

Optimization and Metamodeling of Metal Sandwich Panel Structures

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Abstract - The development of new materials and new manufacturing techniques has accelerated during the last several years. Laser welding is one of these technologies and it has facilitated use of steel sandwich panels. The application of such new structures requires fast optimization procedures to obtain optimal design configuration for a given design case. In order to obtain recommendations concerning the best configuration, different metamodeling methods were used for approximation, optimization and analysis of different core sandwich panels. Application of metamodeling methods for optimization included several steps: 1) design of experiments, 2) numerical experiments based on FE calculations, 3) high precision approximations of experimental data using polynomials, locally weighted polynomials and kriging, 4) multi-objective optimization using three criterions.

The results of different metamodeling methods and parameter fitting techniques were compared, with conclusion that kriging gives the best overall approximation results. The metamodel accuracy was increased by introduction of additional parameter inverse proportional to the second moment of area of panel cross section.

Keywords - Metamodeling, Pareto optimization, Sandwich panels, Kriging, Numerical Experiments, Design of Experiments.

I. INTRODUCTION

The development of new materials and new manufacturing techniques has accelerated during the last several years. One of these new ideas is the laser welding technique. The main advantages of laser welding are low welding distortions, high productivity, easy automation and it enables application of the welding processes in places not accessible with traditional welders [1]. These have opened new opportunities in the design of steel sandwich structures. The main benefits of a sandwich structure are caused by the high stiffness and bending strength properties due to the location of the material as far as possible from the neutral axis of the panels [2]. All-metal sandwich panels are made by a process of laser welding of faceplates to core stiffeners. Core stiffeners separate faceplates from the neutral axis of the panel's considerably increasing stiffness with minimal additional weight.

It is estimated that application of sandwich panels in shipbuilding can save up to 34% weight and 50% production costs [1].

The structural analysis of sandwich panels with thin flat faces was undertaken as early as the 1940's, particularly for aeronautical applications. The theoretical foundation and governing differential equations for the analysis of sandwich panels were presented in detail by [3] and [4]. Design

formulations for different core type all-metal sandwich panels filled with core material or empty, and with symmetric or asymmetric faceplates were recently summarised in [5], [6] and [7]. Stiffened panel design often involves optimising for the geometry of the shell and stiffeners, which frequently requires remeshing for acceptable accuracy. The numerical noise introduced from the discretization makes it difficult to use gradient-based optimisation methods and often additional effort is necessary to comply with design constraints [8]. Therefore in order to give recommendations concerning the best configuration for given design, different metamodeling methods were used for approximation, optimization and analysis of sandwich panel configurations. Application of metamodeling methods included several steps: 1) design of experiments, 2) numerical experiments based on FE calculations, 3) metamodel creation of FE models using kriging, locally weighted polynomials and partial polynomials, 4) multi-objective optimization in order to obtain Pareto optimal design set.

II. SANDWICH PANEL CORE TYPES

Currently sandwich panels composed of I-core and V-core stiffeners are among the most extensively used in

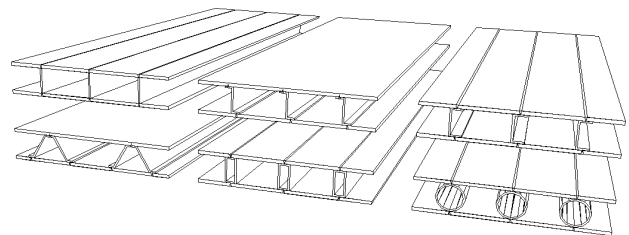


Fig. 1. I, Z, C, V, Osquare and Ocicle (O) core type sandwich panels

manufacturing, however other core-stiffener of Z-core, C-core, Osquare-core, Ocicle-core types as shown in Figure 1, continue to retain interest for further investigation.

III. DESIGN OF NUMERICAL EXPERIMENTS

For calculation and optimization of sandwich panel strength 6 input factors were considered [8], [9], as shown in table 1.

For practical design tasks usually panel width is given and not the distance between stiffeners. However if using panel width B and number of stiffeners n_s , additional constraints have to be met to correctly fit given number of stiffeners for panels of different widths. Therefore the distance between stiffeners and number of stiffeners was chosen as design parameters and

TABLE 1
FACTORS FOR NUMERICAL EXPERIMENTS

	Labels	Boundary	
		Low	High
Panel Length (m)	L	3	7
Panel Height (m)	h	0.004	0.16
Faceplate thickness (m)	t_1	0.002	0.004
Core stiffener thickness (m)	t_2	0.0015	0.004
Distance between stiffeners (m)	s	0.06	0.6
Number of stiffeners	n_s	2	6

additional constrain was evaluated during optimization, which links panel width B , number of stiffeners n_s and distance between stiffeners s :

$$B = n_s \cdot s. \quad (1)$$

We used 6 factor 500 point sequential samplings optimized according to Average Mean Square Error criterion [10]. Experimental points were divided into two sets each containing 250 points. First set was used for approximation as training points and second for validation as test points. This approach was chosen to obtain better estimations of approximation error in order to compare different metamodeling methods and parameter fitting strategies.

