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**TECHNOLOGIES OF COMPUTER**  
**CONTROL**

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**DATORVADĪBAS TEHNOLOĢIJAS**

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**THE APPLICATION OF DATA ENVELOPMENT ANALYSIS APPROACH TO THE  
EFFICIENCY MEASUREMENT OF LATVIAN BANKS'****DATU ČAULAS ANALĪZES METODES PIELIETOJUMS LATVIJAS BANKU  
EFEKTIVITĀTES NOVĒRTĒŠANĀ**

**Arshinova Tatyana** Tatyana Arshinova was born in Riga in 1986. She was educated at the Riga Technical University in Latvia where she earned her Bachelor of Social Science degree in Management in 2006 and her Master of Social Science degree in Management in 2008. Tatyana Arshinova is currently completing her research for a PhD at the Riga Technical University in the field of Mathematical Statistics and its Applications. In 2005 Tatyana Arshinova started working at JSC "Latvijas Krājbanka" as an account manager. Since 2007 she is working as an analyst in the Investment Department at JSC "Latvijas Krājbanka".

*Keywords: DEA (Data Envelopment Analysis); banking efficiency measurement; non-parametric methods; mathematical programming; Decision Making Units (DMUs); returns to scale (RTS); Window analysis*

## **Introduction**

The changes in functioning of Latvian banking system since 1990 have stimulated the development of banking financial management as an original scientific trend. Nevertheless, there still is a lack of united methodology for the solution of some economically significant problems' in this field, among them is the performance evaluation of financial institutions'. Actually the estimation of the level of operating efficiency in the most Latvian banks is realized on the basis of quantitative approach of ratio analysis. Ratio analysis has historically been the standard technique used by industry analysts and management to examine banking performance. Ratios measure the relationship between two variables chosen to provide insights into different aspects of the banks' multifaceted operations, such as liquidity, profitability, capital adequacy, asset quality, risk management, and many others. Any number of ratios can be designed depending on the objective of the analysis, generally for comparisons within the same bank over different time periods, as well as for benchmarking with reference to other banks. Although the traditional ratio measures are attractive to analysts due to their simplicity, there are several limitations that must be considered. For example, the analysis assumes comparable units, which implies constant returns to scale (Smith 1990). Each of the indicators yields a one-dimensional measure by examining only a part of the organization's activities, or combining the multiple dimensions into a single, unsatisfactory number. Moreover, the seemingly unlimited number of ratios that can be created from financial statement data are often contradictory, thus ineffective for the assessment of overall performance. This overly

simplistic analytical approach offers no objective means of identifying inefficient units and requires a biased separation of the inefficient and efficient levels. One of the most significant demerits of ratio analysis is failure to account for multidimensional input and output processes that makes this approach inadequate for reliable efficiency evaluations. [1, pp.350-351]

Methods of frontier analysis ensure a principally different approach to the problem of performance measurement. They provide an opportunity of complex analysis of banking efficiency level for a certain period of time and comparison of it among investigated banks. This multidimensional approach meets the requirements to the banking performance evaluation methodology. The objective of author's research is to improve and supplement the methodology of efficiency measurement of Latvian banks' on the basis of non-parametric approach of Data Envelopment Analysis (DEA). Processing short-term liabilities into long-term assets, banks are ensuring redistribution of income of economical subjects' inside of the country. Thus, the stability and growth of economy of the country is depending on the development and efficiency of the banking sector. In the circumstances of unstable macroeconomical environment and competition, customers' deposits remain one of the main sources of financing of credit institutions'. Fluctuations of interbank offered rates' in 2009 and distrust in Latvian money market confirmed a vital role of customers' deposits' for ensuring the stability of banking sector. In this connection, the author analyzed the efficiency of a set of Latvian banks', assuming deposits as an output. The objects of the research are members of Latvian banking sector, their efficiency level is

analyzed over the time period from 2003 to 2008. Evaluating the performance on the basis of DEA approach, the author included into the set of investigated objects' the credit institutions that take leading positions on the Latvian deposit market: JSC "Swedbank", JSC "DnB Nord Banka", JSC "Aizkraukles Banka", JSC "Parex banka", JSC "SEB Banka", JSC "Latvijas Krājbanka", JSC "Mortgage Bank", JSC "Rietumu Banka", JSC "Norvik Banka", JSC "GE Money Bank" with the exception of branches' of foreign banks' (Allied Irish Bank p.l.c. Latvia Branch, Nordea Finland Latvia Branch, Danske Bank Latvia Branch).

### Methods of frontier data analysis

The progress of production technology and increase of production volumes have stimulated the development of performance measurement methodology. In the second part of the 20th century there were introduced methods of frontier data analysis that provided a qualitatively different approach to the problem. According to the methodology of methods' of frontier data analysis, the efficiency score of investigated DMU's is calculated as a distance from the point that defines the production process of a decision making unit (DMU) to the certain efficiency frontier. Entities that are functioning on the efficiency frontier are considered to be absolutely technically efficient; inefficiency of other DMU's is increasing together with extension of the distance to the efficiency frontier. The value of efficiency score is fluctuating from zero to one.

Methods of frontier analysis may be divided into two groups: parametric (Stochastic Frontier Approach (SFA), Distribution-Free Approach (DFA), Thick Frontier Approach (TFA)) and non-parametric (Data Envelopment Analysis (DEA), Free Disposal Hull (FDH)) methods.

In accordance with parametric approaches, the efficiency frontier is constructed on the basis of econometric modelling, usually in form of Cobb-Douglas (log-linear) production function. Econometric analyses include two error components: an error term that captures inefficiency ( $u_i$ ) and a random error ( $v_i$ ). Parametric methods have significant advantages – they provide the possibilities to use panel data, to distinguish the random noise from inefficiency and to calculate the standard error of efficiency

measurement results'. Nevertheless, the stochastic approaches of performance measurement presume the comparison of investigated DMUs' efficiency to the theoretically developed benchmark frontier, therefore the optimal combinations of inputs' and outputs' sometimes are not achievable practically. The application of parametric methods' also requires observance of the restrictions imposed on the distributional assumptions on the inefficiencies and random error.

In contrast to the econometric approaches, non-parametric methods are based on the hypothesis that the efficiency frontier is generated from the empirical results' of the most efficient DMU's i.e. benchmarks' that „float” on the piecewise linear frontier. The level of technical efficiency of these DMU's is 100%. However, the level of scale efficiency that defines the optimality of output and input proportions' may have different values even among absolutely technically efficient DMU's. While mathematical, non-parametric methods require few assumptions when specifying the best-practice frontier, they generally do not account for random errors.

