On the Figures of Merit for Planar Spiral Inductors

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Abstract - Several expressions of figures of merit (FOM) for different planar spiral inductors (PSI) are presented and analysed. Some of these FOM were found in scientific papers and the other ones were proposed by us. Most of these FOM are heavily dependent on the outer diameter; therefore they do not allow us to examine influence of the other design parameters on PSI overall performance. To prevent this disadvantage FOM independent on the outer diameter was derived during the investigation. Such FOM, for example, enables scaling of PSI parameters to be applied.
Keywords: inductors, figure of merit, overall performance.

I. INTRODUCTION.

Planar spiral inductors (PSI) are essential passive elements in radio-frequency (RF) and microwave integrated circuits [1]. They are used, for example, in LC-passive filters, voltage-controlled oscillators, low-noise amplifiers, switched mode power supplies (SMPS) and so on [6]. PSI are mainly implemented as integrated planar elements in RFIC, but they can also be used as ones on PCB.

A planar spiral inductor consists of thin metal spiral (usually made of Al), placed on substrate. Silicon is more often used as substrate material (typically with resistivity of 0.1 – 20 Ω·cm) [1].

The basic characteristics of PSI are the inductance (L), the quality factor (particularly the maximum Q-factor (Q_{max}) at the frequency f_{max}), the self-resonant frequency (f_{res}), and also the design parameters (number of turns, the metal track width (w), the track spacing (s), the inner diameter and the outer diameter (d_o), the thickness of spiral, etc.). Shapes of spirals can be different: hexagonal, octagonal, rectangular, circular, a.o. Rectangular spirals are more often used for planar inductors because of technological advantages [10]. Geometry of a rectangular spiral inductor is shown in Fig.1. Fig.2 depicts typical frequency dependence of the quality factor for PSI.

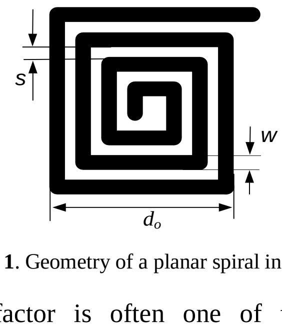

Figure 1. Geometry of a planar spiral inductor [6]

The quality factor is often one of the most critical parameters of PSI due to its typically low values that is problematic for many applications [3,6]. If substrate is made of low resistivity silicon (LRS), PSI Q-factor does not usually exceed 10 units [3]. In order to increase the quality factor substrates made from glass, GaAs, high-resistivity silicon (HRS) are used [1]. Micromachining technology can also be applied to increase the Q-factor and the self-resonant frequency of PSI on substrates made of LRS [1]. This technology consists of silicon etching under metal spiral or spiral lifting above substrate [1,3]. In this case Q-factor can reach tens of units.

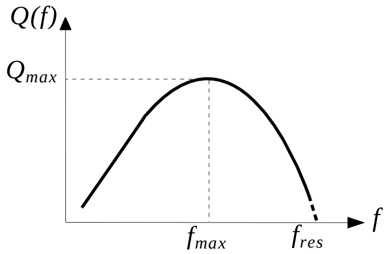

Figure 2. Typical frequency dependence of Q-factor

In past few decades, great efforts have been devoted to the optimisation, modelling and design of planar spiral inductors [2]. Computer software and different useful models for PSI were developed as well as approximate analytical expressions were derived for main PSI parameters. However, it is still difficult to optimise, design and analyze planar spiral inductors [2]. In order to make it easier, it can be useful to apply certain figures of merit (FOM), characterizing the overall performance of PSI.

II. ESSENCE OF FIGURE OF MERIT

Figures of merit are used in various fields of science and engineering, including electronics. For example, one can mention several simplest FOM applied in various fields of electronics as follows [8,9]: clock rate of a CPU of a computer, contrast ratio of a LCD, quality factor of an inductor, resolution of the CCD in a digital camera, etc. One of the possible definitions of FOM is as follows [8,9]: a figure of merit is a numerical quantity based on one or more characteristics of a system, component, device, procedure or material of the same type that can be used to characterize their performance.

Let us show one of the possible general expressions of figure of merit:

$$FOM = \prod_{i=1}^{N} P_i^{k_i}\ , \qquad (1)$$

where, P_i are the parameters (of a given material, device, etc.) concerned; k_i are the influence coefficients assigned for each parameter; N is number of the parameters used in expression for FOM. P_i can depend on the other parameters. Values of the influence coefficients for each parameter (P_i) depend on fields of applications of a given material, component, etc.