Numerical experiments were carried out using ANSYS software. Panels were loaded with uniform 3 kPa load and additional 1 kN concentrated force in the centre point. These loading conditions were chosen according to DNV guidelines [9] for ship deck certification.

Numerical values of the panel's global deflection, local deflection - between the stiffeners, stresses and nodal reactions at the sample points of sequential design were calculated. All responses of FE calculations are summarized in table 2.

TABLE 2
RESPONSES OF NUMERICAL EXPERIMENTS

Response	Label	Units
Maximal deformation of bottom plate	DEF_BOT	m
Length dividend with difference of top and bottom plate maximal deformation	DEF_DIF	
Equivalent maximal stresses of top plate	EQV_TOP	Pa
Equivalent maximal stresses of bottom plate	EQV_BOT	Pa
Shear stress	SHEARE_CORE	Pa
Reaction force	REACT_Y	N
Total mass	MASS	kg

IV. APPROXIMATION OF NUMERICAL EXPERIMENTS

Several approximation methods were used for metamodel creation of FE models: Polynomial regression [11], locally weighted polynomials (LWP) [12] and Gaussian process regression or kriging [13], [14].

To improve accuracy of metamodels, additional input parameter - inverse proportional to the second moment of area was introduced. It is well known that second moment of area is the governing term in deflection calculation of the beams, therefore addition of this input parameter in the metamodeling procedure increases the prediction accuracy of the metamodel. This parameter can be obtained as function of other input parameters and is not independent, therefore in optimization stage additional constraint was introduced to comply with this relationship.

For estimation of metamodel precision the relative Mean Square Error was used according to formula:

$$RRMSE = 100 \frac{\sqrt{\frac{1}{n} \sum_{i=1}^n (f_i - \hat{f}_i)^2}}{\sqrt{\frac{1}{n} \sum_{i=1}^n (f_i - \bar{f})^2}} = 100 \frac{RMSE}{\sigma}, \quad (3)$$

f_i - response value, \hat{f}_i - prediction value for i -th test point, \bar{f} - mean of response and σ - standard deviation of response in test points [15].

Metamodel fitting is crucial for creation of accurate metamodels and in fact is another global optimization problem. Metamodel fitting for different methods included determination of covariance function and estimation of optimal hyper parameters for kriging, bandwidth selection for locally weighted polynomials and choice of terms for polynomials.

Creation of kriging model is relatively time consuming. It includes determination of optimal hyper parameters and calculation of the inverse of covariance matrix. For kriging model we used covariance function in the form of:

$$\text{cov}(\mathbf{x}, \mathbf{x}') = \prod_{k=1}^m \exp \left[-\frac{1}{2} \left(\frac{\|x^k - x'^k\|}{\theta_k} \right)^p \right], \quad (4)$$

where x and x' are points of experiments and θ and p are hiperparametr.

For finding of optimal hyper parameter values we used optimization strategy based on BFGS (Quasi-Newton's) method combined with multi start global optimization [15]. We used relative Leave One Out Cross Validation criterion as metamodels quality criterion according to formula:

$$CV = 100 \frac{\sqrt{\frac{1}{n} \sum_{i=1}^n (\hat{f}_{-i} - f_i)^2}}{\sqrt{\frac{1}{n} \sum_{i=1}^n (f_i - \bar{f})^2}}, \quad (5)$$

where \hat{f}_{-i} – prediction value for i -th point, when approximation is created without using this point.

It was possible to save computational time for kriging model calculations employing the fact that all responses had identical experimental designs and thus covariance matrix, therefore it was possible to calculate inverse of the covariance matrix in each optimization step just once for all responses [15].

The RRMSE and CV of metamodel precision of different methods for I-Core panels are compared in table 3 and it indicates that precision of obtained metamodels are acceptable. The lines in bold corresponds to estimation of approximation error in test points, that have not been used for

metamodel creation. Data in table 3 indicate, that kriging gives best results, except for response 4 (EQV_BOT). Using all 500 experimental points for metamodel creation, kriging had considerably better results compared to other methods for all responses, with relative cross validation error in range of 1-4%.