### The CCR DEA Model

The CCR DEA model was developed by Charnes, Cooper and Rhodes in 1978 to evaluate the performance of Decision Making Units (DMUs). To allow for applications to a wide variety of activities, the term DMU might be used to refer to any entity that is to be evaluated in terms of its abilities to convert inputs into outputs. These evaluations can involve governmental agencies and non-profit organizations as well as business firms. The evaluation can also be directed to educational institutions and hospitals as well as branches for which comparative evaluations of their performance are to be made.

The production process may be aimed either at minimization of resources' or maximization of production volumes'. The orientation of the model should be aimed at controllable variables. In context of banking, volumes of resources' are usually over control of management, therefore only input-oriented models will be examined in the paper.

The measurement of comparative efficiency is based on the assumption that the performance of each DMU is calculated in comparison to  $n$  investigated DMUs. Each DMU consumes varying amounts of  $m$  different inputs to produce  $s$  different outputs. Specifically,  $DMU_j$  consumes amount  $x_{ij}$  of input  $i$  and produces amount  $y_{rj}$  of output  $r$ . It is necessary to assume that  $x_{ij} \geq 0$  and  $y_{rj} \geq 0$  and further to assume that each DMU has at least one positive input and one positive output value. Primarily the DEA model was expressed in fractional, i.e. ratio-form. In this form the ratio of outputs to inputs is used to measure the relative efficiency of the  $DMU_j = DMU_0$  to be evaluated relative to the ratios of all of the  $j = 1, 2, \dots, n$  DMUs. The CCR construction can be interpreted as the reduction of the multiple-output/multiple-input situation (for each DMU) to that of a single 'virtual' output and 'virtual' input. For a particular DMU the ratio of this single virtual output to single virtual input provides a measure of efficiency that is a function of the multipliers. In mathematical programming parlance, this ratio, which is to be maximized, forms the objective function for the particular DMU being evaluated. A set of normalizing constraints (one for each DMU) reflects the condition that the virtual output to virtual input ratio of every DMU, including  $DMU_j = DMU_0$ , must be less than or equal to unity. The mathematical programming problem may thus be stated as (see Formula 1):

$$\begin{aligned} \max h_0(u, v) &= \sum_r u_r y_{r0} / \sum_i v_i x_{i0} \\ \text{subject to} \\ \sum_r u_r y_{rj} / \sum_i v_i x_{ij} &\leq 1 \text{ for } j = 1, \dots, n, \\ u_r, v_i &\geq 0 \text{ for all } i \text{ and } r, \end{aligned} \quad (1)$$

where

$h_0$  – the function of virtual output and virtual input ratio of  $DMU_0$ ;  
 $u_r$  – the output multiplier of  $DMU_0$ ;  
 $v_i$  – the input multiplier of  $DMU_0$ ;  
 $y_{r0}$  – the output of  $DMU_0$ ;  
 $x_{i0}$  – the input of  $DMU_0$ ;  
 $y_{rj}$  – outputs of  $1, 2, \dots, n$  DMUs;  
 $x_{ij}$  – inputs of  $1, 2, \dots, n$  DMUs.

The above ratio form yields an infinite number of solutions; if  $(u^*, v^*)$  is optimal, then  $(\alpha u^*, \alpha v^*)$  is also optimal for  $\alpha > 0$ . However, the transformation developed by Charnes and

Cooper (1962) for linear fractional programming selects a representative solution [i.e., the solution  $(u, v)$   $\sum_{i=1}^m v_i x_{i0} = 1$  for which ] and yields the equivalent linear programming problem in which the change of variables from  $(u, v)$  to  $(\mu, v)$  is a result of the Charnes-Cooper transformation (see Formula 2):

$$\begin{aligned} \max z &= \sum_{r=1}^s \mu_r y_{r0} \\ \text{subject to} \\ \sum_{r=1}^s \mu_r y_{rj} - \sum_{i=1}^m v_i x_{ij} &\leq 0 \\ \sum_{i=1}^m v_i x_{i0} &= 1 \\ \mu_r, v_i &\geq 0, \end{aligned} \quad (2)$$

where

$z$  – the CCR input-oriented function of  $DMU_0$  (multiplier form);  
 $\mu_r$  – the output multiplier of  $DMU_0$ ;  
 $v_i$  – the input multiplier of  $DMU_0$ ;  
 $y_{r0}$  – the output of  $DMU_0$ ;  
 $x_{i0}$  – the input of  $DMU_0$ ;  
 $y_{rj}$  – outputs of  $1, 2, \dots, n$  DMUs;  
 $x_{ij}$  – inputs of  $1, 2, \dots, n$  DMUs;  
 $\varepsilon$  – non-Archimedean element smaller than any positive real number.

Model that is expressed by Formula 2 can be solved by its dual problem (see Formula 3):

$$\begin{aligned} \theta^* &= \min \theta \\ \text{subject to} \\ \sum_{j=1}^n x_{ij} \lambda_j &\leq \theta x_{i0} \quad i = 1, 2, \dots, m; \\ \sum_{j=1}^n y_{rj} \lambda_j &\geq y_{r0} \quad r = 1, 2, \dots, s; \\ \lambda_j &\geq 0 \quad j = 1, 2, \dots, n, \end{aligned} \quad (3)$$

where

$\theta^*$  – the optimal value of dual variable  $\theta$  of  $DMU_0$ ;  
 $\theta, \lambda_j$  – dual variables of  $DMU_0$ ;  
 $y_{r0}$  – the output of  $DMU_0$ ;  
 $x_{i0}$  – the input of  $DMU_0$ ;  
 $y_{rj}$  – outputs of  $1, 2, \dots, n$  DMUs;  
 $x_{ij}$  – inputs of  $1, 2, \dots, n$  DMUs.

This last model is sometimes referred to as the "Farrell model" because it is the one used in

Farrell (1957). In the economics portion of the DEA literature it is said to conform to the assumption of "strong disposal" because it ignores the presence of non-zero slacks. In the operations research portion of the DEA literature this is referred to as "weak efficiency."

