

**COMPUTATIONAL MODEL FOR DETERMINATION
OF VAPOR BUBBLE GROWTH SPEED MAXIMUM
IN SUPERHEATED LIQUIDS**

**MATEMĀTISKAIS MODELIS TVAIKA BURBULĪŠU
VEIDOŠANĀS ĀTRUMA MAKSIMUMA NOTEIKŠANAI
PĀRKARSĒTOS ŠĶIDRUMOS**

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Introduction

Theoretical and experimental investigations of liquids boiling – condensation phenomena (phase change of 1-st kind) have more than fifty years long history due to extremely high importance of these processes in different branches of technologies and engineering sciences. Practical applications of obtained results includes development of high effective and compact heat exchangers, are solving safety and exploitation regimes prediction problems of steam boilers, nuclear reactors and similar equipment operation with very high thermal loads, cooling of microelectronics devices, as well are the in basis of many others technical solutions. Liquids boiling – condensation processes are the most intensive and effective kinds of heat transfer, but at the same time very poor understood phenomena yet.

Problem analysis and main tasks of investigations

Actuality of liquids boiling processes researches indirectly is confirmed with large amount and permanently increasing in last time quantity both theoretical, both experimental investigations in this field – these are carried out in leading technical universities and scientific institutes in a lot of countries. Quite recently questions of situation and future development tendencies in liquid boiling were discussed in *W.M.Rohsenov* scientific laboratory symposium [1], held in Massachusetts Institute of Technology- one of the leading engineering university of USA.

Detailed analysis of present situation in field of boiling investigations finds out and emphasized a few main tasks for the future development. Necessarity of heat transfer researches such technology branches as elaboration and development of new technologies, processes and materials there were highlighted in reports [2,3]. Two main priorities were appointed especially:

1. High potential of advanced computational technic and mathematical apparatus (direct numerical simulation etc.) in boiling – condensation processes investigations both in pool liquid, both on solid heating surfaces. These tools successfully can be used for studies of vapor phase formation and evolution history as well as for determination of heat transfer factors in phase change processes;

2. Investigations of boiling – condensation in molecular and nanometric levels to get complete characteristics of phase change processes in micro- and macro scale. Authors [3,4] are expecting the basic role of nucleate boiling investigations as the primary problem in the theory and experience of phase phenomena in whole. All advanced technical means and theoretical methods have to be used for researches in this field, example, optical methods with digital data processing, infrared and liquid crystals high speed thermography, magnetic resonance, laser induced fluorescence etc.

Besides purely theoretical phase change (evaporatization – condensation) studies, very high importance is getting elaboration and promotion of special heating surfaces with advanced geometry and materials. The main task of such kind of surfaces is artificial nucleation sites formation with possibility to change their density onto surface area unit, heat flux optimization, process behavior prediction and control possibilities.

As it was shown by delegate authors of symposium [1], the present situation of direct numerical simulation and experimental technic don't allows yet to get absolutely exact correlations between elementary processes and macroscopic characteristics of boiling phenomena. While radical changes will occur in this field, it seems actually to made very carefull analysis of all available up to this time experimental data and look for new methods to arrange results of these investigations. A sample of such analysis for nucleate boiling we can find, example, in report [4].

For quite a long time the problems of vapor phase formation and development history in superheated liquids and on the solid surfaces were studied by authors of the given paper. Some aspects of mutual influence of solid heating surface with boiling liquids in microscopic level are summarized in article [7]. An experimental investigations of nucleate boiling by means of holographic interferometry combined with high speed photography and photoelectronic amplifier were carried out in [8,9]. Analysis of latest special literature shows actuality of such kind researches in nowadays as well.

A model of vapor bubble dynamic history

Summing up and processing the results and conclusions of investigations [7,8,9,10], an analytical method was elaborated for vapor phase (bubble) formation and evolution characteristics. Especially the offered method can be useful for modelling and analysis of vapor bubble growth dynamic studies in the initial stage of evolution, where pure experimental investigations are inconvenient with traditional technic and methods.

An analysis of vapor bubble evolution in the initial stage [10] shows priority of heat and mass transfer processes on the ``liquid-vapor`` interface – thermal resistance and phase change resistance in dependence system parameters. Thermal balance of growing vapor bubble can be written as:

$$4\pi R^2 \frac{dR}{d\tau} \rho_v h_{fg} = 4\pi R^2 k [T_{wall} - T_{\infty}(P_{\infty})], \quad (1)$$

$$\text{where: } k = 1/(1/\alpha + \delta/\lambda_l), \quad (2)$$

k – total heat transfer index from liquid to vapor inside of bubble, $W/(m^2 K)$;

δ – thickness of thermal boundary layer around the bubble, m ;

α – heat transfer index on ``liquid-vapor`` interface, $W/m^2 K$;

λ_l – thermal conductivity index of liquid, W/m ;

h_{fg} - latent heat of phase change J/kg ;

ρ_v – density of vapor, kg/m^3 ;

R – radius of vapor bubble, m ;

P_{∞} – system pressure, Pa .