Box plots of approximation errors for all responses of I-Core panels are shown in figure 2. It can be seen, that kriging approximations have best mean, median and standard deviation (the dotted line corresponds to mean value and unbroken line to median) compared to other methods.

Similar results were obtained for panels of other core types.

TABLE 3
COMPARISON OF APPROXIMATION ERRORS OF DIFFERENT METHODS FOR DIFFERENT RESPONSES OF I-CORE PANEL

Method	Criterion	Number. of training points	Number. of test points	DEF _BOT	DEF _DIF	EQV _TOP	EQV _BOT	SHEAR CORE	MASS
Kriging	CV	250		1.08%	2.27%	5.58%	3.32%	5.33%	0.059%
Kriging	RRMSE	250	250	1.99%	3.92%	6.61%	5.11%	5.17%	0.08%
Kriging	CV	500		1.10%	1.90%	4.51%	2.66%	3.33%	0.015%
Polyn.	CV	250		5.56%	10.98%	8.61%	2.82%	7.28%	0.94%7
Polyn.	RRMSE	250	250	6.81%	9.04%	8.98%	3.54%	7.67%	0.897%
LWP	CV	250		5.52%	6.52%	8.06%	2.49%	7.28%	0.146%
LWP	RRMSE	250	250	6.94%	6.94%	9.11%	2.96%	7.67%	0.148%
LWP	CV	500		3.49%	4.47%	7.87%	3.12%	6.46%	0.082%

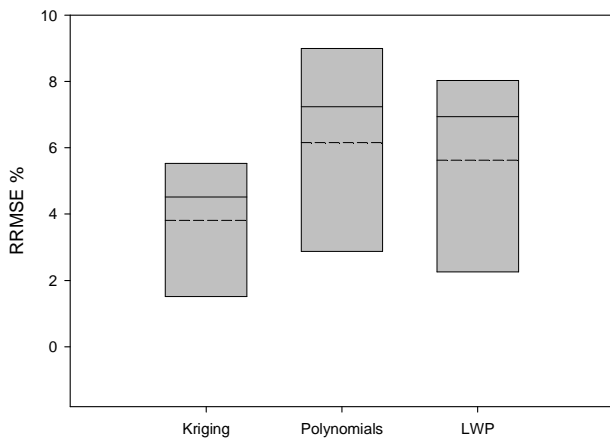


Fig. 2. Approximation error of I-Core responses using 250 training points

V. OPTIMIZATION OF I-CORE PANELS EMPLOYING METAMODELS

In this paper optimization procedure only for I-Core panels will be demonstrated, however optimization of other core types and other configurations were similar.

The multiobjective optimization problems were based on 3 criterions: panel weight, relative deflection (length relation to bottom plate deflection) and production costs. Criterions are summarized in table 4. Pareto optimal configurations were obtained during multiobjective optimization [16] using

software tool EDAOpt created in Machine and Mechanisms Dynamics Research Laboratory of Riga Technical University [17].

TABLE 4
OPTIMIZATION CRITERIONS

Criterion	Label
Mass (kg)	y_7
Length relation to bottom plate deflection	$-L/Def_Bottom$
Production costs (€)	$C = 0.7y_7 + n(5 \times 10^5(t_1 + t_2))L - n(500(t_1 + t_2) + 4)L$

Functional expression of panel production costs were taken from [8] and [18]. It consists of material and welding costs, which are function of number of stiffeners, panel length and faceplate width. The painting, cutting and other costs were not considered. Production costs were estimated in prices of 2003 in Euros.

In the process of optimization panel width B and length L was fixed, assuming that it was defined in project specification.

Optimization constraints were chosen according to DNV guidelines [9] for ship deck certification and are summarized in Table 5.

Additional equality constraints that were used during optimization are:

$$s = B / n_s, \quad (6)$$

$$x_7 = f(x_2, x_3, x_4, x_5), \quad (7)$$

$$L = 6 \text{ m}, \quad (8)$$

where $B=2$ m, f is formula of second moment of area of panel cross-section, and L is constraint for panel length.

TABLE 5
OPTIMIZATION CONSTRAINTS

Name	Label	Constraint	Units
Deflection relation to panel width	DEF_DIF	$\geq s/200$	mm
Equivalent maximal stresses of top plate	EQV_TOP	$\geq -160e6$	Pa
Equivalent maximal stresses of bottom plate	EQV_BOT	$\leq 160e6$	Pa
Shear stress	SHEAR_CORE	$\leq 60e6$	Pa

TABLE 6
OPTIMIZATION PARAMETERS

Name	Labels	Boundaries		Step
		Low	High	
Panel height	h	0.004	0.16	Continuous
Top and Bottom plate thickness	t_1	0.002	0.004	0.0005
Core stiffener thickness	t_2	0.0015	0.004	0.0005
Number of stiffeners	n_s	2	6	1

To obtain Pareto optimal designs of criterions (table 4), considering optimization constraints (table 5) and equality constrains (6), (7) and (8) 4 input parameters were varied (h , t_1 , t_2 , n see table 6). It was assumed that parameters t_1 and t_2 (plate and stiffener thickness) are discrete and can be varied with step 0.0005 m [18].

