

**RIGA TECHNICAL UNIVERSITY**

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**LATVIAN GEOID MODEL AND ITS DEVELOPMENT**

**Summary of Doctoral Thesis**

**Riga 2010**

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## GENERAL CHARACTERISTICS OF THESIS

### Topicality of Thesis

All geodetic measurements made by classical and also space geodesic methods are carried out in the environment of the Earth's gravitational field. It is necessary to know the gravitational field parameters exactly when a geoid is being determined, a vertical deviation is being calculated, the results of geodetic measurements are being reduced to a common coordinate system, as well as when satellite motion is being described. Our purpose is to obtain a maximally accurate fitting of a position into the previously defined geodetic coordinate system as a result of spatial measurements, and it is important to maintain this mathematical alignment in the future. For the height component reference of the spatial three-dimensional coordinate system, a digital equipotential surface, also known as a *geoid*, is applied; it is applied for all further practical work. It provides a unified reference mathematical surface. Today even the formation of any local geodetic reference network for survey works for diverse economical purposes is carried out by the use of the economically most advantageous and fastest DGPS/DGNSS methods. This alone justifies the requirements for a high precision geoid model, so that the height component of measurements can be reduced to the traditionally used mean sea level reference surface. The more accurate the geoid model at our disposal is, the more accurate the results of the geodetic measurements will be.

The general topicality of the subject of the doctoral thesis and the expansion of the practical application of the developed original model attracts increasing interest from colleagues and makes it necessary to justify the research work already carried out. While the elaborated model is being developed further towards its application in the latest aerospace technologies, the traditional ground gravimetric measurements are not receding into the background. They provide a succession of measurements and a base for theoretical studies of gravimetric values. In this work, all the activities performed which are used for the correction of geodetic systems and data shall be precisely documented.

### Aims of Thesis

The present thesis aims to develop a scientifically justified mathematical model of maximally high precision describing the gravitational field and its anomalies for the territory

of the Republic of Latvia. As a result of calculations, to adjust – the geoid – defined as the most accurate descriptor of the Earth's surface, for further use within the territory of Latvia when survey works of diverse kinds are performed.

### **Work tasks**

In order to accomplish the aims of the Thesis, work tasks are defined as follows:

- To choose the gravitational field development theory and technology that is best suited to the territory of the country.
- To carry out an analysis of the information sources which may eventually be applicable, to assess the volume of additional information necessary, and to evaluate the ways and possibilities of obtaining this.
- To develop formulas for the approximation of the geoid of the Republic of Latvia and to calculate the geoid model.
- To perform an evaluation of the accuracy of the developed model and to attach it to the geodetic network.

### **Scientific novelty of Thesis**

In the Thesis, the author has for the first time created a way of obtaining a theoretical and practical basis for obtaining a Latvian geoid model, and this has been successfully implemented in the Thesis.

The new geoid model for height recalculation obtained as a result of the respondent's work – the LV'98 GPS-geoid, is attached to the common network with the neighbouring countries, Lithuania and Estonia. There are no gaps at state borders because the stations of the geodetic network of neighbouring countries were used for the fitting of heights, and stations of the unified absolute gravimetric base network were used for the reduction of gravimetric measurements as well. The generalization of absolute and relative gravimetric results in accordance with all available resources has been followed by creative modelling work with GRAVSOFTE software. During the modelling process, I detected that we have an insufficient coverage of gravimetric stations in the territory of the Baltic Sea. Altimetric measurements made by the ERS-1 satellite were then used additionally in the modelling, and was done for the first time ever in the territory of the Baltic Sea. These were then included in the calculations and the accuracy of our geoid model was thereby improved by 1.3 cm.

After using all of the diverse gravimetric data and checking their mutual correlation, the most accurate geoid model possible was calculated; this is linked to the verified national geodetic stations of the highest order. Thus the model obtained by the respondent serves not only as a height reference equipotential surface in the work with DGPS/DGNSS measuring instruments, but also gives a precise evaluation on the mutual compatibility and accuracy of national geodetic networks of various types. This determines, in which specific directions additional geodetic measurements should be made, in order to get a geoid model of still higher accuracy.

The Thesis is a completed unassisted elaboration, and the results described therein have material significance in the geodesy sub-branch of the civil engineering branch of science.

### **Practical value of Thesis**

The development of a practical model has been carried out for a completed cycle starting with the collection of gravimetric data, their digitization and the determination of an absolute gravimetric value up to the calculation of a new geoid model. It is always possible to repeat a completed calculation cycle, if geodetic data of a higher precision are available.

The author of the Thesis is himself directly participating in the absolute, relative, as well as airborne gravimetric measurements. He has performed the processing of data and their preparation for modelling. He is working with the GRAVSOFIT programming package, as well as performing the necessary programming work. He is carrying out wide-scale comparisons with the best-known geoid models in geodesy, is analysing their suitability for the territory of Latvia and is assessing their compliance with his developed model. also in this way the quality of the national geodetic reference network is assessed and international circulation of data is provided. By participating in international projects, he is also analysing the qualitative further development of a solution of the Latvian geoid model. Such opportunities are possible through a more precise definition of the national geodetic reference network, the completion of levelling data processing and their linking with DGPS/DGNSS observations. Further updates of global gravitation models are expected from the results of the new space missions. However, all the measurement data obtained with the geoid model developed by the author are precise in this system. They will be documented and retained in order to secure the conversion from one system used in praxis to another system.