III. FIGURES OF MERIT FOR PSI

As mentioned before, one of the central problems of PSI is low Q-factor values. Many scientific papers are devoted to this problem. That is why the Q-factor is considered to be the main figure of merit [4,10] for planar spiral inductors. However, use of this single parameter, firstly, is often not enough to characterize the overall performance of planar inductors; secondly, main parameters of PSI (Q_{max}, f_{max}, A, L, f_{res}) can be conflicting (increasing Q_{max} can lead to decreasing f_{res}). Therefore it is of importance to use more characteristics in expressions for FOM.

In [4, 10] the following FOM were suggested:

$$FOM1 = Q_{max}L/A; \qquad (2)$$

$$FOM2 = Q_{max}f_{res}/A,$$

where A is the occupied area of PSI. Analysing, for example, (2) one can observe that it is particular case of (1) where all the influence coefficients are equal to ±1.

By using main PSI parameters, we can suggest four more FOM including Q_{max} and A:

$$FOM3 = Q_{max}f_{max}/A;$$

$$FOM4 = Q_{max}/A;$$

$$FOM5 = (Q_{max}Lf_{res})/A;$$

$$FOM6 = (Q_{max}Lf_{max})/A.$$

Calculated values of FOM1…FOM6 for several planar spiral inductors with different d_o are tabulated in Table 1. Note, that in this table the occupied area of PSI is assumed to be equal to d_o^2 for any shape of spiral.

TABLE 1
FOM1...FOM6 VALUES FOR SOME PSI

Type of PSI	Ordinary	Ordinary	Ordinary	Ordinary	Ordinary	Ordinary
Shape of spiral	Octagonal	Octagonal	Square	Square	Circular	Square
Substrate material	Si	Si	Si	Si	Si	FR4
Substrate resistivity, Ω·cm	10	10	10	10	2000	10^8
d_o, mm	0.14	0.22	0.3	0.4	0.61	16
Reference	2	2	7	7	10	Our
FOM1, nH/mm^2	952	556	493	315	484	165
FOM2, GHz/mm^2	4690	1228	404	215	307	0.2
FOM3, GHz/mm^2	1520	443	88	35	109	0.05
FOM4, mm^{-2}	372	155	62	39	41	0.6
FOM5, Hz·H/mm^2	12007	4411	3231	1717	3627	64
FOM6, Hz·H/mm^2	3892	1588	701	284	990	13

The results shown in Table 1 demonstrate that FOM1…FOM6 values tend to decrease if d_o increases. Significant difference in FOM1…FOM6 values can be observed when comparing PSI with d_o=0.14 mm versus d_o=16 mm (in case of FOM1 it is less pronounced) despite the fact that these PSI utilize different substrates (Si 10Ω·cm versus FR4). It is evidence of strong influence of d_o on FOM1…FOM6 values.

To select high-performance PSI for a given application, FOM1…FOM6 have to be corrected by means of introducing influence coefficients for each parameter. It is necessary because different devices, where PSI are used, have their own particular emphasis or requirement on each individual parameter of PSI, higher absolute values of the influence coefficients should be assigned to the more critical parameters. For instance, in the case of output filters of SMPS integrated circuits, it is important for figures of merits to take into account three basic characteristics of PSI: Q_{max}, L and occupied area A, because efficiency of SMPS depends on Q_{max}, output ripple depends on L, but occupied area affects the cost of the filter. Using (1) and (2), one can write useful figure of merit to compare PSI overall performance for the aforementioned application:

$$FOM_{SMPS} = Q_{\max}^{k_1} L^{k_2} A^{k_3}, \qquad (3)$$

where k_1,k_2,k_3 are the influence coefficients (k_1,k_2>0, k_3<0).

IV. SEARCH FOR FOM INDEPENDENT ON THE OUTER DIAMETER

The fact that the abovementioned FOM values are heavily dependent on d_o does not make it possible to analyze the effect of the other design parameters on PSI overall performance. Therefore the main task is to determine figure of merit independent on the outer diameter at least to first approximation.

To derive such FOM we resort to the analysis of analytical expressions for Q_{max}, L and f_{max} [5]. As the result of this, one can write down the following:

$$L \sim d_o;\ f_{res} \sim 1/d_o;\ Q_{\max} \sim d_o\sqrt{f_{res}};\ Q_{\max} \sim \sqrt{d_o};\ A \sim d_o^2. \qquad (4)$$

Using (2) and (4), the proportionality between FOM1 and d_o can be determined:

$$FOM1 \sim \frac{d_o\sqrt{f_{res}}\,d_o}{d_o^2} = \sqrt{f_{res}} = \frac{1}{\sqrt{d_o}}.$$

Now it is possible to get FOM independent on the outer diameter to a first approximation:

$$FOM7 = \sqrt{d_o} \cdot FOM1 = Q_{\max} L / d_o^{3/2}.$$

To demonstrate usefulness of FOM7 we calculated its values for the PSI mentioned in Table 1. The results are shown in Table 2. The standard deviation values (σ_n) normalised by arithmetic-mean values (AMV) for calculated FOM1…FOM7 are shown in Table 3. Analysis of the results presented in both tables reveals the absence of strong influence of the outer diameter on FOM7 values.