By virtue of the dual theorem of linear programming we have  $z^* = \theta$ . Hence either problem may be used. One can solve the dual linear program, to obtain an efficiency score. Setting  $\theta = 1$  and  $\lambda_k^* = 1$  with  $\lambda_k = \lambda_o^*$  and all other  $\lambda_k^* = 0$ , a solution of dual problem (see Formula 3) always exists. Moreover this solution implies  $\theta^* \leq 1$ . The optimal solution,  $\theta^*$ , yields an efficiency score for a particular DMU. The process is repeated for each DMU. i.e., solving the model, expressed by Formula 3, with  $(X_o, Y_o) = (X_k, Y_k)$ , where  $(X_k, Y_k)$  represent vectors with components  $x_{ik}, y_{rk}$  and, similarly  $(X_o, Y_o)$  has components  $x_{ok}, y_{ok}$ . DMUs for which  $\theta^* < 1$  are inefficient, while DMUs for which  $\theta^* = 1$  are boundary points. Some boundary points may be "weakly efficient" because we have non-zero slacks. This may appear because alternate optima may have non-zero slacks in some solutions, but not in others. However, we can avoid this effect by invoking the following linear program in which the slacks are taken to their maximal values (see Formula 4).

$$\begin{aligned} & \max \sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+ \\ & \text{subject to} \\ & \sum_{j=1}^n x_{ij} \lambda_j + s_i^- = \theta^* x_{io} \quad i = 1, 2, \dots, m; \\ & \sum_{j=1}^n y_{rj} \lambda_j - s_r^+ = y_{ro} \quad r = 1, 2, \dots, s; \\ & \lambda_j, s_i^-, s_r^+ \geq 0 \quad \forall i, j, r, \end{aligned} \quad (4)$$

where

$s_i^-$  – input slacks;  
 $s_r^+$  – output slacks;  
 $\theta^*$  – the optimal value of dual variable  $\theta$  of DMU<sub>0</sub>;  
 $\lambda_j$  – the dual variable of DMU<sub>0</sub>;  
 $y_{r0}$  – the output of DMU<sub>0</sub>;  
 $x_{i0}$  – the input of DMU<sub>0</sub>;  
 $y_{rj}$  – outputs of 1,2...n DMUs;  
 $x_{ij}$  – inputs of 1,2...n DMUs.

It shall be noted that the choices of  $s_i^-$  and  $s_r^+$  do not affect the optimal  $\theta^*$  which is determined from model expressed by Formula 3. These

developments lead to the following definitions of DEA efficiency:

**DEA Efficiency:** The performance of DMU<sub>0</sub> is fully (100%) efficient if and only if both (i)  $\theta^* = 1$  and (ii) all slacks  $s_i^- = s_r^+ = 0$ .

**Weakly DEA Efficiency:** The performance of DMU<sub>0</sub> is weakly efficient if and only if both (i)  $\theta^* = 1$  and (ii)  $s_i^- \neq 0$  and/or  $s_r^+ \neq 0$  for some  $i$  and  $r$  in some alternate optima.

It is to be noted that the preceding development amounts to solving the following problem in two steps (see Formula 5):

$$\begin{aligned} & \min \theta - \varepsilon \left( \sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+ \right) \\ & \text{subject to} \\ & \sum_{j=1}^n x_{ij} \lambda_j + s_i^- = \theta^* x_{io} \quad i = 1, 2, \dots, m; \\ & \sum_{j=1}^n y_{rj} \lambda_j - s_r^+ = y_{ro} \quad r = 1, 2, \dots, s; \\ & \lambda_j, s_i^-, s_r^+ \geq 0 \quad \forall i, j, r, \end{aligned} \quad (5)$$

where

$\theta, \lambda_j$  – dual variables of DMU<sub>0</sub> (CCR input-oriented envelopment model);  
 $\varepsilon$  – non-Archimedean element smaller than any positive real number;  
 $s_i^-$  – input slacks;  
 $s_r^+$  – output slacks;  
 $y_{r0}$  – the output of DMU<sub>0</sub>;  
 $x_{i0}$  – the input of DMU<sub>0</sub>;  
 $y_{rj}$  – outputs of 1,2...n DMUs;  
 $x_{ij}$  – inputs of 1,2...n DMUs.

The  $s_i^-$  and  $s_r^+$  are slack variables used to convert the inequalities in Formula 3 to equivalent equations. Here  $\varepsilon > 0$  is a so-called non-Archimedean element defined to be smaller than any positive real number. This is equivalent to solving the dual problem in two stages by first minimizing  $\theta$ , then fixing  $\theta = \theta^*$  as in Formula 1, where the slacks are to be maximized without altering the previously determined value of  $\theta = \theta^*$ . Formally, this is equivalent to granting "preemptive priority" to the determination of  $\theta^*$  in Formula 2. In this manner, the fact that the non-Archimedean element  $\varepsilon$  is defined to be smaller than any positive real number is accommodated without having to specify the value of  $\varepsilon$ . [2, pp.8-12]

The CCR efficiency score is indicative of the overall efficiency level of investigated DMUs.

## The BCC DEA Model

The BCC DEA model in its present form was introduced by Banker, Charnes and Cooper in 1984. The BCC efficiency measurement algorithm is concerned with the concept of returns to scale (RTS) that has been widely studied within different scientific contexts. In the literature of classical economics, returns to scale have typically been defined only for single output situations. RTS are considered to be increasing if a proportional increase in all the inputs results in a more than proportional increase in the single output. Let  $\alpha$  represent the proportional input increase and  $\beta$  represent the resulting proportional increase of the single output. Increasing returns to scale prevail if  $\beta > \alpha$  and decreasing returns to scale prevail if  $\beta < \alpha$ . Banker (1984), Banker, Charnes and Cooper (1984) and Banker and Thrall (1992) extended the RTS concept from the single output case to multiple output cases using DEA.

The input-oriented BCC model evaluates the efficiency of  $DMU_j$  by solving the following (envelopment form) linear program (see Formula 6):

$$\begin{aligned} \min \theta_o - \varepsilon \left( \sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+ \right) \\ \text{subject to} \\ \sum_{j=1}^n x_{ij} \lambda_j + s_i^- = \theta x_{io} \quad i = 1, 2, \dots, m; \\ \sum_{j=1}^n y_{rj} \lambda_j - s_r^+ = y_{ro} \quad r = 1, 2, \dots, s; \\ \sum_{j=1}^n \lambda_j = 1 \quad j = 1, 2, \dots, n; \\ \lambda_j, s_i^-, s_r^+ \geq 0 \quad \forall i, j, r, \end{aligned} \quad (6)$$

where

$\theta_o, \lambda_j$  – dual variables of  $DMU_o$  (BCC input-oriented envelopment model);

$\varepsilon$  – non-Archimedean element smaller than any positive real number;

$s_i^-$  – input slacks;

$s_r^+$  – output slacks;

$y_{ro}$  – the output of  $DMU_o$ ;

$x_{io}$  – the input of  $DMU_o$ ;

$y_{rj}$  – outputs of 1, 2...n DMUs.