The model described in the paper has already successfully served in everyday surveying works in Latvia for 10 years and is still continuing to do so. This indicates that the developed model has withstood the test of time and is very useful for surveyors, geodesists and scientists. The greatest beneficiaries are professionals of the geodesy branch and users of the calculated geoid model.

### **Theses put forward for *viva voce***

- The scientifically justified development of a mathematical model describing the gravitational field of the Republic of Latvia and its anomalies has been completed. A new theory and development algorithm for geoid modelling has been obtained.
- In the territories without gravimetric data in the Baltic Sea, an improvement of the precision of the geoid model by 1.3 cm has been obtained by the use of satellite altimetric data for the first time in the world, providing a gravimetric geoid model of the accuracy of 6 – 8 cm with wide possibilities for practical application in the entire territory of the country, for the first time in the history of Latvia.
- A gravimetric reference value has been established with an accuracy of 5  $\mu\text{Gal}$  in three stations in Latvia, which were specially selected by the author, and a Potsdam value correction has been established -14.0 mGal.
- Digitised gravimetric data are included in the data base ( $\approx 12\,000$  points) that provides for the calculation of a precise geoid model.

### **Composition and volume of Thesis**

The Doctoral thesis is independent scientific research, and contains an introduction, 6 chapters, a final part, conclusions and references. The Thesis volume includes 151 printed pages, 52 figures, 7 tables, literature references, and contains 112 publication titles, and a supplement.

### **Approbation and publications of Thesis**

Results of the present thesis have been reported and discussed in more than 7 domestic and 9 international conferences. The course of the scientific development and the main results of the Thesis have been disclosed in 15 publications published in international journals and



compilations of proceedings. The work was performed at the Department of Geomatics of the Riga Technical University, the State Land Service (SLS), the Danish National Surveying and Cadastre Centre (KMS) and the Latvian Geospatial Information Agency during the period from 1996 until 2010.

## STRUCTURE OF THESIS

The present thesis comprises **six chapters**, where in the first chapters, theoretical fundamental principles and underlying problems have been brought to the forefront while practical solutions and the results achieved are described in the further chapters. In the introduction of the Thesis, the approach to the problem that is related to the use of gravimetric data is described, and the aims of the Thesis, the work tasks, and the scientific and practical value of the Thesis is formulated.

In the **first chapter**, the necessity of the definition of the geodetic reference network is considered and the relationship of this network to gravitational field observations is explained. In the **second chapter**, geodynamic processes and the direct impact thereof onto the vertical, i.e., the height system, which forces changes on the geoid model as well, are described. The Baltic Sea serves as a demonstrative surface for our geoid; the shape of its surface fluctuates or changes with time. In order to define and use the height system locally and globally, one common gravitational model of the Earth is necessary. Such a global model is used in the Thesis; it serves as a universal reference equipotential surface, and a more comprehensive description of it is given in the **third chapter**. Further, in the **fourth chapter**, the characteristics and basic mathematical relationships of the normal gravitational field, as well as methods for the direct absolute measurement of gravitational field are presented, which have been performed at three specially established absolute points in Latvia. The above-mentioned measurements were made at the same time in all three Baltic States and provide a base for further gravimetric works. In the realization of these works, national level experts were applying relative measurement methods on their own account. The establishment of a denser densifying gravimetric network and practical relative gravimetric measurements in the territory of Latvia, which include airborne gravimetric measurements as well, are described in the **fifth chapter**. In the case of new measurements, control values of the gravimetric field have been determined. We are then able to evaluate objectively a large number of relative gravimetric measurements from the Soviet time. In the **sixth chapter**, the scientific result based on gravimetric measurements, which has been used in further studies and also in today's praxis, has been described. The above-mentioned geoid model LV'98 has been mathematically attached to the specially selected geodetic base stations of the territory of Latvia. Such an end product is applied widely in surveying works, when works are performed

by the DGPS/DGNSS method. The GRAVSOFT program package was applied for the attainment of these scientific objectives.

The calculated original geoid model is attached to the Thesis as a supplement.

### **First chapter.**

In the first chapter of the doctoral thesis, **the** scientific approach to the problem is described, which concerns the significance of gravimetric data and the use thereof in geodesy.

In total, there are six international geodetic base stations in Latvia; two of which are situated in Riga due to historical circumstances. If this implementation of the ETRS89 coordinate system is compared with the previous one, the geographical distribution of stations is much more symmetric than the previous one. Originally in Kurzeme, there was only one station – Arājs – and the Irbene geodetic station in Ventspils district did not exist yet. As the geographic layout of stations described by the author of the Thesis is implemented, a geodetic reference system that is more reliable and stable is provided. It provides reference values for the control of both the global and local height reference equipotential surface, as well as for the adapting of the situation to the field.

Living on a dynamic earth, the first principle is to record the precise time of all observations made. Secondly, a defined and hopefully common valid reference epoch must be assigned to all the geodetic reference networks. It must also be fixed when classical surveying methods are used and a connection to adjacent geodetic monuments is made, although the crustal deformation is negligible within the tectonic plate in most cases. However, at the same time DGPS/DGNSS methods utilizing permanent stations several hundreds of kilometres away will in the near future need to use crustal deformation and height reference surfaces fixed in time – accurate geoid models . Thus thirdly, the three-dimensional deformations in the geodetic reference network must be modelled and adjusted. This is possible today and is very fortunate, as precise gravimetric measurements, as well as continuous observations through permanent GPS/GNSS stations have been available for several decades. In accordance with the above-mentioned specific subjects, the Nordic Geodetic Commission (NKG) ad hoc groups have been established and are working; the situation is being analysed, and scientifically justified decisions for further work are being made there.

