ADAPTIVE CONTROL OF THE ELECTRICAL SOLAR SAIL BASED ON PARAMETRIC IDENTIFICATION

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Keywords: electrical solar sail, adaptive control, parametric identification, identifier

1. Introduction

The electric sail is a new space propulsion concept which uses the solar radiation momentum for producing thrust. The electric sail is somewhat similar to the more well-known solar radiation pressure sail which is often called simply the solar sail.

A full-scale electric sail consists of a number of long, thin conducting wires. The spacecraft contains a solar-powered electron gun (typical power a few hundred watts) which is used to keep the spacecraft and the wires in a high (up to 20 *kV*) positive potential. The electric field of the wires extends a few tens of meters into the surrounding solar wind plasma. Therefore the solar wind ions "see" the wires as rather thick, about 50 *m* wide obstacles. A technical concept exists for deploying (opening) the wires in a relatively simple way and guiding or "flying" the resulting spacecraft electrically. The implementation details are presently under study [1].

The dynamic pressure of the solar wind varies but is on average about 2 *nPa* at Earth distance from the Sun. This is about 5000 times weaker than the solar radiation pressure. Due to the very large effective area and very low weight per unit length of a thin metal wire, the electric sail is still efficient, however. A 20 *km* long electric sail wire weighs only a few hundred grams and fits in a small reel, but when opened in space and connected to the spacecraft's electron gun, it can produce a one square kilometre effective solar wind sail area which is capable of extracting 1-2 *mN* force from the solar wind. For example, by equipping a small, 200 *kg*, spacecraft with 100 such wires, one may produce acceleration of about 1 mm/s^2. After acting for one year, this acceleration would produce a significant final speed of 30 *km/s.* Small payloads could be moved quite fast in space using the electric sail, a Pluto flyby could occur in less than five years, for example. Alternatively, one might choose to move medium size payloads at ordinary 5-10 *km/s* speed, but with lowered propulsion costs because the mass that has to be launched from Earth is small in the electric sail. A schematic view of the deployment phase of a spinning electric sail is presented in Figure1a. Only twelve wires have been drawn for simplicity. Figure 1a shows a dummy of the electrical solar sail consisting of twelve sections, and Figure 1b shows electrical solar sail that is unrolling monostable [2]. There are the following necessary servers of the solar sail: body – resolver; solar panel; wire; antenna and electron gun. The resolver is a technical block which consists of the devices of solar sail control system. A full-scale electric sail consists of a number (50-100) of long (e.g., 20 *km*), thin (e.g., 20 μm) conducting tethers (wires). Table 1 [3] shows the mesh mass **m**, obtained acceleration at $1AU$ under nominal solar wind conditions a, approximate final velocity $\mathbf{v_{final}}$= $\sqrt{[2a(1AU)]}$, total electric power consumption **P**, and fraction of power consumed, distributed in the wire mesh rather than the voltage source. The first column is the mesh size **L**, and the second column is the wire diameter **2r**. In all the cases considered, the last column value remains small. (In the last row it is 10%). When this parameter is small, the acceleration is independent of the mesh size and the scaling of the system is trivial. Obtaining large accelerations and final velocities requires using thin wire, which is technologically more challenging than using a thicker wire.