The added constraint  $\sum_{j=1}^n \lambda_j = 1$  introduces an additional variable,  $\mu_o$ , into the (dual) multiplier problem. This extra variable makes it possible to

effect returns to scale evaluations (increasing, constant and decreasing). So the BCC model is also referred to as the VRS (Variable Returns to scale) model and distinguished from the CCR model which is referred to as the CRS (Constant Returns to Scale) model.

The dual multiplier form of the BCC model is represented in Formula 7:

$$\begin{aligned} \max z_o &= \sum_{r=1}^s \mu_r y_{ro} - \mu_o \\ \text{subject to} \\ \sum_{r=1}^s \mu_r y_{rj} - \sum_{i=1}^m v_i x_{ij} - \mu_o &\leq 0 \quad j = 1, 2, \dots, n; \\ \sum_{i=1}^m v_i x_{io} &= 1; \\ \mu_r, v_i &\geq \varepsilon > 0, \mu_o \quad \forall \end{aligned} \quad (7)$$

where

$z_o$  – the BCC input-oriented function of  $DMU_o$  (multiplier form);

$\mu_o$  – the returns to scale region defining variable of  $DMU_o$ ;

$\mu_r$  – the output multiplier of  $DMU_o$ ;

$v_i$  – the input multiplier of  $DMU_o$ ;

$y_{ro}$  – the output of  $DMU_o$ ;

$x_{io}$  – the input of  $DMU_o$ ;

$y_{rj}$  – outputs of 1, 2...n DMUs;

$x_{ij}$  – inputs of 1, 2...n DMUs;

$\varepsilon$  – non-Archimedean element smaller than any positive real number.

The above formulations assume that that  $x_{ij}, y_{rj} \geq 0 \quad \forall i, r, j$ . All variables in Formula 7 are also constrained to be non-negative - except for  $\mu_o$ , which may be positive, negative or zero with consequences that make it possible to use optimal values of this variable to identify RTS. When a  $DMU_o$  is efficient in accordance with the definition of DEA efficiency, the optimal value of  $\mu_o$ , i.e.,  $\mu_o^*$ , in Formula 7, can be used to characterize the situation for Returns to Scale (RTS). RTS generally has an unambiguous meaning only if  $DMU_o$  is on the efficiency frontier – since it is only in this state that a tradeoff between inputs and outputs is required to improve one or the other of these elements. However, there is no need to be concerned about the efficiency status in the analyses because efficiency can always be achieved as follows. If a  $DMU_o$  is not BCC efficient, there is a possibility

to use optimal values from Formula 6 to project this DMU onto the BCC efficiency frontier via Formulas 8, 9:

$$\hat{x}_{io} = \theta_o^* x_{io} - s_i^{-*} = \sum_{j=1}^n x_{ij} \lambda_j^*, \quad i = 1, \dots, m; \quad (8)$$

$$\hat{y}_{ro} = y_{ro} + s_r^{+*} = \sum_{j=1}^n y_{rj} \lambda_j^*, \quad r = 1, \dots, s, \quad (9)$$

where

$\hat{x}_{io}$  – virtual inputs of DMU<sub>0</sub>;  
 $\hat{y}_{ro}$  – virtual outputs of DMU<sub>0</sub>;  
 $\theta_o^*, \lambda_j^*$  – optimal dual variables of DMU<sub>0</sub> (BCC input-oriented envelopment model);  
 $x_{i0}$  – the input of DMU<sub>0</sub>;  
 $y_{r0}$  – the output of DMU<sub>0</sub>;  
 $s_i^{-*}$  – the optimal input slacks;  
 $s_r^{+*}$  – the optimal output slacks;  
 $y_{rj}$  – outputs of 1, 2...n DMUs;  
 $x_{ij}$  – inputs of 1, 2...n DMUs.

These are sometimes referred to as the "CCR Projection Formulas" because Charnes, Cooper and Rhodes (1978) showed that the resulting

$\hat{x}_{io} \leq x_{io}$  and  $\hat{y}_{ro} \leq y_{ro}$  correspond to the coordinates of a point on the efficiency frontier. They are of the point used to evaluate **DMU<sub>0</sub>** when Formula 6 is employed. The returns to scale regions might be determined on the basis of the theorem developed by Banker and Thrall (1992) who identify RTS with the sign of  $\mu_o^*$  in Formula 7 as follows:

(i) Increasing RTS prevail at  $(\hat{x}_{io}, \hat{y}_{ro})$  if and only if  $\mu_o^* < 0$  for all optimal solutions.

(ii) Decreasing RTS prevail at  $(\hat{x}_{io}, \hat{y}_{ro})$  if and only if  $\mu_o^* > 0$  for all optimal solutions.

(iii) Constant RTS prevail at  $(\hat{x}_{io}, \hat{y}_{ro})$  if and only if  $\mu_o^* = 0$  for at least one optimal solution. [3, pp.42-46]

## The DEA Window Analysis approach

The methodology mentioned above provided the possibility of cross-sectional analysis, i.e. each DMU was observed once during a certain period of time. Nevertheless, it is significant to analyze dynamics and fluctuations of efficiency level over time in the processes of decision-making and strategical planning. It is achievable with the

help of Malmquist index or DEA Window Analysis approach. In the scientific paper the author uses Window analysis method, because it is also helpful for the situation when there is an insufficient number of DMUs in comparison to the number of relevant inputs and outputs in the model. This technique works on the principle of moving averages (Charnes et al. 1994B, Yue 1992), and is useful to detect performance trends of a unit over time. Each unit in a different year is treated as if it were a "different" unit. In doing so, the performance of a unit in a particular year is compared with its performance in other periods in addition to the performance of the other units. This means that the units of the same DMU in different years are treated as if they were independent of each other - but comparable. A notable feature of this technique is that there are **nk** units (DMUs) in each window where **n** is the number of units in a given time period (and it is the same in all time periods), and **k** is the width of each window (equal for all windows). This feature is extremely important in the case of a small number of DMUs and a large number of inputs and outputs since it increases the discriminatory power of the results. This is accomplished by dividing the total number of time periods, **T**, into a series of overlapping periods or windows, each of width **k** (**k** < **T**) and thus having **nk** units. Hence the first window has **nk** DMUs for the time periods **{1,...,k}**, the second one has **nk** DMUs and the time periods **{2, ...,k+1}**, and so on and the last window consists of **nk** DMUs and the time periods **{T-k+1, ...,T}**. In all, there are **T-k+1** separate analyses where each analysis examines **nk** DMUs. An important factor is the determination of the window size. If the window is too narrow, there may not be enough DMUs in the analysis and thus not enough discrimination in the results (which also depends on the number of DMUs and variables in the model). On the other hand, too wide a window may give misleading results because of significant changes that occur over periods covered by each window. The best window size is usually determined by experimentation. [1, pp.372-373]

The author will introduce the application of DEA Window Analysis approach to the efficiency measurement of Latvian banks' in the chapter "The efficiency measurement results of Latvian banks' on the basis of Window Analysis approach".