Research of the gravimetric field and of its impact on geodetic observations is significant in order to create a contemporary geodetic reference system, to establish a geodetic and gravimetric network, to implement new coordinate and geopotential height systems for geophysical problem studies, the solution of navigation tasks and other tasks of the branch.

### **Second chapter.**

The zero line of the land uplift goes through Latvia; thus it is divided into two different parts. North-westwards, in the north-western part of Latvia, i.e., towards the Cape of Kolkasrags, we can observe land uplift, but towards the south-east, i.e., around Daugavpils, the land goes down. These are the consequences of an opposing process that had taken place in the ice age under the thick ice shield, when the land surface had gone down as result of this pressure. Geoid changes are also directly related to these vertical changes of the Earth's crust, because deformations of a large mass are taking place. Calculations show that geoid rise comprises about 6.5% of the Earth's surface vertical deformation value.

At present, in all there are 1300 benchmarks included in the national levelling network. The nodal and endpoints of the lines are marked with 77 fundamental benchmarks. An extremely large amount of the benchmarks, 55%, are wall benchmarks. According to the levelling catalogues, bridges, culverts and the foundations of buildings with different stability were used mostly for monumenting with wall benchmarks. The balance between stability of the benchmarks and the accuracy of measurements is the subject of further critical discussions. An analysis of the results of the above mentioned inspection of the levelling network shows that up to 30% of the total number of benchmarks has been destroyed. The number of fundamental and ground benchmarks destroyed is relatively small. The reasons for this are probably the more successful choice of locations and the deep setting. The safeness of wall benchmarks depends on the purpose of the use of the buildings. It should be noted that it is not the precision of levelling but the stability of levelling benchmarks and the large number of destroyed benchmarks which are the main reasons for the new relevening.

Sea level is still an important surface for crustal deformation studies. A time series of sea level data represents the sum of the vertical movements of the station and the eustatic rise of the sea level. From the sea level time series and from repeated levelling, the absolute uplift of land can be calculated, if the the corrections from the eustatic rise of sea level, geoid rise and change of the sea surface topography are added. The secular sea level change can be studied from the satellite measurements of absolute land uplift from and apparent land uplift from sea level data.

Gravity change and elevation change are correlated geophysical quantities. With repeated gravity measurements and repeated levelling the relation between the gravity change and land uplift can be studied.

The reference system of new quality for gravimetric network began in 1995, when three absolute gravimetric points, Rīga, Pope and Višķi, were measured by an expert from the Finnish Geodetic Institute. They serve as base for the national gravimetric network of Latvia.

The transformation between height systems can be done using the well-known equation:

$$h - H - N = 0 \quad , \quad (1)$$

where  $h$  – geometrical (geodetic) height  
 $H$  – physical (absolute) height  
 $N$  – (quasi)geoid height.

Theoretically, the fixed reference datum used for heights is the equipotential surface that we call a geoid. In the present stage, there are no methods to determine  $W_0$  for zero initial points of levelling with an accuracy comparable to the potential differences from geometrical levelling. Because a geoid is not available, for physical access the mean sea level (MSL) as the average of the instantaneous sea level over sufficient periods of observations is used instead. The heights are transferred to inner land using levelling networks that are connected to the mean sea level through tide gauge observations.