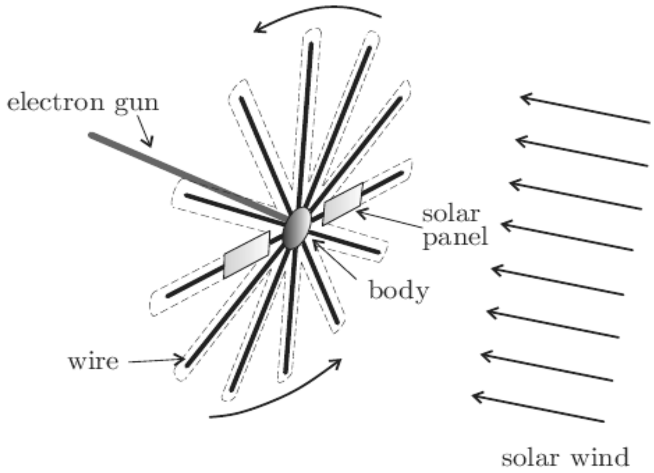


Figure 1a. Electrical solar sail scheme with 12 sections of wires

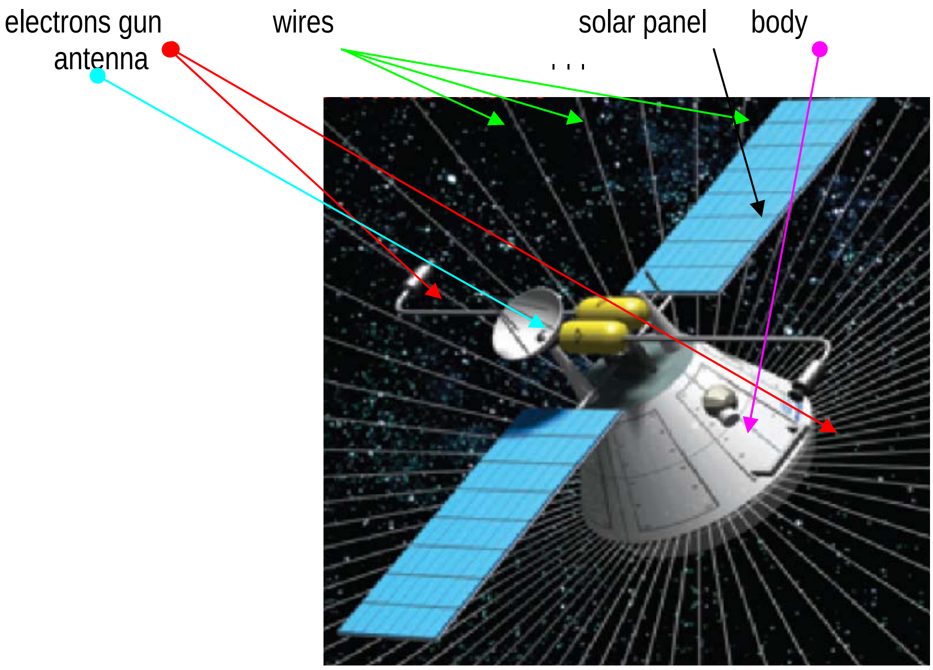


Figure 1b. Electrical solar sail is unrolling monostable

Table 1

Electric sail configurations

L, km	2r, μm	m, kg	a, m/s^2	V_{final}, km/s	P	P_{wire}/P, %
30	10	250	7*10^{-3}	46	80 W	0.06
30	5	63	0.027	91	40 W	0.11
60	2.5	63	0.11	183	80 W	1
200	2.5	700	0.11	183	1 kW	10

The problem is to find the optimal control law $\mathbf{u}(t)$ (where $t \in (t_0, t_f)$), which minimizes the time t_f necessary to transfer the spacecraft from an initial state $x_0 = (r_0, v_0)$ to a final state $x_j = (r_j, v_j)$ by maximizing the performance index $J(\mathbf{u})$. To obtain the optimal control law $\mathbf{u}(t)$ in the domain of feasible controls $\mathbf{U}$, different authors are using the Pontryagin's maximum principle, whose essence is that at a certain instant of time the Hamiltonian is an absolute maximum [4].

In this paper we consider the design of the optimal control law $\mathbf{u}(t)$ by selecting the domain of feasible controls $\mathbf{U}$, in which at a certain instant of time the functional $J(\mathbf{u})$ is minimized.

$$J(\mathbf{u}) = \frac{1}{2} < \mathbf{e}_1(T), \mathbf{F}\mathbf{e}_2(T) > + \frac{1}{2}\int_0^T [< \mathbf{e}(t), \mathbf{Q}(t)\mathbf{e}(t) > + < \mathbf{u}(t), \mathbf{R}(t)\mathbf{u}(t) >]dt, \qquad (1)$$

where $\mathbf{F}$ – *(m, m)* constant positively semi-determined matrix; $\mathbf{Q}(t)$ – *(m, m)* positively semi-determined matrix; $\mathbf{R}(t)$ – *(r, r)* positively determined matrix; T – eventual time. Two vectors dot product is indicated with $<\mathbf{e}_1, \mathbf{e}_2>$. Indeed, optimal control does exist and is explicitly defined [5] by

$$\mathbf{u}(t) = \mathbf{R}^{-1}(t)\mathbf{B}^T(t)[\mathbf{p}(t) - \mathbf{K}(t)\mathbf{x}(t)]. \qquad (2)$$