## The application of DEA approach to the efficiency measurement of Latvian banks'

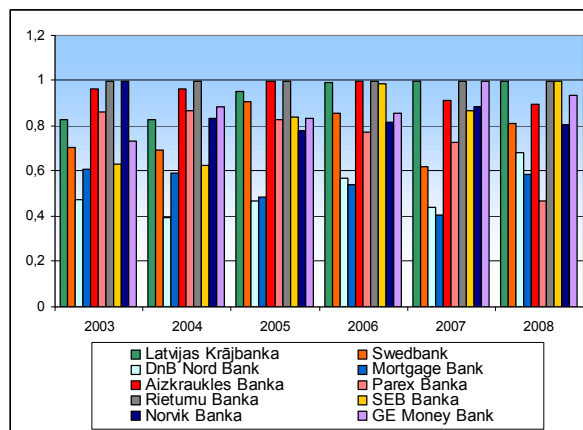
### Methodology of the research

Analyzing the efficiency level of Latvian banks' it is necessary to take into account that the number of members' of banking sector is relatively small. For the elaboration of a reliable efficiency measurement model the number of DMU's should exceed the total number of variables' at least 2-3 times. In order to apply the methodology of frontier data analysis, such as DEA, to the performance evaluation of Latvian banks', it is necessary either to increase the number of investigated banks', realizing cross-country banking analysis, or to reduce the total number of variables', investigating every process of output production separately. Due to differences in banking management, demographical factors' and customer structure in different countries, the implementation of cross-country banking performance analysis is bound up with many restrictions. In this connection, there is developed a concept of efficiency measurement of the deposit production process of Latvian banks' in the research: deposits are assumed to be an output while total assets, interest expense and personnel costs are defined as inputs.

The application of DEA approach requires the determination of assumptions, concerning orientation measures of the model and the concept of returns to scale (RTS). The banking production process may be aimed either at minimization of resources' (input-oriented) or maximization of production volumes' (output-oriented). It is emphasized in the international researches that the orientation of the model should be aimed at controllable variables. Usually volumes of resources' are considered to be over control of banking management, therefore there is applied the assumption of input orientation in the research. The efficiency of investigated banks' will be analyzed, using both concepts of constant (CRS) and variable returns to scale (VRS). Since the CRS approach represents the total (overall) efficiency level, it is considered to be the basic concept while the VRS approach will be applied to the analysis of the sources of inefficiency, providing the possibility to distinguish between technical and scale inefficiency.

## Efficiency measurement results of Latvian banks' on the basis of CCR and BCC models'

The results of banking efficiency measurement on the basis of CCR input-oriented model (see Formula 5), assuming deposits as an output, are represented in Figure 1.



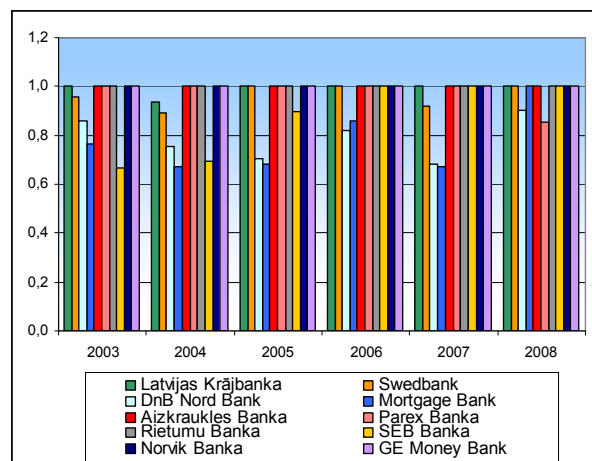
**Fig. 1.** Dynamics of banking deposit efficiency CCR score on the basis of DEA CRS approach (cross-sectional data)

According to the results' of the research, the average overall deposit efficiency level is fluctuating from 76,9% in 2004 to 83,8% in 2006, although there is observable the redistribution of leaders' positions' among investigated banks'. JSC "Rietumu Banka" has demonstrated the best result, operating on the efficiency frontier during all periods of the observation. The long-term stability of the efficiency level of "Rietumu Banka" is indicative of its ability to maximize the volume of output using minimal volumes' of inputs' and to ensure optimal proportions' of output and inputs' in the process of production, thus of both 100% technical and scale efficiency in comparison to the set of investigated banks'. The maximal overall efficiency of "Rietumu Banka" is confirmed by its successful strategy and consistent management activities. The target customer group of JSC "Rietumu Banka" consists of juridical persons' and private customers' with high level of income. At the current moment the minimal amount of the term deposit of JSC "Rietumu Banka" is 10 000 USD, but the minimal amount of more flexible deposit types (deposits with accessible principal, market interest deposits, growing interest rate deposits and savings accounts) reaches 100 000 USD.



JSC “Swedbank” and JSC “GE Money Bank” have improved their performance on the deposit market during the investigated period. JSC “Swedbank” was least efficient in 2007, when increase of total assets’ and interest expenses’ did not accordingly affect the volume of total deposits’. Nevertheless, JSC “Swedbank” improved its efficiency in 2008, reducing the personnel costs and volumes’ of deposits’. JSC “Aizkraukles Banka” was fully efficient in 2005 and 2006, JSC “Norvik Banka” – in 2003, but both credit institutions have lost their leading positions by the end of the investigation period. In order to clarify the main sources of inefficiency among banks’ and propose versions of their elimination, the author applied the input-oriented BCC approach (Formula 6) to the measurement of technical banking deposit efficiency (see Figure 2).

Since the overall efficiency is calculated as a product of technical and scale efficiency, the BCC efficiency score under the assumption of variable returns to scale always exceeds the CCR efficiency score under the assumption of constant returns to scale. On the basis of BCC approach the efficiency level of investigated banks’ is considered to be independent on scale effects, thus, it represents only one component of the performance – technical efficiency.