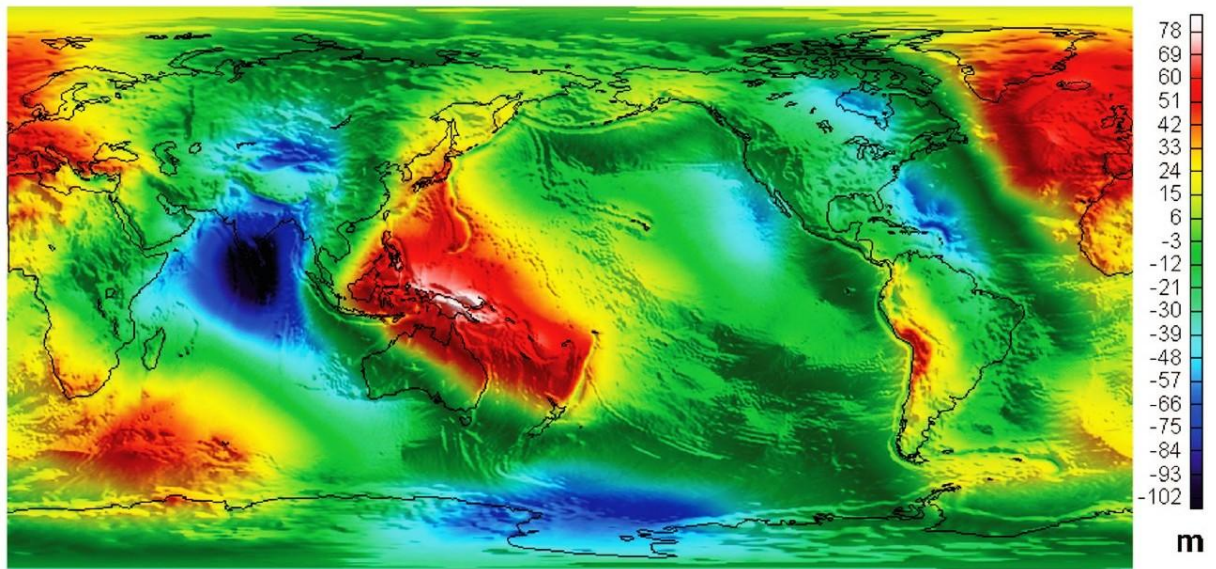
### **Third chapter.**

Gravitational potential  $V$  for many applications of the Earth's geopotential models is expressed in spherical harmonic expansion, where potential coefficients in this expansion are determined by different techniques. During the last 40 years, considerable improvement of estimation of potential coefficients has taken place in two general ways. Firstly, the highest degree in expansion is expanded to a much higher degree using additional satellite data and land gravitational data, wherewith the resolution of models is improved. Secondly, the accuracy of coefficients is being improved without interruption by using the inclusion of additional data that improves geographical coverage and accuracy in the course of time.

In the last decade of the 20th century, the need for an improved determination of geoid undulations has become more topical. The central need concerns the conversion of ellipsoidal

heights from heights determined by GPS to orthometric/normal heights. The aim to determine a globally defined geoid that could serve as a reference surface for the global vertical datum is also related to the improvement of the geoid.

The most important terrestrial gravity acquisitions include the airborne gravimetric data over parts of Greenland, the Arctic and the Antarctic, which were measured by the Sea Explorations Laboratory of the USA, as well as other data obtained in the framework of cooperation projects. These measures of aggregation have improved and densified the data on many terrestrial regions of the world. Among these noteworthy geographic regions are Alaska, Canada, parts of South America and Africa, South-East Asia, Eastern Europe and the former Soviet Union. In general, the density of gravity points used in Europe is sufficient for the obtaining of 30'x30' mean free air anomalies.

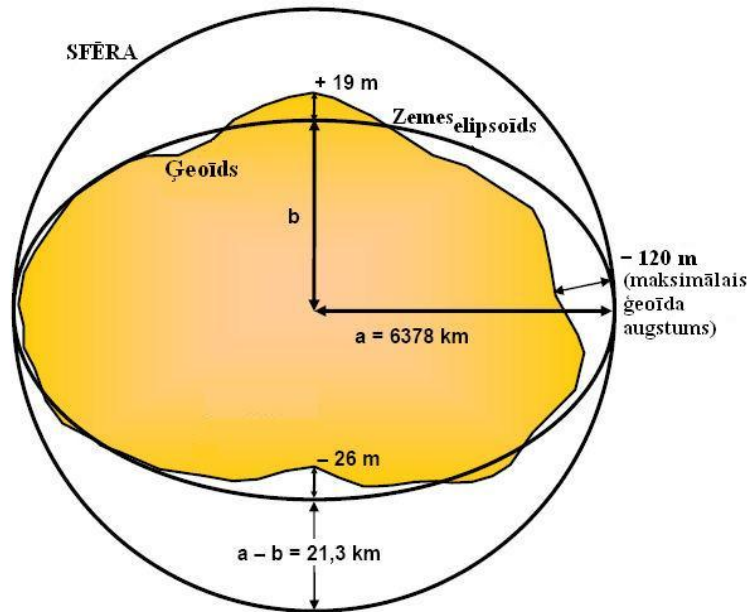


**Fig. 1** Earth geoid model EGM96, from +85m up to -107m

Although the development of EGM96 is a noteworthy milestone, work is going on to improve the model for specially dedicated and also general applications. When the CHAMP, GRACE and GOCE space missions for studies of the gravitational field are evaluated, the ocean geoid modelling should also be improved in order to more fully use the accuracy of 3-4 cm reached by the synoptic TOPEX/POSEIDON altimetry. Thus we continue to improve our understanding on the circulation of oceans and the new effects of geopotential model with the lapse of time.

#### Fourth chapter.

When the gravitational field of Earth is studied, the mathematical model most corresponding to it – the normal gravitational field of the Earth caused by the selected equipotential ellipsoid should be selected first. This mathematical ellipsoid is also used in order to describe the shape of the Earth. The normal gravitational field of the Earth is considerably simpler than the real one and its equipotential surfaces and lines of force can be used to position points of surface. The normal gravitational field is described by the normal gravitational potential. But the difference between the real gravitational field of the Earth and the normal field is expressed by the perturbed potential. We can see great differences between the selected equipotential rotational ellipsoid and the sphere.



**Fig. 2** The shape of the Earth as a sphere, ellipsoid or geoid

The normal gravity on the ellipsoid surface  $\gamma_0$  is expressed by the K. Somilgiana formula:

$$\gamma_0 = \frac{a\gamma_e \cos^2 B + b\gamma_p \sin^2 B}{\sqrt{a^2 \cos^2 B + b^2 \sin^2 B}}, \quad (2)$$

where  $\gamma_e$  and  $\gamma_p$  – respectively is the normal gravity on the ellipsoid equator and its poles,

$B$  – geodetic latitude.

Contemporary absolute measurements of gravity acceleration in the starting gravimetric points, which we call zero class points, are performed with an accuracy of a few microgals. Nowadays, such a high precision can be reached by special ballistic equipment. It is necessary that absolute gravity measurements in the gravimetric zero class points are repeated periodically. The preferred period between the repeated measurements is from 5 until 10 years. Many special requirements should be applied to the site of the gravimetric zero class point and to its arrangement. Points shall be established at sites that are stable from a geological viewpoint where fluctuations of groundwater levels are minimal. It is advisable to establish boring wells for measurements of groundwater level at gravimetric points.

