \quad \tau \geq \tau_0, \quad (4)$$

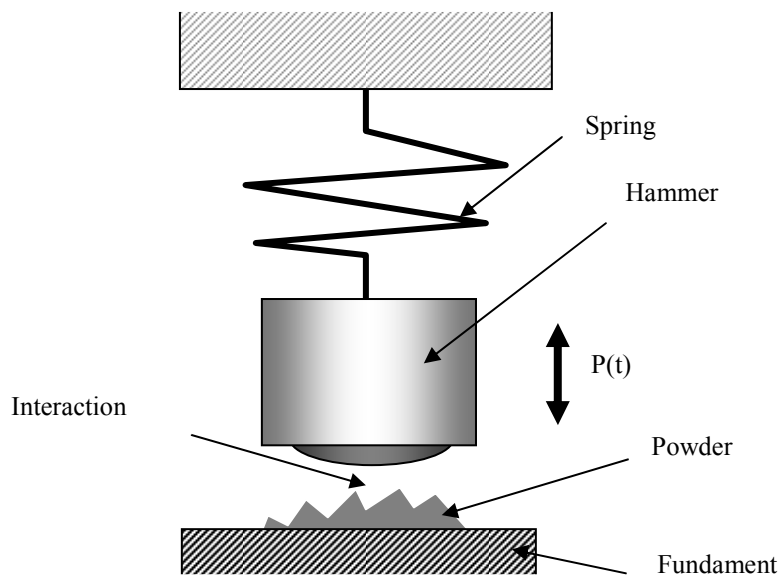


Figure. 1. Base system for powder material crush

where  $v_0; A_0$  - limits of velocity and displacement;  $z_0; \tau_0$  - maximal number of impulses and minimal time between them.

For solution of this task of optimization, various methods of optimal control theory can be used, for example:

- the maximum principle of Portraying;
- the classical method of variation calculus;
- method of moments;
- dynamic programming;
- mixed method (usually initially the optimization phase, followed by variation of parameters).

Thus in the second task of algorithm we get the ideal (optimal) control law or ideal (optimal) control trajectory in time domain or phase plane.

The next task is to get new shapes of real control action. They can be near to optimal control and must be stable, for example, like adaptive control actions, as shown below.

### 3. Analysis of ideal control actions

In the second task the obtained optimal control law can't be always realized in real systems. Therefore usually it is determined as ideal (non-real). This ideal control law may be useful for two purposes:

1. For evaluation of existing systems with respect to ideal law.
2. For synthesis of principally new structural systems with a control law close to ideal control law. This task is a foundation for new system synthesis. Here scientific approach must be open and very interesting new structures may be invented, for example, structures with adaptive control law.

#### 3.1. Example of existing system evaluation

Let us examine the optimal control law in a very simple case when one degree of freedom system is without spring (5):

$$m\ddot{x} = P(t). \quad (5)$$

The optimal force  $P(t)$  for maximal amplitude has constant level and changes the sign in the middle point of motion:  $u = \pm u_0$ ;  $x = 0$  (Fig. 2.). Mathematically it may be described as (6):

$$m\ddot{x} = P_0 \frac{\sin\left(\frac{2t}{T_0} \pi\right)}{\left|\sin\left(\frac{2t}{T_0} \pi\right)\right|}. \quad (6)$$

Here  $T_0 = \sqrt{\frac{8mA}{P_0}}$ , where  $T_0$  - time of optimal motion found from (5.). If we suppose that a user decides to approximate the obtained control law by harmonic function (Fig. 2.):

$$P(t) = P_0 \cdot \sin\left(\frac{t}{T_0 \cdot C} \pi\right), \text{ or}$$

$$m\ddot{x} = P_0 \cdot \sin\left(\frac{t}{T_0 \cdot C} \pi\right), \quad (7)$$

where  $C$  - constant.

Because the limits of  $P_0$  and displacement  $2A$  are fixed, the constant  $C$  of efficiency must be found by integration of equation (7):

$$T_0 \cdot C = \sqrt{\frac{4\pi mA}{P_0}}.$$

Therefore from  $T_0 = \sqrt{\frac{8mA}{P_0}}$  and last equation we have:

$$C = \sqrt{\frac{\pi}{2}}, \text{ or } C = 1.2533$$

The efficiency is about 25 %.

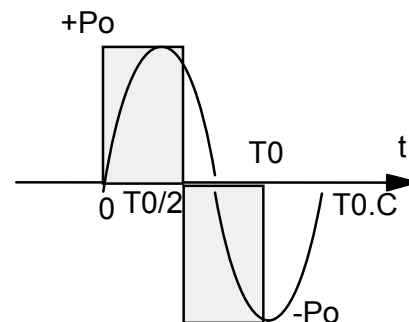


Figure 2. Approximation real control by harmonic function

#### 4. Synthesis of a new structural schemes

For solving this task we can use:

- full information about optimal control law;
- results of analysis of ideal control law;
- principles of acting of similar structural schemes;
- possible number of additional bodies;
- possible kinds of constraints;
- methods of forming of internal and external interaction in electro - mechanical systems, etc.