**Fig. 2.** Dynamics of banking deposit efficiency BCC score on the basis of DEA VRS approach (cross-sectional data)

According to the results’ of the research, there is observable the increase of BCC efficiency score during the investigation period; in 2008 eight of ten banks’ were functioning on the efficiency frontier. For example, the technical efficiency score of JSC “SEB Banka” was 66,84% in 2003,

but in 2008 the credit institution has improved its results, being 100% BCC efficient. The comparatively low level of technical efficiency of JSC “DnB Nord Bank” (68,15% in 2007) and JSC “Mortgage Bank”(67,12% in 2007) is indicative of their inefficient operational activities or management, i.e. other banks are able to produce the equal volume of output, using the lesser volumes of inputs’. Nevertheless, both banks have improved their performance in 2008, having 90,49% and 100% BCC technical efficiency score respectively.

The efficiency measurement results on the basis of CCR and BCC approaches reflect that the main source of inefficiency among investigated banks is scale inefficiency. To prevent the scale inefficiency, the approach of Data Envelopment Analysis provides a possibility to determine the returns to scale regions (RTS) on the efficiency frontier. Knowledge of RTS regions’ allows defining, whether the increase or decrease of input/output volumes’ will result the higher scale efficiency of the bank (see Table 1).

**Table 1.**

Deposit efficiency scores and RTS in 2003

Bank	CCR efficiency		BCC efficiency		Scale score
	Score	Reference	Score	RTS	
<b>2003</b>					
Latvijas Krājbanka	0.83	Rietumu Banka	1.00	incr.	0.83
Swedbank	0.71	Rietumu Banka, Norvik Banka	0.96	decr.	0.74
DnB Nord Bank	0.47	Rietumu Banka	0.86	incr.	0.55
Mortgage Bank	0.61	Rietumu Banka	0.77	incr.	0.79
Aizkraukles Banka	0.97	Rietumu Banka	1.00	incr.	0.97
Parex Banka	0.86	Rietumu Banka	1.00	decr.	0.86
Rietumu Banka	1.00	-	1.00	const.	1.00
SEB Banka	0.63	Rietumu Banka	0.67	decr.	0.95
Norvik Banka	1.00	-	1.00	const.	1.00
GE Money Bank	0.73	Rietumu Banka	1.00	incr.	0.73
<b>Average</b>	<b>0.78</b>		<b>0.93</b>		<b>0.84</b>

All analyzed credit institutions with the exception of JSC “DnB Nord Bank” and JSC “Mortgage Bank” have demonstrated BCC efficiency score above 80% that is considered to be an indicator of technical efficiency. However, the CCR overall efficiency level is fluctuating

from 39,59% to 100% among banks during the period of the observation.

In 2003 JSC “Rietumu Banka” and JSC “Norvik Banka” were 100% CCR efficient, operating on the constant returns to scale segment of the efficiency frontier. However, JSC “Rietumu Banka” is the most frequent referent, thus having the largest  $\lambda_j$  value. The deposit efficiency of JSC “SEB Banka” and JSC “Latvijas Krājbanka” was increasing during the period of the observation; in 2008 both credit institutions are considered to be fully efficient. Analyzing the results of efficiency measurement on the basis of BCC approach, the author stated that JSC “SEB Banka” was operating on the decreasing returns to scale segment of the efficiency frontier over the time period from 2003 to 2007. Nevertheless, volumes of output and inputs’ were consecutively increased due to growing crediting activity of the bank. In 2008 JSC “SEB Banka” reduced both volumes of output and resources’ and thus achieved functioning under the conditions of constant returns to scale (see Table 2). On the contrary, JSC “Latvijas Krājbanka” was operating on the increasing returns to scale segment of efficiency frontier and constant increasing of deposit volumes’ coupled with increasing of input volumes’ resulted the full scale efficiency of the bank. Analogically, JSC “GE Money Bank” was functioning on the increasing returns to scale region; its efficiency reached its peak in 2007. However, in 2008 the overall CCR efficiency decreased to 93.43% due to the reduction of inputs’ and output under the conditions of increasing returns to scale (see Table 2). JSC “DnB Nord” and JSC “Mortgage Bank” were functioning with the lowest deposit efficiency level on the increasing returns to scale piecewise linear frontier segment. However, the improvement of their performance would be achievable by increasing of volumes’ of both production and resources. The overall efficiency score of JSC “Parex Banka” collapsed to 46,90% in 2008. Inadequate increase of personnel costs’ and interest expense has resulted the lowest overall and scale efficiency score of the bank. Analyzing the dynamics of average efficiency score among Latvian banks’, it should be emphasized that the level of average overall CCR efficiency is more significantly influenced by scale inefficiency, which been stable in 2003 and 2008, remaining at the level of 83-84%.

**Table 2.**

Deposit efficiency scores and RTS in 2008

Bank	CCR efficiency		BCC efficiency		Scale score
	Score	Reference	Score	RTS	
2008					
Latvijas Krājbanka	1.00	-	1.00	const.	1.00
Swedbank	0.81	SEB Banka	1.00	decr.	0.81
DnB Nord Bank	0.68	SEB Banka	0.90	incr.	0.75
Mortgage Bank	0.59	Rietumu Banka, SEB Banka	1.00	incr.	0.59
Aizkraukles Banka	0.90	Latvijas Krājbanka	1.00	decr.	0.90
Parex Banka	0.47	Latvijas Krājbanka, Rietumu Banka	0.85	decr.	0.55
Rietumu Banka	1.00	-	1.00	const.	1.00
SEB Banka	1.00	-	1.00	const.	1.00
Norvik Banka	0.81	Latvijas Krājbanka	1.00	incr.	0.81
GE Money Bank	0.93	Latvijas Krājbanka	1.00	incr.	0.93
<b>Average</b>	<b>0.82</b>		<b>0.98</b>		<b>0.83</b>

To improve the performance of inefficient banks’, it is important to determine, which proportions of resources’ will maximize the overall efficiency level (see Table 3).

**Table 3.**

CCR virtual input volumes in 2008 (thsd. LVL)

	Virtual input (Total assets), reduction (%)	Virtual input (Personnel costs), reduction (%)	Virtual input (Interest expense), reduction (%)
Latvijas Krājbanka	679 849.00 (0.00%)	11 369.00 (0.00%)	22 260.00 (0.00%)
Swedbank	3 756 617.14 (27.71%)	28 060.30 (18.86%)	136 726.17 (25.74%)
DnB Nord Bank	1 054 556.23 (45.44%)	7 877.08 (31.72%)	38 381.72 (50.83%)
Mortgage Bank	554 976.02 (41.45%)	5 590.69 (41.45%)	15 307.19 (87.83%)
Aizkraukles Banka	830 063.39 (15.59%)	13 881.01 (11.68%)	27 178.40 (10.43%)
Parex Banka	1 602 944.19 (53.10%)	23 554.76 (53.10%)	47 169.57 (62.49%)
Rietumu Banka	1 117 276.00 (0.00%)	12 526.00 (0.00%)	26 515.00 (0.00%)
SEB Banka	2 985 046.00 (0.00%)	22 297.00 (0.00%)	108 644.00 (0.00%)
Norvik Banka	408 775.49 (19.41%)	6 835.88 (31.30%)	13 384.36 (38.08%)
GE Money Bank	237 602.73 (6.57%)	3 973.39 (57.15%)	7 779.72 (16.47%)

The DEA approach provides a possibility to calculate the volumes of virtual inputs', i.e. optimal input volumes'. According to the data in Table 3, the CCR projection requires significant reduction in inputs, especially for JSC „DnB Nord Bank“, JSC „Mortgage Bank“ and JSC „Parex Bank“.