Tab. 1

Reduction of gravimetric measurement data of 1995

Gravimetric station	$g_h$ , μGal	$W_{zz}$ , μGal/m	$\Delta g_0$ , μGal	$g_0^{95}$ , μGal
RIGA	981650943.3	-210.6	175.6	981651119.0
POPE	981667249.9	-317.7	267.2	981667517.1
VIŠŅI	981567863.7	-313.2	261.2	981568124.9

A valuable absolute gravimetric point has been established in the anteroom of the Pope Evangelical Lutheran Church. It is constructed in moraine soil at a depth of 2 metres with iron reinforcement wires deposited therein. The total volume of the concrete in this support construction is 3.2 m<sup>3</sup>. It is the only absolute gravimetric point in the Baltic States constructed in a church. This work was justified by the consideration that the point should be protected and should remain intact for the longest possible time. The author of this thesis has taken part in the choice of this site, in the work of establishing the point and at observations as well. The external view of the gravimetric support with a geodetic cast-iron mark embedded in concrete in its centre is shown in Fig 3.





**Fig. 3** The support of the Pope gravimetric point with the mark and the author of the thesis

### **Fifth chapter.**

After the conclusion of the Danish – Baltic sectoral programme in geodesy in 1999, decisions were passed concerning the development of the national geodetic network of Latvia in general, as well as concerning the renewal of the national gravimetric network. Initially, during the implementation of the sectoral programme, a unified basis for the renewal of the gravimetric network in the territory of the three Baltic States was already created. This work is very time-consuming and requires scrupulous work. Three absolute gravimetric points were measured in each country previously; this served as the basis for further relative measurements.

It is important, that first order points can be easily reached by car, that they are protected, durable, and that the conditions for measurements are good. When locations are chosen, the existing infrastructure of the specific site and its further development trends are taken into consideration. A stable, horizontal surface is necessary for the execution of observations; new gravimetric point sites are coordinated with the existing and designed national geodetic networks as well. The locations of points are chosen so that not only gravity acceleration can be measured there, but that the point can be included in the national height

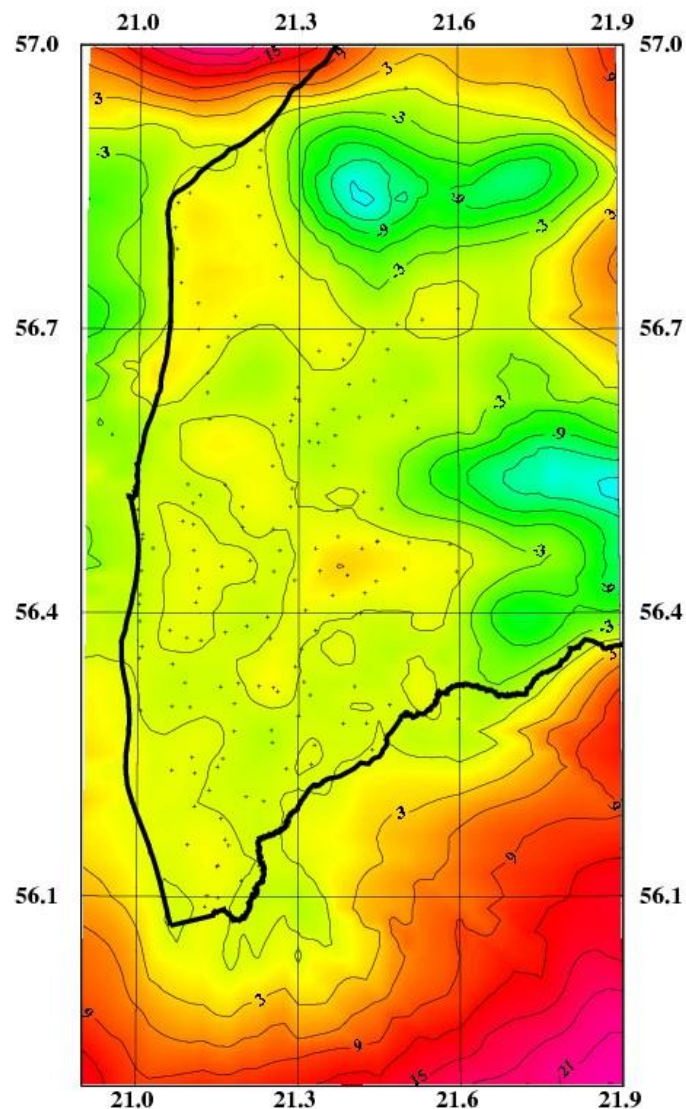
network too or that a levelling mark is at close range. The points of the gravimetric reference network should be evenly distributed covering the entire territory of the country.

After the conclusion of the Danish – Baltic sectoral programme, in the framework of which, the correspondence of digitised gravimetric data on land was checked as well, the issue of obtaining real measurement data in the Baltic Sea was considered urgent as well. Although the information on gravimetric anomalies determined from the ERS-1 satellite altimetric measurements was available at that time, the Nordic Geodetic Commission decided to nevertheless implement the airborne gravimetric measurement project, in which all Nordic Countries and the three Baltic States are taking part, mainly by taking organizational measures, like coordinating flight permission with the competent authorities of corresponding states. Flight routes were elaborated and mutually coordinated. Liepāja was chosen as the only airport in the Baltic States, where the landing and plane onboard gravimeter fitting to the national network was performed during the measurements. On the Latvian side, it was provided by the author of this thesis from the relative gravimeter CG-3 acquired in the framework of the sectoral programme. The absolute gravimetric point of the renewed basic network and of the Soviet era network in the centre of Liepāja City located at the furnace of the former sanatorium was used as a starting point for fitting.