True symmetric positively determined *(n, n)* matrix $\mathbf{K}(t)$ is the solution of matrix differential Riccati equation:

$$\dot{\mathbf{K}}(t) = -\mathbf{K}(t)\mathbf{A}(t) - \mathbf{A}^T(t)\mathbf{K}(t) + \mathbf{K}(t)\mathbf{B}(t)\mathbf{R}^{-1}(t)\mathbf{B}^T(t)\mathbf{K}(t) - \mathbf{C}^T(t)\mathbf{Q}(t)\mathbf{C}(t) \qquad (3)$$

with boundary condition $\mathbf{K}(T) = \mathbf{C}^T(T)\mathbf{F}\mathbf{C}(T)$.

The second part of the problem definition for dynamic system optimal control is providing its time-optimal high speed. Thus the requirements for the chosen quality control functional (1) expand. Additional requirements, on the one hand, are to consider time variations of the dynamic system's parameters (coefficients of the matrices $\mathbf{A}(t)$ and $\mathbf{B}(t)$), and adaptation of control algorithm that forms control signals (2) vector both to the changes of the system's internal parameters and to the variable in time system's external influences, on the other hand.

It is planned to achieve realization of these additional requirements by including an identifier into control contour of the dynamic system [6, 7].

The following tasks are set for the identifier algorithm:

- solution of parametric identification problem for the controlled dynamic system, i.e. calculation of the current values of state matrix $\mathbf{A}(t)$ and control matrix $\mathbf{B}(t)$ in terms of the measured consequences of output vector $\mathbf{Y}(t)$ and control vector $\mathbf{U}(t)$ in the defined time interval;
- calculation of gain matrix $\mathbf{K}(t)$ in each renovated interval of matrices $\mathbf{A}(t)$, $\mathbf{B}(t)$;
- modification of the control algorithm in the required time interval by both modifying system parameters and changing external disturbing factors.

2. Structural diagrams of the control system of the electrical solar sail

2.1. Equations of motion of the electrical solar sail

Let the reference frame $I = (e_x, e_y, e_z)$ be an inertial right-handed Cartesian coordinate frame. The equations of solar sail motion in the I-frame are [1]:

$$\dot{\mathbf{r}} = \mathbf{v}, \qquad (4)$$

$$\dot{\mathbf{v}} = -\frac{\mu}{r^3}\mathbf{r} + a \quad , \tag{5}$$

where $\mathbf{r} = (r_x, r_y, r_z)$ is the sailcraft position and $v = (\dot{r}_x, \dot{r}_y, \dot{r}_z) = (v_x, v_y, v_z)$ the sailcraft velocity, $r = |\mathbf{r}|$, μ is the sun's gravitational parameter, and a is the acceleration acting on the wires. Within this model of the electrical solar sail, the optimal control problem must be solved using relations (1), (2) and (3)

For simplicity of analysis of the equations of electrical solar sail motion, there are considered equations of inertial right-handed motion for coordinate x, e. g.

$$\dot{r}_x = v_x , \tag{6}$$

$$\dot{v}_x = -\frac{\mu}{r^3} r_x + a . \tag{7}$$

We obtain the transfer function of the electrical sail using the Laplace transform:

$$G_p(s) = \frac{V(s)}{Z(s)} = \frac{a}{s^2 + \frac{\mu}{r^3}} . \tag{8}$$

2.2. Structural diagrams of the control system of the electrical solar sail

Figure 2 shows the functional structure of the solar sail control system which consists of n sections (wire).