New structural schemes can be found in the medium of three kinds of strongly non-linear (impact) systems. They are:

1. Systems with external or internal excitation as a time function.
2. Systems with external or internal excitation as a function of phase co-ordinates only.
3. Systems with mixed excitations (parts of excitation are functions of time and other parts are functions of phase co-ordinates).

Between those last two systems (2. and 3.) the adaptive control (self-controlling) systems may be set. They ensure a stable motion of strongly non-linear (impact)

systems (if in working regime some parameters of a system and excitations have disturbances).

#### 4.1. Example of synthesis of vibration system with time excitation

From optimal control law Fig. 2. we can synthesize a vibration system with amplitude  $A$  in a following form (8):

$$m\ddot{x} = -c \cdot x - b \cdot \dot{x} - P0 \cdot \frac{\sin(\frac{\pi \cdot t}{2 \cdot T0})}{\left| \sin(\frac{\pi \cdot t}{2 \cdot T0}) \right|}, \quad (8)$$

were  $m$  - mass,  $c$  – stiffness of spring;  $b$  – damping coefficient,  $T0$  – period of motion,  $t$  – time.

In this case without damping ( $b = 0$ ) the system must be starting with special initial conditions  $(0; V0)$ , to have the optimal motion. (Fig. 3.).

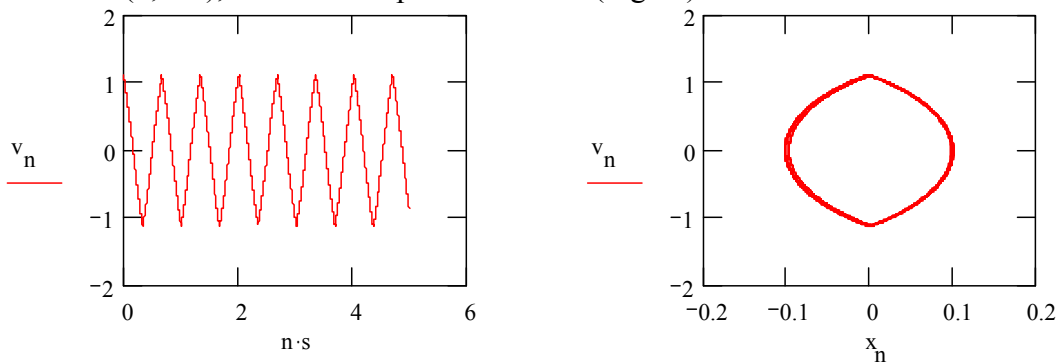


Figure 3. Motion in time domain and phase plane when  $b = 0$

If the initial conditions change  $(-0,01; V0-0,8)$ , the motion is unstable (Fig. 4.).

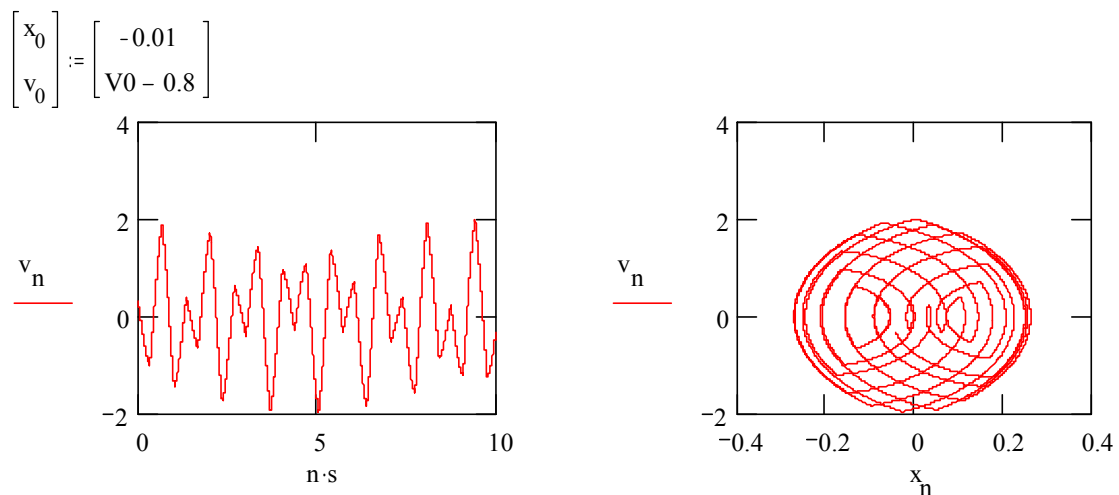


Figure 4. Unstable motion ( $b = 0$ )

For motion stabilization damping  $b \neq 0$  must be used (Fig. 5).