The relative RMSE (2) of approximation of different responses in Pareto optimal points is given in Table 7.

Only one out of 4 variable parameters were continuous therefore Pareto points formed isolated sets. The 3D charts from different viewing angles of criterion values and obtained optimal Pareto points for given optimization case are shown in Figure 3.

Pareto points in 3D charts of figure 4 indicate that choosing designs of panels with increased stiffness, increases panel weight and production costs. However it is possible to reduce

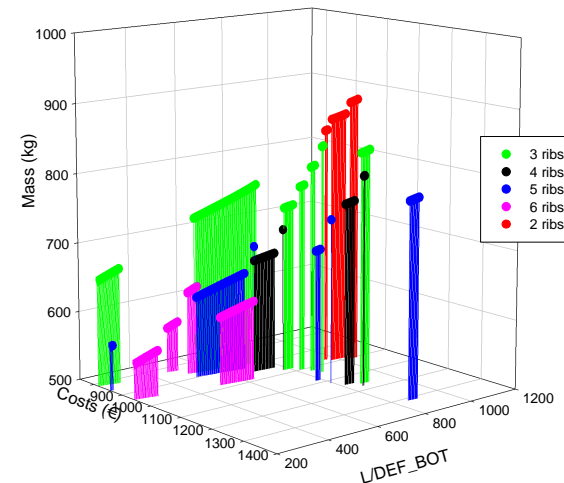
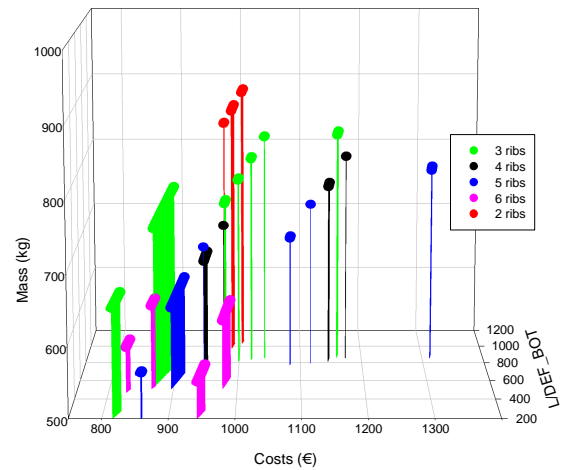


Fig. 3. I-Core Pareto points

panel weight by adding supplemental stiffeners. In such case, however, production costs increases due to welding costs of additional stiffeners.

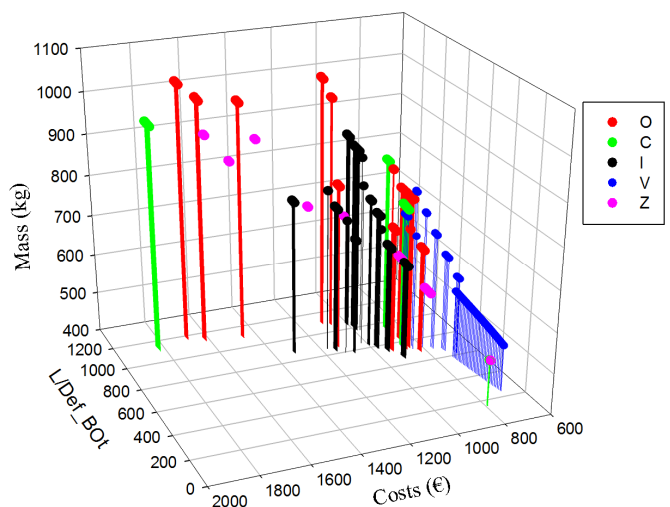


Fig. 4. Different core metal sandwich panel optimal solutions

In some cases there may be more than one possible optimal solution for given stiffness. For example for deflection of $L/DEF_BOT=540$ there are three optimal solutions as shown in table 8. Data in table 8 indicate that first case has less weight and higher costs, but second case more weight, but lower costs. In this example weight is reduced, because thinner stiffeners should be used, but costs increases due to thicker plates, that increase welding costs.