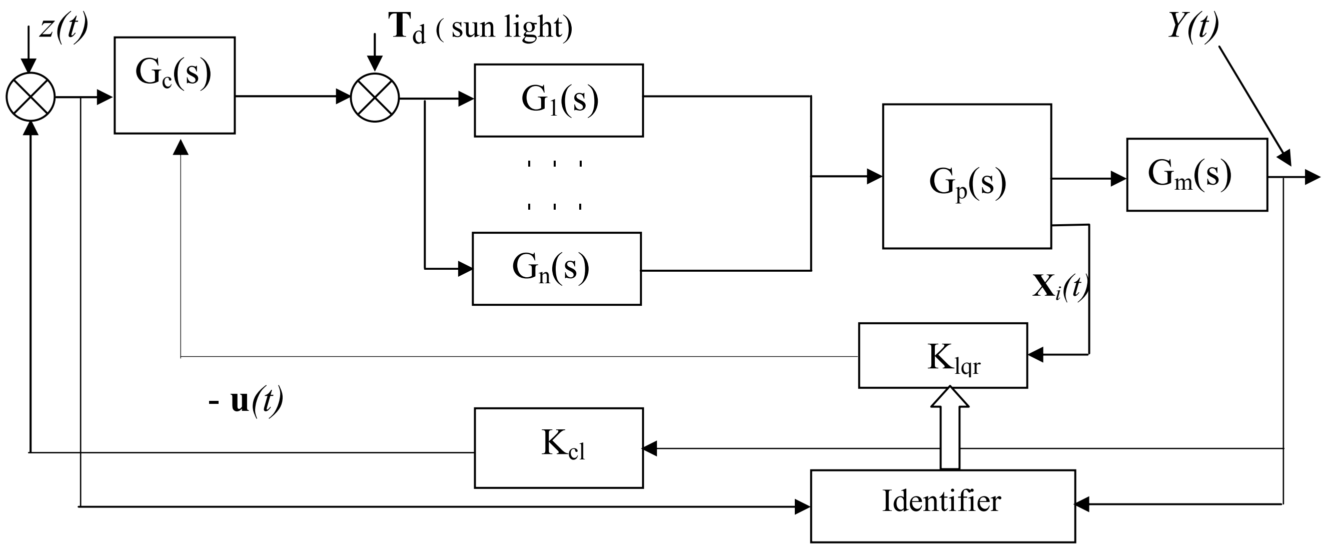


Figure 2. Structural diagram of the control system of the electrical solar sail

In this scheme:

z(t), Y(t) – input and output signal for the simulation of the model of the solar sail;

u*(t)* – control signal from the feedback control system;

$\mathbf{X}_i(t)$ – vector state space of the control system;

$\mathbf{T}_d$ – external action for the electrical solar sail (sun light);

$G_c(s) = \frac{k_0}{s}$ – transfer function of the controller of the solar sail;

$G_i(s)$ – transfer function of the solar sail of one section of wire (i=1,...,n);

$G_p(s) = \dfrac{a}{s^2 + \dfrac{\mu}{r^3}}$ – transfer function of the model of the electrical solar sail;

$G_m(s) = \dfrac{k_d}{s+d}$ – transfer function of the mechanism for control of the solar sail;

K_{lqr} – matrix coefficients of the optimal control system of the solar sail;

k_{cl} – coefficient of the feedback control system of the solar sail.

Identifier is the device which contains algorithms for computing coefficients of matrices $\widehat{\mathbf{A}}$, $\widehat{\mathbf{B}}$ of the solar sail control system.

Each section would be unroll after the solar sail proves to be orbital space. For solving that task we consider each section of the control system. Figure 3 shows a functional structure of one section of the solar sail.