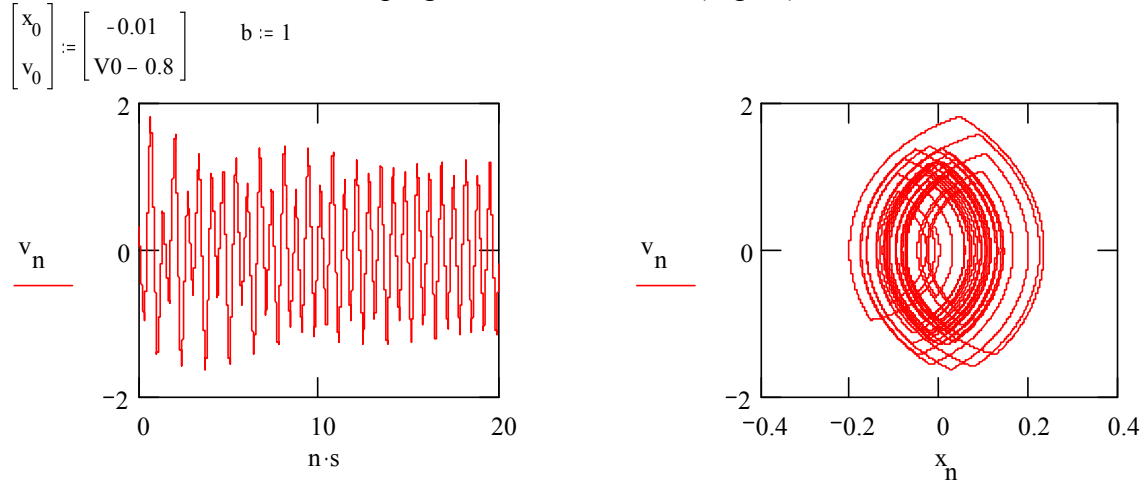


Figure 5. Motion with damping

Despite damping, the motion is not so good because transition motion is very long (in this example about 10 seconds) and trajectories in phase plane are crossed many times.

#### 4.2. Example of synthesis of a vibration system with excitation as function of phase coordinates

Results of modeling of such adaptive system are shown in Fig. 6. - Fig. 8. The system is controlled by variation in both phase co-ordinates: displacement ( $x$ ) and velocity ( $v$ ). The main conclusion based on obtained results is that system is very stable and transient motion ends fast enough.

$$\begin{bmatrix} x_{n+1} \\ v_{n+1} \end{bmatrix} := \begin{bmatrix} x_n + s \cdot v_n \\ v_n + \frac{s}{m} \cdot \left[ \left( -c \cdot x_n - b \cdot v_n \right) - P0 \cdot \frac{x_n}{|x_n|} \cdot 0.5 \cdot \left( 1 + \frac{0.5 \cdot V0 - |v_n|}{|0.5 \cdot V0 - |v_n||} \right) \right] \end{bmatrix}$$