During the process of the digitising of available gravimetric maps of the scale 1:200 000, the volume of available information was very variable. The volume of available information levels for each separate map sheet was rather different. There were gravimetric anomalies of solely one type, i.e., e.g., Bouguer anomalies, available for many territories, as well as the gravimetric reference system and density of soil used for calculations of anomalies which were known for these territories. Such a situation existed in the entire territory of Kurzeme, where the gravimetric measurements were performed in the 1960's, when the mean value for the density of soil used, was  $2.3 \text{ g cm}^{-3}$ . The most recent measurements for the territory of Latvia were available from the 1980's, when the mean value for the density of soil  $2.67 \text{ g cm}^{-3}$  was used having regard to all values fixed by the present international gravimetric standard network IGSN1971.

For the evaluation of the accuracy of the gravimetric map, Bouguer anomalies were determined from the map using reduction formulas and gravity acceleration was calculated  $g_{71}$ . Liepāja district is the specially chosen test region, where 171 geodetic points were measured by relative gravimeters. The greatest differences between the calculated and

measured values of gravity acceleration can reach +1.5 mGal or –1.5 mGal; the average value is practically close to 0.0 mGal, as is shown in the next figure.



**Fig. 4** Differences of Bouguer anomalies in Liepāja district, c.i. 3 mGal

In order to adopt a new height system for Latvia, a geoid model included, the available results of work, which were acquired in several previous international projects, e.g., the "European Unified Vertical Network" (EUVN) and the "Baltic Sea Level" (BSL) shall be used. Latvian geodesists have also engaged actively in these international initiatives. Thus, we acquire fundamental knowledge about the Baltic Sea surface, the Baltic region geoid, geodynamic process and the possibilities for the use of the GPS/GNSS method.

After the completion of the mathematical processing of the results of measurements, EUVN network points' geodetic coordinates in the ETRS 89 system and normal heights,

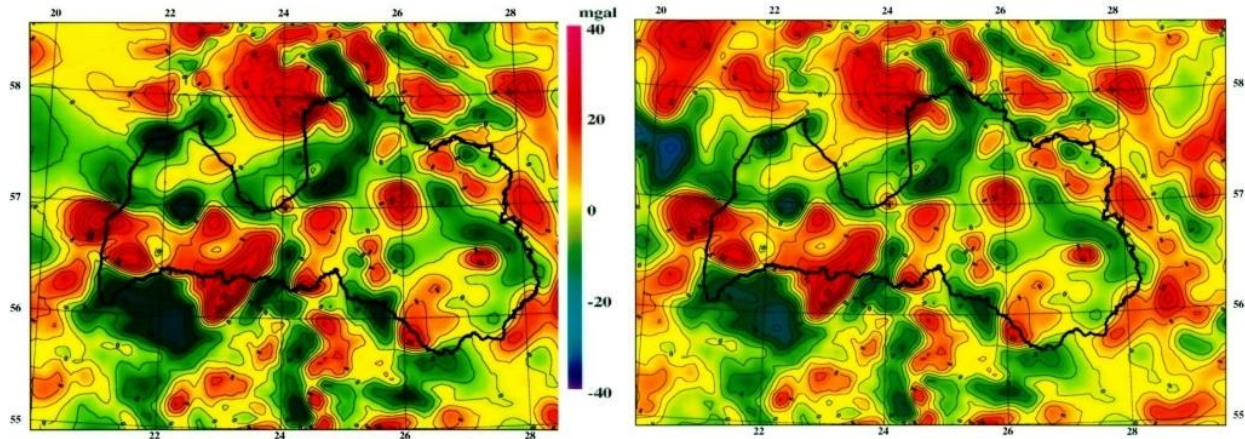
which were linked to the Unified European Levelling Network UELN95/98 and Amsterdam zero point, i.e., European Vertical Reference System (EVRS) were obtained. This creates the preconditions for the adoption of a new national height system for Latvia. GPS stations at RIGA, IRBENE and the entire LatPOS network which is one of the European geodetic networks have become geodetic points of higher accuracy for Latvia.

Further, as we are aiming to increase the accuracy of determination of (quasi)geoid heights, it is necessary to complete the establishment of vertical and gravimetric networks of the territory of Latvia, which has already commenced, connecting them with the reference points of GPS/GNSS.

### **Sixth chapter.**

The national GPS network consisting of several thousands points is completed, and now intensive densifying of this network is in process and coordinates are being determined for thousands of points every year; the method for the calculation of height above sea level from the determined ellipsoid heights becomes more topical. When the height of a point measured by GPS/GNSS measurements is calculated, one additional parameter is required – a geoid height that is determined using a digital geoid model. The obtained result is determined by the accuracy of the applied digital geoid model. The values of its errors and their distribution depends on the amount of geodetic information and its accuracy.

The scientific novelty of the Thesis is the determination of gravimetric anomalies for the territory of the Baltic Sea from the results of altimetric satellite measurements. When gravimetric data are being modelled, all of the available gravimetric measurements are being analysed. In the Baltic Sea, gravimetric measurements were not available in wide regions. Therefore, free-air gravimetric anomalies determined from altimetric data and appropriately prepared were used in order to fill these gaps. When a gravimetric field is depicted with or without new gravimetric anomalies, we can perceive differences at once; we see this in the next image.