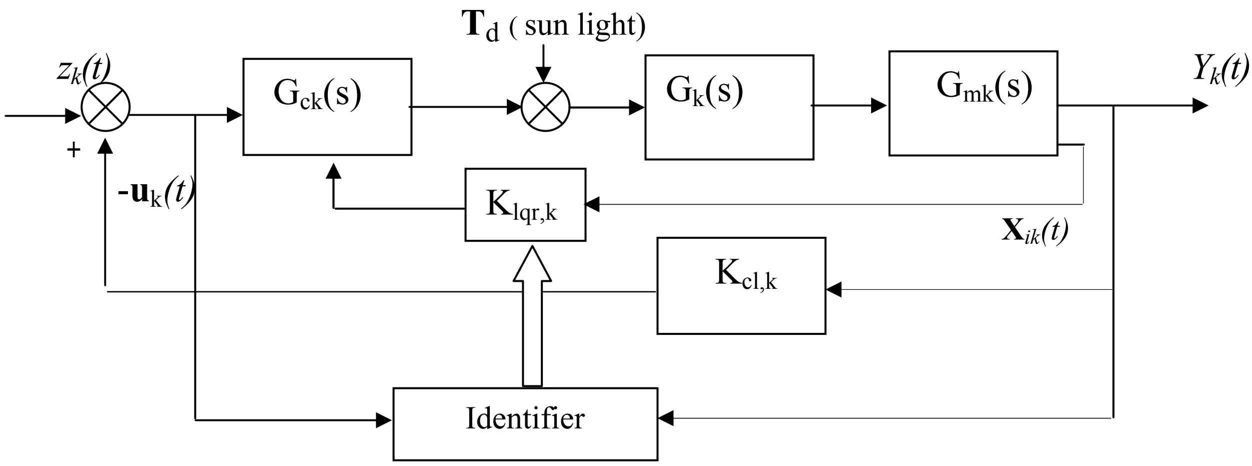


Figure 3. Structural diagram of one section (wire) of the control system of the electrical solar sail

In this scheme:

$z_k(t), Y_k(t)$ – input and output signal for the simulation of the one section model of the solar sail;
$\mathbf{u}_k(t)$ – control signal from the feedback control system of the one section of the solar sail;
$\mathbf{X}_{ik}(t)$ – vector state space one section of the solar sail of wire of the control system;
$\mathbf{T}_d$ – at the internal force (sun light) act of the every section solar sail of wire;

$G_{ck}(s) = \dfrac{k_i}{s}$ – the transfer function of the controller of the solar sail of wire;

$G_k(s) = \dfrac{k_a}{s+a}$ – the transfer function of the reel one section of the solar sail of wire;

$G_{mk}(s) = \dfrac{k_b}{s+b}$ – the transfer function of the mechanism for control of the reel one section of wire;

$K_{lqr,k}$ – matrix coefficients of the optimal control system of one section of the solar sail of wire;

$k_{cl,k}$ – coefficient of the closed-loop of the control system of one section of the solar sail of wire.

Here the identifier is the device which consists of algorithms for computing the coefficients of matrices $\widehat{\mathbf{A}}, \widehat{\mathbf{B}}$ for the control system of one section of the solar sail.

3. Design of the optimal control law for one section of wire of the solar sail

According to the structural diagrams of one section (wire) shown in Figure 3, the transfer function of the closed-loop of the control system of one section is:

$$G_{cl}(s) = \frac{G_{ol}(s)}{1 + G_{ol}(s)H(s)} = \frac{\dfrac{k_i}{s}\,\dfrac{k_a}{(s+a)}\,\dfrac{k_b}{(s+b)}}{1 + \dfrac{k_i k_a k_b}{s(s+a)(s+b)}H(s)} = \frac{k_m}{s^3 + m_{11}s^2 + m_{12}s + k_m}, \tag{9}$$

where $G_{ol}(s)$ – transfer function of the open-loop of the control system of one section;

$G_{cl}(s)$ – transfer function of the closed-loop of the control system of one section;

$H(s) = k_{cl,k}$ – coefficient of the closed-loop of the control system of one section, let $H(s) = 1$;

$k_m = k_i k_a k_b$; $m_{11} = a + b$; $m_{12} = ab$ – the coefficients of the transfer function $G_{cl}(s)$.