### *The efficiency measurement results of Latvian banks' on the basis of Window Analysis approach*

The previously introduced efficiency measurement results were obtained using cross-sectional data. The multidimensional assessment of banking performance might be realized on the basis of Window Analysis approach.

According to the methodology, described in the chapter “The DEA Window Analysis approach”, firstly it is necessary to select the window size. In order to investigate the impact of macroeconomical environment on the stability and efficiency of banking sector in 2008, it is significant to compare the banking performance in 2007 and 2008. Thus, the selected window size  $k = 2$ , the number of investigated banks'  $n = 10$ , the total number of observations' is 20.

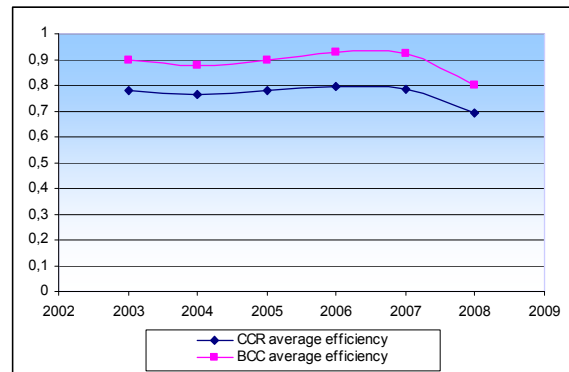
**Table 4.**  
CCR deposit efficiency scores of JSC  
“Aizkraukles Banka” with 2 year windows

Time periods	2003	2004	2005	2006	2007	2008
2003-2004	0.965	0.949				
2004-2005		0.964	0.983			
2005-2006			1	0.96		
2006-2007				1	0.906	
2007-2008					0.912	0.832
Average CCR efficiency score	0.965	0.957	0.992	0.98	0.909	0.832

The CCR and BCC efficiency scores of other investigated banks' were calculated analogically to the approach, represented in Table 4. The results of average banking overall (CCR) and technical (BCC) deposit efficiency level are depicted in the Figure 3:

During the time period from 2003 to 2007 the overall and technical deposit efficiency level of investigated banks' is fluctuating insignificantly: the BCC average efficiency score from 87,62% to 92,77%, the CCR average efficiency - from 76,4% to 79,28%. However, in 2008 both

technical and overall banking performance declined to 79,97% and 69,28% respectively. It is to be emphasized that there is a difference in the results, calculated using the Window Analysis approach, and the results' obtained from cross-sectional data.



**Fig. 3.** The CCR and BCC average deposit efficiency

The CCR and BCC average deposit efficiency scores in 2008 using cross-sectional data exceed the one calculated on the basis of Window Analysis methodology approximately by 10%. The approach of Window Analysis provided a possibility to compare the efficiency of investigated banks' in 2008 to their performance in 2007, indicating the trend of its decline. If there is observable the overall increase or decrease of the efficiency level among all investigated DMUs, it is recommended to use the multidimensional specifications of DEA like Malmquist index and Window Analysis, however the cross-sectional analyses of efficiency can still be useful for the purposes of operational planning.

### *The simulation of bank merger*

Due to the unstable macroeconomical environment, the further development of Latvian banking sector might be concerned with mergers and acquisitions. The first credit institution that was nationalized is JSC „Parex Banka“. According to the decision of the Cabinet of Ministers of the Republic of Latvia on 8 November 2008, JSC „Mortgage Bank“ has taken over the control JSC „Parex Banka“ by acquisition of 51% of its shares. On 3 December 2008 JSC „Mortgage Bank“ took over additional shares of JSC „Parex Banka“, becoming the owner of 84,83% of the shares. At the current

moment both credit institutions' are functioning independently, without joining up their resources. However, according to the results' of deposit efficiency measurement, both banks were operating inefficiently in 2008. It is to be emphasized that banks are operating on different returns to scale segments of the efficiency frontier in 2008: JSC „Parex Banka“ was functioning on the decreasing RTS region while JSC „Mortgage Bank“ – on the increasing returns to scale region.

The performance of investigated banks' can be improved, merging their input and output volumes in optimal proportions (see Table 5):

**Table 5.**  
CCR virtual input volumes of JSC “Mortgage Bank” and JSC “Parex Banka” in 2008 (thsd. LVL)

	Virtual input (Total assets), reduction (%)	Virtual input (Personnel costs), reduction (%)	Virtual input (Interest expense), reduction (%)
Mortgage Bank	554 976.02 (41.45%)	5 590.69 (41.45%)	15 307.19 (87.83%)
Parex Banka	1 602 944.19 (53.10%)	23 554.76 (53.10%)	47 169.57 (62.49%)

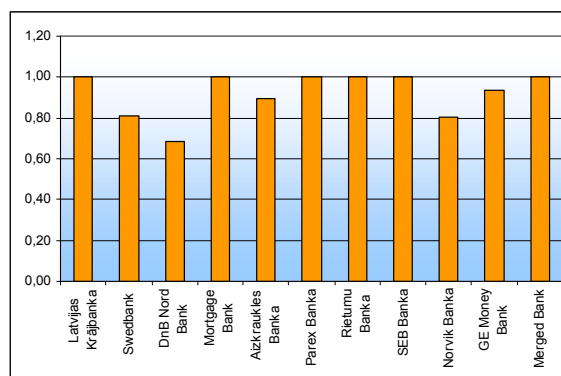
To simulate the merger of JSC “Mortgage Bank” and JSC “Parex Banka” the volumes of virtual weighted virtual inputs' and outputs should be summarized (see Table 6).