Let us discuss what FOM7 can give us. Firstly it gives possibility to compare the overall performance of PSI of different dimensions without taking into account the impact of d_o. For the purpose of justifying this statement, we calculated AMV of FOM1…FOM7 for PSI on different substrates (Table 4). Values of parameters for these FOM were defined by making the analysis of experimental data obtained both from many scientific papers (more than 50 publications were analysed) and from our experimental results (we designed and produced several experimental PSI with different outer diameters on FR4 substrates). As expected, FOM7 values in Table 4 confirms that PSI on substrate made of dielectrics as well as micromachined ones, are the best in terms of their overall performance.

TABLE 2
FOM7 VALUES FOR THE SAME PSI AS IN TABLE 1

Type of PSI	Shape of spiral	Substrate	d_o, mm	FOM7, nH/mm$^{3/2}$
Ordinary	Octagonal	Si, 10Ω·cm	0.14	356
Ordinary	Octagonal	Si, 10Ω·cm	0.22	261
Ordinary	Square	Si, 10Ω·cm	0.3	270
Ordinary	Square	Si, 10Ω·cm	0.4	199
Ordinary	Circular	HRS	0.61	377
Ordinary	Square	FR4	16	660

TABLE 3
σ_n NORMALIZED BY AMV OF FOM1-FOM7 VALUES FOR PSI FROM TABLE 1

FOM1	FOM2	FOM3	FOM4	FOM5	FOM6	FOM7
0.54	1.39	1.42	1.06	0.99	1.16	0.46

TABLE 4
AMV VALUES OF FOM1…FOM7 FOR PSI ON DIFFERENT SUBSTRATES

Type of PSI	Ordinary	Ordinary	Ordinary	Micromachined
Substrate	Si, ≤20Ω·cm	HRS	Glass, FR4	LRS, HRS
Number of PSI	30	6	4	8
d_o, mm	0.14…1.5	0.45…1.5	1.5…40	0.15…1
AMV of FOM1, nH/mm^2	476	442	207	482
AMV of FOM2, GHz/mm^2	745	234	28	816
AMV of FOM3, GHz/mm^2	362	101	23	114
AMV of FOM4, mm-^{2}	107	50	7	69
AMV of FOM5, Hz·H/mm^2	3415	2720	351	4176
AMV of FOM6 Hz·H/mm^2	1325	947	122	578
AMV of FOM7, nH/mm$^{3/2}$	289	325	621	665

Secondly, FOM7 allows applying scaling of PSI parameters. It means that using special scaling factors may permit one to transform parameters of large planar spiral inductors to parameters of smaller ones or vice versa.

Thirdly, use of FOM7 could help us to investigate influence of the skin effect in the spiral on frequency dependence of Q-factor. Thus, if one determines that there figure of merit independent on the outer diameter for set of PSI exists, one finds out if it is necessary to take the skin effect into consideration or not.

Further development of the obtained FOM7 is to derive figures of merit dependent only on one parameter of PSI (e.g. substrate resistivity). This task is much more difficult than one solved above. If one derives FOM dependent, for example, only on substrate resistivity, one will examine how substrate resistivity and spiral resistance influence Q(f).

V. CONCLUSIONS

To characterize overall performance of PSI it is not enough to use one of main PSI parameters. That is why FOM, in the form of combinations of main parameters, were presented in this paper. These FOM with the influence coefficients assigned for each parameter can help designers to choose high-performance PSI geometrical and process parameters for a given device. During the analysis of these FOM it was found out that they are heavily dependent on the outer diameter. This, in turn, hinders us to analyze the influence of the other design parameters on the overall performance of PSI. That is why FOM weakly dependent on d_o was derived. This FOM make it possible to compare the overall performance of PSI on different substrates without taking into account effect of d_o, to examine influence of the skin effect on frequency dependence of Q-factor as well as to apply PSI parameters scaling. Additionally it is believed that application of this FOM as well as derivation of new ones dependent only on one parameter would give us a possibility, for instance, to determine PSI layouts with minimum energy losses.

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