T_{wall} - heating wall temperature, K

T_{sat} – saturation temperature accordingly system pressure, K .

Assume border conditions $k \rightarrow \alpha$, when $\delta \rightarrow 0$. From the theory of molekular kinetic theory we can apply equation:

$$\alpha = \alpha_{lv} \frac{\sigma_l h_{fg}^2}{(2\pi \frac{K}{m} T_{sat})^{0.5} T_{sat}}. \quad (3)$$

In equation (3):

α_{lv} – index of ``evaporatization – condensation`` on the liquid-vapor interface, shows relation between number of substance molecules crossing vapor-liquid border in both directions. Index is dimensionless, depends upon kind of liquid, system parameters and numerical values of α_{lv} can be find in special technical literature;

σ_l – surface tension factor of liquid, m^2/s ;

m – molecular weight of substance, $kg/kmol$;

K – Boltzmann factor, J/K ;

T_{sat} - saturation temperature of vapor, depends from system pressure.

Temperature T_{sat} is function of pressure inside of vapor bubble $P = f(R)$ and can be calculated:

$$T_{sat} = T_{\infty}(P_{\infty}) + \Delta T(R, T_{wall}), \quad (4)$$

Taking into account variable pressure inside of bubble, maximal temperature difference will be:

$$T_{wall} - T_{\infty} = (T_{wall} - T_{sat\infty}) \left(1 - \frac{R_{cr}}{R}\right) = \Delta T_{\infty} \left(1 - \frac{R_{cr}}{R}\right), \quad (5)$$

where: $R_{cr} = 2\sigma_l / (P_{wall} - P_{\infty})$ – critical radius of vapor nuclei.

Thickness of thermal boundary layer δ as function of bubble radius and variable temperature inside can be obtained from equation (1) and (2), considering linear temperature distribution nearly liquid-vapor interface:

$$\delta = \frac{2h_{fg}\rho_v R}{3c_p \rho_l (T_{wall} - T_{\infty})}. \quad (6)$$

Inserting value of δ in equation (1) and with some mathematical transformations we can obtain expression for vapor bubble growth speed ($dR/d\tau$, m/s) determination:

$$\frac{dR}{d\tau} = \frac{\alpha \lambda_l \Delta T_{\infty} (1 - R_{cr} / R)}{\rho_v h_{fg} \left[\lambda_l + \alpha \frac{2 h_{fg} \rho_v R}{3 \Delta T_{\infty} c_p \rho_l (1 - R_{cr} / R)} \right]}, \quad (7)$$

Equation (7) can be transformed in dimensionless form:

$$\frac{dX}{dFo} = Bi * Ja * \frac{1}{X} \frac{(1 - 1/X)^2}{(1/X - 1/X^2 + 2Bi/(3Ja))}, \quad (8)$$

The following dimensionless numbers are used in (8):

$X = R / R_{cr}$ – dimensionless radius of vapor bubble;

$$Fo = \frac{a_l \tau}{R_{cr}^2} \text{ – Fourier's number;} \quad (9)$$

$$Bi = \frac{\alpha R_{cr}}{\lambda_l} \text{ – Biot number;} \quad (10)$$

$$Ja = \frac{c_p \rho_l \Delta T_{max}}{h_{fg} \rho_v} \text{ – Jacob's number.} \quad (11)$$

Equation (8) gives possibility to calculate vapor nuclei growth dynamic in dimensionless form ($dX / dFo = f(X)$) as well to simulate bubble behavior in initial stage of bubble evolution. One sample of mathematical model simulation is shown on Fig.1. for the most common heat transfer agent – water , atmospheric pressure in system, initial superheating of liquid = 10K.