**Table 6.**  
CCR virtual input volumes of the Merged Bank in 2008 (thsd. LVL)

	Virtual input (Total assets)	Virtual input (Personnel costs)	Virtual input (Interest expense)	Total deposits
Merged Bank	2 157 920.21	29 145.45	62 476.76	1 527 697.02

Evaluating the deposit efficiency, the volumes of inputs' and output of JSC „Parex Banka“ and of JSC „Mortgage Bank“ are included into the data set in their virtual, i.e. weighted, values that ensure the full 100% CCR efficiency score. The output values of both credit institutions' are taken in their real values: 302 208 thousand LVL for JSC „Mortgage Bank“ and 1 225 488 thousand LVL for JSC „Parex Banka“ (the total sum of deposits' of JSC „Parex Banka“ does not

include the 676 398 thousand LVL government deposit that is concerned with nationalization of the bank). The results of bank merger simulation are depicted in the Figure 4. As it was determined, JSC „Parex Banka“ and JSC „Mortgage Bank“ have demonstrated the full CCR efficiency.



**Fig. 4.** The results of bank merger simulation in 2008 using the CCR projection

The efficiency score of the merged bank is 99,93%. This result allows to suppose that merger of two CCR inefficient banks' that are operating on decreasing and increasing returns to scale regions leads to the formation of CCR efficient DMU.

## Conclusions

The scientific paper is devoted to the efficiency measurement problem of Latvian banks'. The author observed numerous restrictions' that are concerning data analysis, specification of models' and interpretation of the results' of performance evaluations'. To assess the deposit efficiency level of Latvian banks', the author used the non-parametric frontier approach of Data Envelopment Analysis. The standard methods of efficiency measurement, such as quantitative ratio analysis, ratings and regression analysis do not provide the possibility of multidimensional efficiency assessment. The methodology of DEA is considered to be a sophisticated tool for efficiency measurement that allows the investigation of complex production processes among a set of Decision Making Units (DMUs). According to the information that is available to the author, Latvian banks currently do not apply methods of

frontier data analysis, including DEA, to the measurement of their efficiency level.

The author calculated the deposit efficiency scores of a set of Latvian banks' using CCR and BCC approaches. JSC "Rietumu Banka" has demonstrated the best result, operating on the efficiency frontier during all periods of the observation. It was stated that the main source of inefficiency among investigated banks' is scale inefficiency. On the basis of BCC approach there were determined the returns to scale regions of investigated banks' that allowed defining, whether the increase or decrease of input/output volumes' will result the higher scale efficiency score. The author also has calculated the volumes of weighted virtual inputs' that provide the possibility to maximize the overall efficiency score. Analyzing the change in the efficiency on the basis of Window Analysis approach, it was stated that there is observable the decrease of both BCC and CCR efficiency in 2008 in comparison to the previous year by 12,3% and 9,07% respectively. In the scientific paper DEA method was also applied to the simulation of JSC „Parex Banka“ and JSC „Mortgage Bank“ merger in order to calculate the optimal volumes of inputs' and output for the merged bank and its efficiency score.

According to the obtained results of efficiency measurement of Latvian banking sector, the author recommends to improve the methodology of efficiency measurement of Latvian banks' on the basis of non-parametric approach of DEA.

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## Aršinoва T. Datu čaulas analīzes metodes pielietošana Latvijas banku efektivitātes novērtēšanā

*Banku darbības efektivitātes novērtēšana ir bijusi par daudzu pētījumu objektu produktivitātes analīzes jomā. Sakarā ar bankas ražošanas procesu sarežģītību, efektivitātes novērtēšanas metodei ir jānodrošina daudzdimensionālās novērtēšanas iespēju. Tas var tikt realizēts, izmantojot robežas datu analīzes metodes, tādas kā DEA.*

*Zinātniskajā rakstā ir izstrādāta pieeja banku depožu efektivitātes novērtēšanai uz datu čaulas analīzes metodes pamata. Izmantojot mūsdienu informācijas tehnoloģijas, tika izanalizēta Latvijas banku izlases efektivitātes līmeņa dinamika, neefektivitātes cēloņi, optimālie resursu un produkcijas apjomi neefektīvajām bankām, kā arī novērtētas banku apvienošanās sekas. Uz alternatīvās DEA logu analīzes (Window Analysis) pieejas tika aprēķināts Latvijas banku vidējais efektivitātes līmenis, kurš ir atkarīgs no laika faktora ietekmes. Lai noteiktu visefektīvāk funkcionējošās bankas (nozāres etalonus), tika aprēķināta tehniskā, mēroga un kopējā efektivitāte. Autors rekomendē praktiski izmantot šos rādītājus stratēģiskajā un operacionālajā plānošanā,*

*lēmumu pieņemšanas procesā, nozares etalonu noteikšanā un riska vadībā.*

**Arshinova T. The Application of Data Envelopment Analysis Approach to the Efficiency Measurement of Latvian Banks'**

*The assessment of banking performance has been an object of many researches in the field of productivity analysis. Due to the complex nature of banking production processes, the required efficiency measurement approach should provide the possibility of multidimensional assessment. It is achievable using methods of frontier data analysis such as DEA.*

*In the scientific paper there is developed a concept of banking efficiency measurement on the basis of Data Envelopment Analysis assuming deposits as an output. Using the current information technology, there are analyzed dynamics of a set of Latvian banks' efficiency level, sources of inefficiency, optimal volumes of output and inputs for inefficient banks', assessed the effects of bank merger. On the basis of the alternative DEA Window analysis approach, there is calculated the average efficiency level of Latvian banks', which is dependent on the time factor influence. In order to determine the most efficient banks (benchmarks), there are calculated levels of technical, scale and overall efficiency. The author recommends to use these indicators practically in strategic and operational planning, decision-making process, benchmarking and risk management.*

**Аршинова Т. Применение метода оболочечного анализа данных для оценки эффективности латвийских банков**

*Оценка эффективности банков является предметом многих исследований в области анализа продуктивности. В связи со сложностью специфики банковских производственных процессов, метод оценки эффективности должен обеспечить возможность многомерной оценки. Это достижимо при использовании методов граничного анализа, например DEA.*

*В научной статье разработана концепция оценки эффективности привлечения банковских депозитов на основе метода оболочечного анализа данных DEA. При использовании современных информационных технологий проанализирована динамика уровня эффективности группы латвийских банков, источники неэффективности, оптимальные объемы ресурсов и продукции для неэффективных банков, а также оценены следствия слияния банков. На основе альтернативного подхода Window analysis рассчитан средний уровень эффективности латвийских банков, который является зависимым от влияния фактора времени. С целью определения наиболее эффективно функционирующие банки (эталон) был рассчитан уровень общей технической*

*эффективности, а также уровень эффективности от масштаба. Автор рекомендует практически использовать эти показатели в стратегическом и операционном планировании, в процессе принятия решений, при определении эталонов отрасли и в риск-менеджменте.*