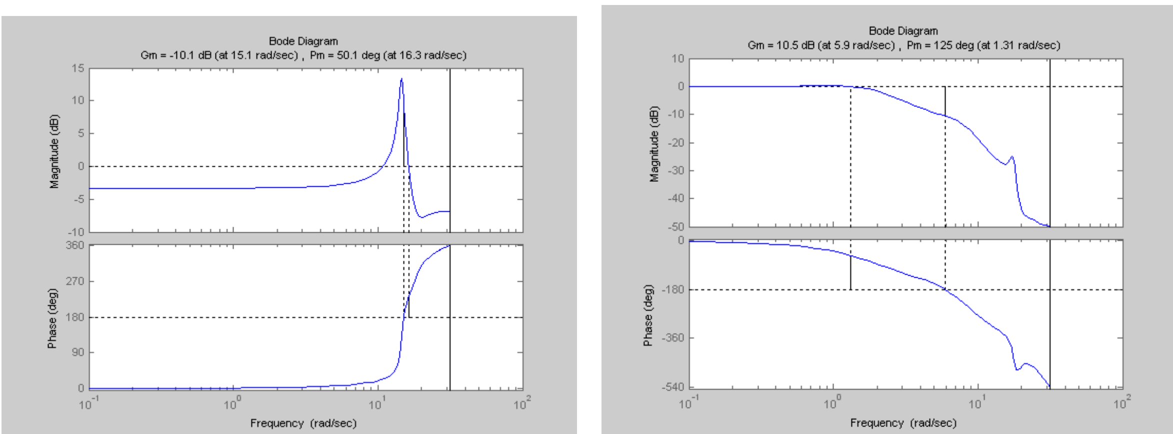
To design the optimal control law of the matrix $K_{lqr,k}$ using of MATLAB command, we find

$K_{lqr,k}$ = [8.4513; 12.9153; 7.2081; 1.1125].

Then we find the transfer functions of the controller for every coordinate of the one section wire

$G_{ck}(s)$, which are: $G_{ck1}(s) = \dfrac{8.451}{s}$; $G_{ck2}(s) = 12.92$; $G_{ck3}(s) = 7.208$; $G_{ck4}(s) = 1.112$.

The dynamical characteristics (Bode Diagram) of the one section wire of the electrical solar sail shown in Figure 4 (the left-hand part) confirm that the closed-loop system of the one section wire is unstable.



Gm=-10.1dB; Pm = 125 deg; Wcg = 5.9 rad/sec.; Wcp = 1.31rad/sec

Gm=10.5 dB; Pm = 125 deg; Wcg = 5.9 rad/sec.; Wcp = 1.31rad/sec

Figure 4. The Bode diagrams of the one section of wire

To design the optimal control law which would guarantee the stability of the control system we use the algorithms of parametric identification to compute coefficients of matrices **A** and **B.** These coefficients are used to compute matrix coefficients of the optimal control system $K_{lqr,k}$ when solving Riccati equation.

The dynamical characteristics (Bode Diagram) of the one section wire shown in Figure 4 (the right-hand part) confirm that the closed-loop system of the one section wire is stable when the algorithms of parametric identification are used in the control system. For the system parameters in the control algorithm used, we obtain:

Gm=10.5 dB; Pm = 125 deg; Wcg = 5.9 rad/sec.; Wcp = 1.31rad/sec.

Hence, the controlled system is stable.

4. Design of the optimal control law of the electrical solar sail

According to the structural diagrams shown in Figure 2, the transfer function of the open-loop of the control system of the electrical solar sail is:

$$G_{ol}(s) = G_c(s)G_{12w}(s)G_p(s)G_m(s) = \frac{k_0}{s}\frac{12k_m}{s^3 + m_{11}s^2 + m_{12}s + k_m}\frac{g}{s^2 + \frac{\mu}{r^3}}\frac{k_d}{s+d} =$$

$$= \frac{12g\,k_0k_dk_m}{s^7 + f_1s^6 + f_2s^5 + f_3s^4 + f_4s^3 + f_5s^2 + f_6s}. \tag{10}$$