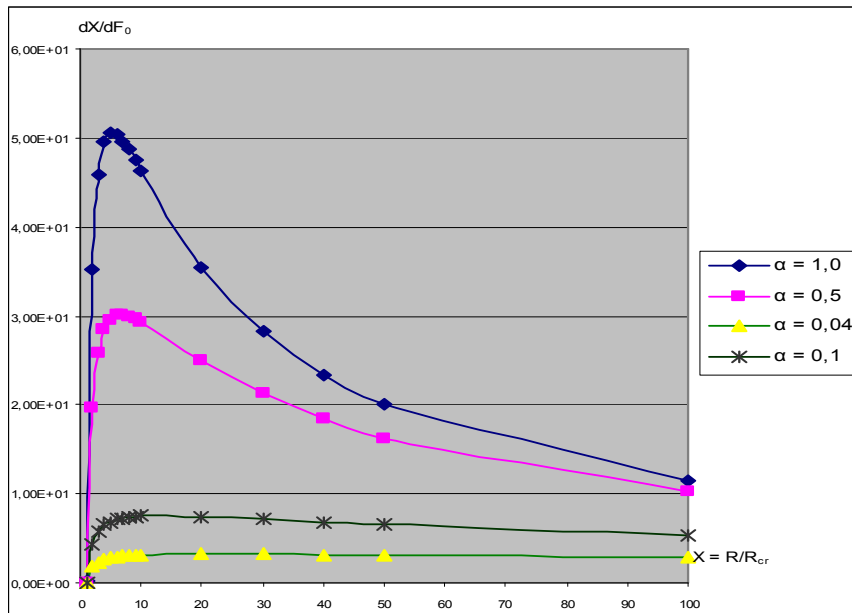


Fig.1. Dimensionless vapor bubble growth speed $dX/dFo = f(X)$. Water, system pressure $P=101.3kPa$, superheating $10K$; different values of "evaporation-condensation" factor α_v values ($\alpha_v = 0,04; 0,1; 0,5; 1,0$).

Conclusions

Numerical simulations of equation (8) shows dimensionless vapor bubble growth speed dX/dFo maximum in close vicinity of bubble critical radius R_{cr} values (dimensionless radius $X=R/R_{cr} < 10$). It means important influence of system conditions at vapor nuclei formation stage onto following bubble evolution. Bubble growth speed maximum essentially depends upon physical properties of liquid, system parameters as well as values of "evaporation – condensation" factor. Detailed investigations of vapor phase formation and initial stage of vapor bubble growth are actual to understand boiling phenomena dynamic and heat transfer in macroscale.

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Turlajs D., Grivcovs V., Jaundālders S., Matemātiskais modelis tvaika burbulīšu veidošanās ātruma maksimuma noteikšanai pārkarstos šķidrums

Darbs veltīts aktuālām visintensīvākā siltuma novadīšanas procesa - šķidrumu vārīšanās izpētes problēmām. Pašlaik galvenā uzmanība šķidrumu vārīšanās procesu pētījumos ir veltīta šo procesu izpētei mikroskopiskajā un nanometriskajā līmenī ar mērķi adekvāti aprakstīt parādību makroskopiskos mērogos. Pētījumā izstrādāts matemātiskais modelis tvaika burbulīša augšanas ātruma maksimuma aprēķināšanai burbulīša kritiskā rādiusa apkārtnē. Skaitliskie aprēķini veikti ūdenim ar sistēmas parametriem, kuru intervālā pieejams vislielākais eksperimentālo datu daudzums. Maksimālais tvaika burbulīša augšanas ātrums iegūts intervālā 2...5 kritiskā rādiusa vērtības. Iegūtā vienādojuma bezdimensiju forma ļauj analizēt tvaika burbulīša dinamiku dažādiem šķidrumiem un procesa fizikālajiem parametriem.

Turlajs D., Grivcovs V., Yaundalders S. Computational model for determination of vapor bubble growth speed maximum in superheated liquids

The article deals with problems of investigation of boiling of liquids - the most intensive heat transfer phenomena. There are a lot of research reports of boiling dynamics and heat transfer studies in micro- and nanoscopic levels, carried out with aim to understand phenomena in whole. Simulation model for vapor bubble growth speed maximum calculations in the region of bubble critical radius is elaborated. Numerical calculations were carried out for water in the region of system parameters with the most quantity experimental data available. Maximum of bubble growth speed obtained in region of 2...5 values of bubble critical radius. Dimensionless form of equations allows to perform analysis of vapor bubble dynamic for different liquids and process physical parameters.

Турлайс Д., Гривцов В., Яундалдерс С. Математическая модель определения максимума скорости роста паровых пузырьков в перегретых жидкостях

Работа посвящена актуальным проблемам изучения процессов кипения жидкостей - наиболее интенсивного способа отвода теплоты от твердой поверхности. Тенденции указывают на возрастающую роль исследований кипения в области микро- и наноскопических масштабах, что позволяет понимать процесс в целом. В работе представлена математическая модель для расчета максимума скорости паровых пузырьков в области критического радиуса. Численные расчёты проведены для воды в области физических параметров системы с наибольшим количеством доступных данных. Максимальная скорость роста парового пузырька получена в пределах 2...5 значений критического радиуса. Безразмерная форма уравнения позволяет анализировать процессы для различных жидкостей и физических параметров систем