**Fig. 5** Free-air gravimetric anomalies; c. i. 4 mGal

Attention should be paid to the left side of the images, i. e., the territory in the Baltic Sea. In the image on the left, the surface of the gravimetric field in the above-mentioned territory is unnaturally even. This points to a geophysical contradiction in the gravimetric field and a non-compliance with the situation in reality. This additional work gave an increase in the accuracy of the geoid model by 1.3 cm.

Latvian geoid model calculations were realized with the following logical sequence:

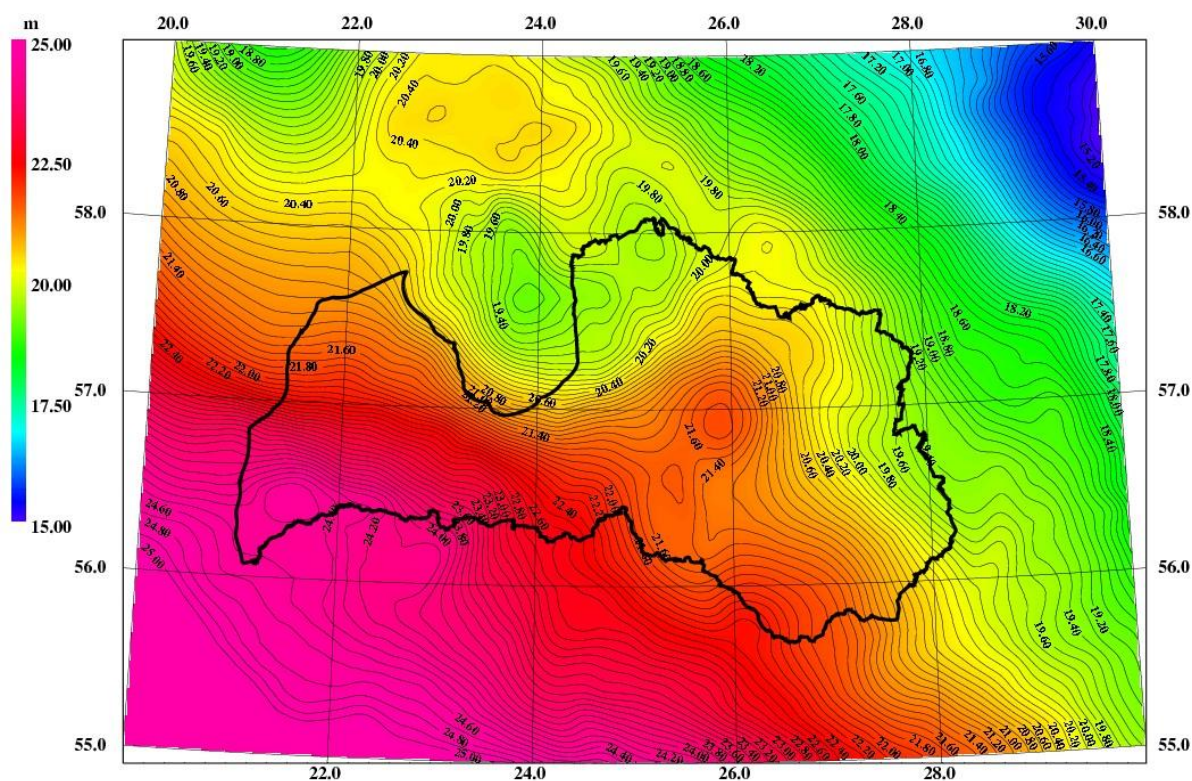
- in the area covered by the model, differences between free-air anomalies calculated according to the EGM96 model and determined from field measurement results were found;
- the obtained free-air anomaly differences were transformed into regular grid data;
- on the basis of this new grid data, the corresponding world geoid model height correction values were calculated for each site according to Stokes' formula;
- after a summarization of the world geoid model and geoid height correction values, the updated gravimetric geoid model was created in the modelling region for calculations;
- in conclusion the calculated gravimetric geoid model was transformed to a height system adopted in the country using heights of points, coordinates of which are determined also in GPS measurements. Thus the calculation cycle is closed with some specific GPS-geoid models, e.g., LV'98.

For geoid determination Stokes' formula is used. It relates geoid height  $N$  to the gravity anomalies  $\Delta g$  as a surface integral as follows:

$$N = \frac{R}{4\pi\gamma} \iint_{\sigma} \Delta g S(\psi) d\sigma , \quad (3)$$

where  $\psi$  – spherical distance and  $S$  – Stokes' function.





**Fig. 6** Heights of calculated geoid LV'98 above GRS 80 ellipsoid; c.i. 10 cm

Characteristics of the accuracy of a digital geoid model covering the territory of the country and calculated in 1998 are compiled combining alternatives – using coordinates of national GPS network points and levelling point heights. Alternative results of calculations provide independent evaluation of the accuracy of a geoid model obtained in a gravimetric way. The root mean square error of this model does not exceed 6 cm in the territory of the country.