Figure 6. Excitation as a function of phase co-ordinates ( $x_n; v_n$ ) only

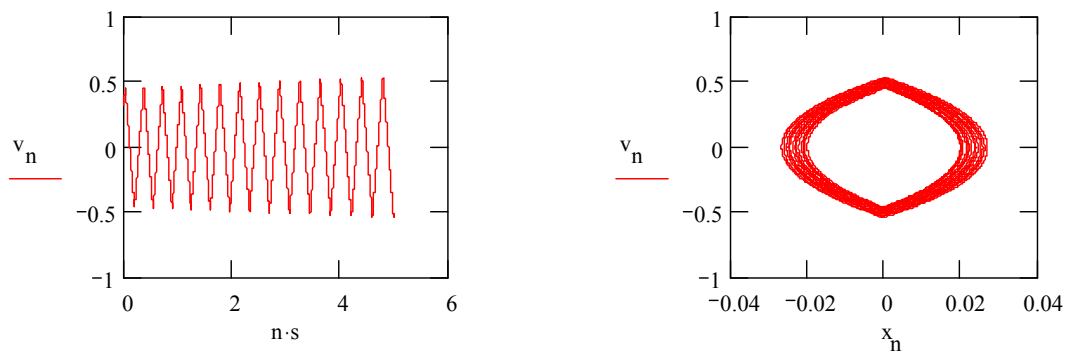


Figure 7. Initial conditions are inside limit cycle

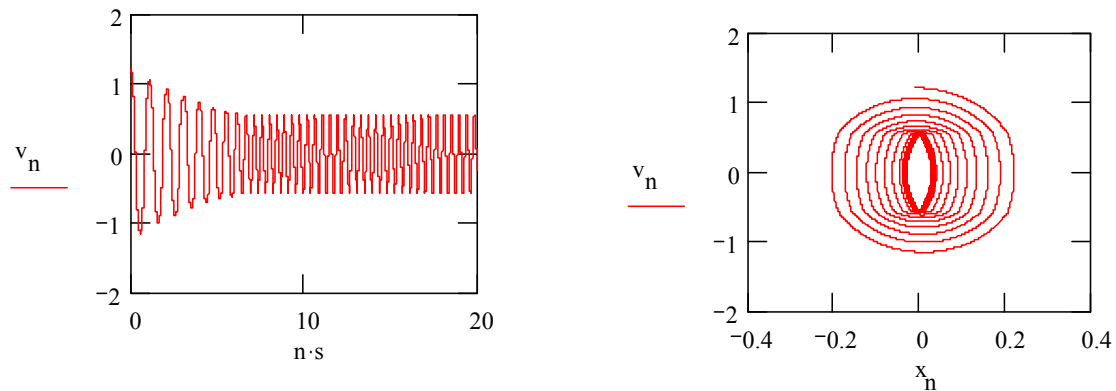


Figure 8. Initial conditions are out of limit cycle. Motion is very stable

## 5. Calculation of optimal parameters

If the synthesis of a system satisfies completely all conditions of optimal control law this task is complete. For other cases the theory and methods of optimal parameters calculation can be used [1, 2].

## References

1. Lavendelis, E. Synthesis of optimal vibro machines - Zinatne, Riga, 1970. – 252 p. (in Russian).
2. Viba, J. Optimisation and synthesis of vibro impact systems - Zinatne, Riga, 1988. – 253 p. (in Russian).

### **Vība J., Lavendelis E. Stipri nelineāru (impulsu) sistēmu optimālās sintēzes metode**

*Parādīts, ka (lai izgudrotu principiāli jaunas adaptīvas dinamiskas sistēmas) stipri nelineāru (triecienu) sistēmu optimālās sintēzes algoritmu var iedalīt šādos piecos formālos etapos: - pētniekam dotā pirmatnējā uzdevuma analīze; - galvenās fundamentālās sistēmas optimizācija; - ideālās vadības iedarbības analīze; - jaunu strukturālu shēmu sintēze; - optimālo parametru atrašana. Otrajā etapā iegūto vadības likumu var izlietot diviem mērķiem: - eksistējošo sistēmu novērtēšanai attiecībā pret ideālo likumu; - principiāli jaunu strukturālu sistēmu sintēzei attiecībā pret tuvinājumu ideālajam likumam. Šis etaps ir bāze jauniem izgudrojumiem. Te zinātniska pieeja dod iespēju izgudrot principiāli jaunas mašīnas un mehānismus. Jaunās strukturālās shēmas jāmeklē trīs stipri nelineāru (triecienu) sistēmu vadības principos: - sistēmās ar vadības laika ierosmi; - sistēmās ar fāzu koordinātu ierosmes vadību; - jauktās sistēmās.*

### **Viba J., Lavendelis E. Methods of optimal synthesis of strongly non-linear (impact) systems**

*Is shown that algorithm of optimal synthesis of strongly non-linear (impact) systems may be dividing into five tasks of formal order to invent a principally new adaptive dynamical system. They are: - preliminary analysis of main goal given to scientist to solve the problem; - optimization of main fundamental system; - analysis of ideal control actions; - synthesis a new structural schemes; - calculation of optimal parameters. In the second task the obtained optimal control law may be useful for two purposes: - for evaluation of existing systems with respect to ideal law; - for synthesis of principally new structural systems with a control law close to ideal control law. This task is a*

*foundation for new system synthesis. New structural schemes can be found in the medium of three kinds of strongly non-linear (impact) systems. They are: - systems with external or internal excitation as a time function; - systems with external or internal excitation as a function of phase co-ordinates only; - systems with mixed excitations.*

***Виба Я., Лавенделис Э. Метод оптимального синтеза сильно нелинейных (импульсных) систем***

*Показано, что для изобретения принципиально новых адаптивных динамических систем алгоритм оптимального синтеза сильно нелинейных (импульсных) систем может быть разделён на пять самостоятельных этапов. Они следующие: - предварительный анализ задачи, которая даётся исследователю для решения проблемы; - оптимизация главной фундаментальной системы; - анализ идеального управляющего воздействия; - Синтез новой структурной схемы; - определение оптимальных параметров. На втором этапе найденный оптимальный закон управления может быть использован для двух целей: - для оценки существующих систем по отношению к идеальному закону; - для синтеза принципиально новых структурных систем с управляющим воздействием, близким к идеальному закону. Этот этап служит основой для синтеза новых структурных систем. Новые структурные схемы должны быть определены в числе трёх видов сильно нелинейных (импульсных) систем: - системы с возбуждением по времени; - системы с возбуждением по фазовым координатам; - смешанные системы.*