Combined results of optimization of different core sandwich panels, are shown in Figure 4. Results in figure 4 indicate, that for 6 m long panels with fixed deflection (form 0.75 to 3 cm) it is effective to use panels with stiffeners of V-Core type. This result can be explained with the fact, that V-Core panels have smaller welding costs per vertical stiffener (each core has two vertical stiffeners, but needs only 3 welding joints).

VI. CONCLUSIONS

The metamodel based multi-objective optimization of different core steel sandwich panels was performed using high

quality metamodels. The developed procedure can be used to effectively obtain optimal configurations of steel sandwich structures and reduce production costs.

Kriging was the most effective approximation method for metamodeling of sandwich panel FE models. The best results of kriging metamodels were obtained using cross validation criterion and determining covariance function and optimal hyper parameter values for each input factor. The problems concerning application of kriging are time consuming calculations of the inverse of covariance matrix and estimation of optimal covariance hyper parameter values.

The metamodel precision was considerably improved introducing additional parameter, which was inverse proportional to the second moment of area of panel cross-section.

TABLE 7

RELATIVE RMSE OF APPROXIMATION IN OPTIMUM POINTS

DEF_BOT	DEF_DIF	EQV_TOP	EQV_BOT	SHEAR CORE	REACT_Y	MASS
1.94%	7.00%	4.90%	3.83%	5.58%	0.17%	1.05%

TABLE 8

OPTIMAL DESIGN CASES OF I-CORE PANEL FOR GIVEN DEFLECTION

h	T ₁	t ₂	N	Mass	Costs	L/DEF_BOT
0.1447	0.0025	0.0015	6	603.19	980.23	540
0.1589	0.002	0.0025	6	627.86	871.50	540
0.1391	0.0025	0.002	5	622.13	900.49	540

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Jānis Auziņš, Jānis Janušovskis, Kaspars Kalniņš, Agrita Kovaļska. Metāla sendviču paneļu konstrukciju metamodelēšana un optimizācija

Jaunu materiālu un ražošanas tehnoloģiju attīstība ir paātrinājusies pēdējos gados. Lāzermetināšanas tehnoloģija ir viena no šādām tehnoloģijām, un tā ir sekmējusi dažādu metāla sendviču paneļu ražošanu un pielietošanu kuģubūvē. Šādu jaunu konstrukciju pielietošanai ir nepieciešamas ātras optimizācijas metodes, lai dotajām specifikācijām iegūtu optimālus projektējuma variantus. Lai dotu rekomendācijas optimālu metāla sendviču paneļu projektējuma variantu izvēlei, šajā darbā tika pielietotas dažādas metamodelēšanas metodes paneļu konstrukciju datoreksperimentu aproksimācijai un optimizācijai. Metamodelēšanas pielietošana paneļu optimizācijai ietvēra sekojošus etapus: 1) eksperimentu plāna izveide, 2) skaitlisko eksperimentu aprēķinus ar galīgo elementu metodi, 3) eksperimentu datu aproksimācija, izmantojot krīginga, globālo un lokālo polinomu metodes, 4) daudzkrīteriālā optimizācija, lai iegūtu Pareto optimālos variantus.

Salīdzināti dažādu metamodelēšanas metožu un to parametru pieskaņošanas varianti, secinot, ka kopumā krīgings dod labākos aproksimācijas rezultātus. Metamodelēšanas precizitāte paaugstināta, ieviešot papildus parametru, kas ir apgriezti proporcionāls paneļa šķēluma inerces momentam.

Янис Аузиньш, Янис Янушевскис, Каспарс Калныньш, Агрита Ковальска. Метамоделирование и оптимизация конструкций стальных сэндвичных панелей

В течение нескольких прошлых лет ускорилось развитие новых материалов и технологий производства. Лазерная сварка - одна из этих технологий, которая обеспечила широкое использование стальных сэндвичных панелей. Применение таких новых структур требует быстрых процедур оптимизации для получения оптимальной конфигурации конструкций. Для аппроксимации, оптимизации и анализа сэндвичных панелей с различными типами сердечников использовались разные методы метамоделирования. Применение методов метамоделирования для оптимизации включает несколько шагов: 1) планирование экспериментов, 2) эксперименты, основанные на вычислениях методом КЭ, 3) аппроксимация экспериментальных данных с высокой точностью, используя глобальные и локальные полиномиальные выражения и метод кригинга, 4) многокритериальная оптимизация для получения оптимального набора конфигурации Парето.

Сравнение различных методов метамоделирования и определения параметров привело к заключению, что в большинстве случаев кригинг дает лучшие результаты аппроксимации. Точность метамоделей была увеличена введением дополнительного фактора обратно пропорционального моменту инерции сечения.