For this model the coefficients of transfer functions are the following:

$$a=1,\ b=1,\ d=1,\ k_0=1,\ k_d=1,\ k_a=1,\ k_b=1,\ k_m=k_ik_ak_b,\ m_{11}=a+b,\ m_{12}=ab,\ q=\frac{\mu}{r^3},$$

$$f_1=d+m_{11},\ f_2=d\,m_{11}+m_{12}+q,\ f_3=m_{11}q+dm_{12}+d\,q+k_m,\ f_4=dk_m+dm_{11}q+m_{12}q,$$

$$f_5=d\,m_{12}\,q+q\,k_m,\ f_6=d\,q\,k_m. \tag{11}$$

The transfer function of the closed-loop of the control system $(H(s)=k_{cl}=1)$ of the electrical solar sail is:

$$G_{cl}(s) = \frac{G_{ol}(s)}{1+G_{ol}(s)H(s)} = \frac{12g\,k_0k_dk_m}{s^7 + f_1s^6 + f_2s^5 + f_3s^4 + f_4s^3 + f_5s^2 + f_6s + f_7}, \tag{12}$$

where $f_7 = 12gk_0k_dk_m$.

Using MATLAB command to design the optimal control law of the matrix K_{lqr}, we find

K_{lqr} = [44.7214; 153.2489; 160.8063; 142.3401; 80.4571; 64.8391; 17.5343; 3.0579].

Then, find the transfer functions of the controller of the electrical solar sail for every coordinate $G_c(s)$ which are:

$$G_{c1}(s) = \frac{44.72}{s},\ G_{c2}(s) = 153.2,\ G_{c3}(s) = 160.8,\ G_{c4}(s) = 142.3,\ G_{c5}(s) = 80.46,$$

$$G_{c6}(s) = 64.84,\ G_{c7}(s) = 17.53,\ G_{c8}(s) = 3.058.$$

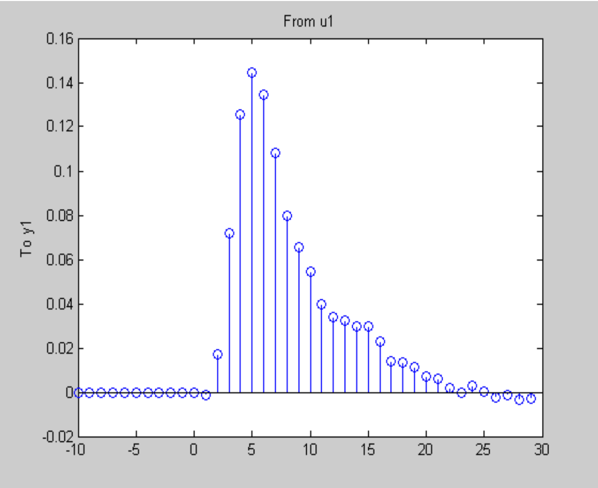
The dynamical characteristics of the electrical solar sail confirm that the closed-loop system of the one section wire is unstable.

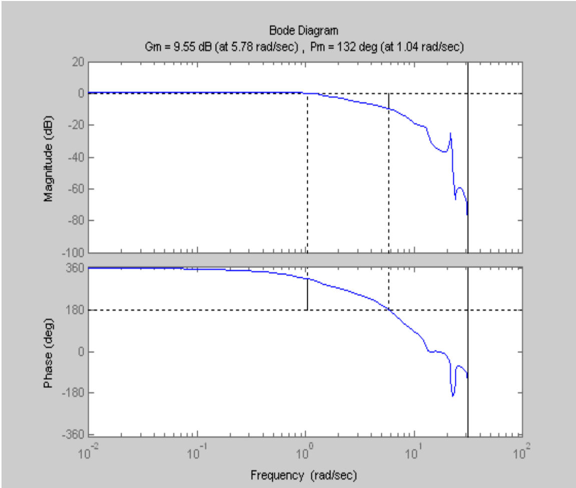
To design the optimal control law which guarantees the stability of the control system we use the algorithms of parametric identification to compute coefficients of matrices **A** and **B.** These coefficients are used to compute matrix coefficients of the optimal control system K_{lqr} when solving Riccati equation.

The dynamical characteristics (impulse response, Bode Diagram) of the electrical solar sail, shown in Figures 5 and 6 confirm that the closed-loop system of the electrical solar sail is stable when the algorithms of parametric identification are used in the control system. So, for the controlled system parameters in the control algorithm, we obtain:

Gm =9.55 dB; Pm =132 deg. Wcg = 5.78 rad/sec, Wcp=1.04rad./sec.

Therefore, the controlled system is stable.





Gm =9.55 dB Pm =132 deg. Wcg = 5.78 rad/sec, W cp=1.04rad./sec

Figure 5. The Impulse response $G_{cl}(s)$

Figure 6. The Bode diagrams of the electrical solar sail

5. Summary

Modelling results of time-optimal adaptive system with a linear-square regulator in a feedback loop and parametric identification algorithm confirm that the synthesized control system of electric solar sail is steady. The comparison of efficiency of the adaptive control system of electric solar sail with the similar systems using Pontryagin's maximum principle confirms its advantage.

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Matvejevs Aleksandrs, Matvejevs Andrejs, Vulfs Ģirts. **Uz parametriskās identifikācijas bāzētā elektriskā saules spārna adaptīva vadība**
Darbā apspriesti dinamisku objektu vadības un identifikācijas optimālu algoritmu pētījuma rezultāti, kas funkcionē ekstremālos apstākļos pie ierobežotas aprioras informācijas. Par pētījuma objekta modeļi tiek izvēlēts elektriskās saules buras modelis, kas funkcionē piezemes orbītās. Elektriskā saules bura sastāv no tievu elektrisko vadu tīkla, ar katra vada diametru ap 10 mikroniem un kopīgu vadu garumu vairāk par 30 000 metriem, un elektroniskās aparatūras kompleksa, kas nodrošina šīs sistēmas ilgstošo funkcionēšanu kosmiskajā telpā. Rakstā tiek aplūkotas objekta struktūras īpatnības un tā vadības sistēmas funkcionālā shēma. Ir analizēti informācijas mērīšanas sistēmas algoritmi, kas nodrošina funkcionējošās sistēmas stāvokļa objektīvu novērtējumu. Tiek piedāvāti optimālo algoritmu dažādi varianti, kas nodrošina vadības sistēmas stabilitāti pie ārēju faktoru iedarbības.

Matvejevs Aleksandrs, Matvejevs Andrejs, Vulfs Ģirts. Adaptive control of the electrical solar sail based on parametric identification
Research results of optimal control and identification algorithms of the dynamic systems in extreme conditions and limited prior information are discussed. The model of electrical solar sail is chosen as an object for research. The electrical solar sail consists of a net of the thinnest electric send-offs with a diameter of 10 microns, whose general length is more than 30 000 meters and complex of electronic equipment, providing the permanent functioning of this system in space. Features of structure and function scheme of the object control system are considered. The algorithms of information and measuring system providing the objective estimation of the system and its stability are analyzed. Modelling results of some variants of optimal algorithms of the control system providing its steady functioning under influence of external factors are examined.

Матвеев Александр, Матвеев Андрей, Вульф Гирт. Адаптивное управление электрического солнечного паруса на основе параметрической идентификации
В работе обсуждаются результаты исследования оптимальных алгоритмов управления и идентификации динамических объектов, функционирующих в экстремальных условиях при ограниченной априорной информации. В качестве объекта исследования выбрана модель «электрический солнечный парус» - выводимое на околоземную орбиту устройство, состоящее из сетки тончайших электрических проводов общей длины более 30 000 метров и диаметром в 10 микрон, и комплекса электронной аппаратуры, обеспечивающей длительное функционирование этой системы в космическом пространстве. Рассматриваются особенности построения структурной и функциональной схемы системы управления таким объектом. Анализируются алгоритмы информационно-измерительной системы, обеспечивающие объективную оценку состояния функционирующей системы. Моделируются варианты построения системы управления, обеспечивающие ее устойчивое функционирование под воздействием внешних факторов. Анализируются результаты моделирования некоторых оптимальных алгоритмов системы управления контролируемого объекта при ограниченной априорной информации о его состоянии.