Geoid heights vary in the territory of Latvia from 25.4 m in Liepāja district at the border with Lithuania at the Baltic Sea down to 19.3 m in Balvi district close to the border with Russia. The maximal gradient of change of geoid height in the direction of a line connecting the above-mentioned places reaches 20 mm/km. This gradient will be smaller in other directions. In the territory of Latvia, the sharpest changes of geoid values can be observed in Kurzeme, not in the rest of the territory of the country, although the surface terrain is more explicit in other parts of Latvia and reaches the maximal height value in Gaiziņkalns.

It will be possible to create an accurate digital geoid model of the territory of Latvia at the next level only after the completion of new gravimetric measurements and of the national vertical geodetic network renewal works.

## CONCLUSIONS

- The purpose set in the doctoral thesis is achieved.

In the course of the performance of the work, a mathematical modelling of the gravitational field and its anomalies for the territory of the Republic of Latvia was performed. Calculations were done to determine the mathematical shape describing the physical surface of the Earth – geoid model – in the territory of Latvia; I have assigned the **LV'98** designation to it. It has been implemented into practical usage in work with DGPS/DGNSS equipment.

- The essence and necessity for a contemporary height reference system application.

Precise information on a gravity field is necessary for the contemporary provision of a height reference system in the country, in particular when geodetic measurements are performed with GPS/GNSS receivers. The above-mentioned receivers measure three Cartesian space coordinates and provide a height above the mathematical ellipsoid surface, but do not measure height above sea level. For the determination of height above sea level it is necessary to know a geoid model value in each of the corresponding points of measurement. The more precise the geoid model, the more precisely we can determine the height over sea level if we are using GPS/GNSS receivers for work. The more precise the results of measurements, the wider is their application, and the benefit arising from them is not only economic, but also very practical and facilitates geodetic work in general. For example, if a geoid model is very precise, we can replace the precise geometrical levelling which is expensive and time-consuming, with the DGPS/DGNSS method for the transfer of heights.

- Provision of data necessary for modelling beyond boundaries of modelling area and in the Baltic Sea. Data exchange.

It is essential that gravimetric data are provided for the modelling area and also beyond its boundaries. There are no cooperation problems concerning data exchange with Lithuania and Estonia. Data concerning all three countries is available and can easily be used in calculations. More dense coverage of gravimetric data is necessary for the territories of

Belarus and Russia, as well as the Baltic Sea territory . In the course of the solution of the problem, I have arrived at the conclusion that the shortage of this data can be made up with satellite **altimetric data in the Baltic Sea**. This attempt was very innovative and successful, when the territory received coverage of gravimetric values after altimetric measurements made by the ERS-1 satellite. I have linked altimetric measurements with available gravimetric data by GRAVSOF software. In the further modelling process, I have established that the use of ERS-1 data provides an improvement of accuracy of the Latvian geoid by 1.3 cm. I am maintaining that this is the first successful instance of application of altimetry results in the world for calculation of local geoid models. The above-mentioned results have been analysed in some of the author's publications.

- Selection of special geodetic points and evaluation of accuracy of geoid model to be reached.

Before the commencement of use of the new geoid model in practical work, the correspondence of the model to the local national height system should be checked and secured. The national geodetic reference network has full responsibility, and it is determinant here. Zero-order and first-order geodetic network points in the entire territory of the country were taken as starting reference points in the beginning. They should be linked to national height network at the same time. Territorial coverage shall be maximally even. The selected points shall contain neither GPS measurement errors, nor levelling errors, because otherwise all these errors will be embedded in the height reference model. When special **co-located DGPS/DGNSS and levelling points are being selected**, the certainty of these acquired values shall be taken into account by performing a comparison with the geoid model and also with geodetic points located in the vicinity. As a result, after a performing of the fitting it is neither a proper geoid, nor a quasi-geoid, although all co-located points have normal heights. It is simpler, if a geographically fitted geoid model is called a GPS-geoid. The author of the thesis has participated in work groups together with Danish, Estonian and Lithuanian geodesists, in which the further course of works and the development of geodetic networks in future were elaborated. Works, which were started during the Danish – Baltic sectoral programme, are continuing today as well. When the **accuracy** of our GPS-geoid **LV'98** model is assessed according to special points, it is not higher than **6-8 cm**. At present, it is not realistic to achieve another result for the entire territory of the country, until a sufficiently reliable and checked network of co-located geodetic points is obtained and gravimetric measurements are completed. The accuracy of a GPS-geoid model, which can be achieved



theoretically, is in the range of 1–2 cm. This provides for mutual integration and fitting of local height networks and geodetic systems in the global scale.

- Integration of geoid models obtained by different methods.

There are two methods of geoid surface determination, which are used in the doctoral thesis. One method is based on gravimetric measurements, and the other one – on co-located points of DGPS/DGNSS and of levelling networks. In the final stage of works, the results of both methods are **combined into one GPS-geoid model** on the condition that it shall fit most precisely for the geodetic reference system used in the country – LKS-92. The name LV'98 is given to the acquired model. Working from the base station network LatPOS both in real time in the field and later in data post-processing, this model of normal height determination is used. It is embedded for optional use in several GPS/GNSS manufacturers' receivers. Both *Trimble* and *Leica* receivers already have this GPS-geoid model in their geodetic calculation software.

- Application of global spatial methods in geoid modelling.

In the future, it will be possible to apply independent fundamental geodetic methods for geoid model updating and repeated control of its values. A Satellite laser location station (SLR) is also operating where the Riga absolute gravimetric point is situated. The former provides an independent assessment of the geodetic model. The technical capabilities of the Irbene Radio Telescope situated in the Ance civil parish of Ventspils district should be used as well, when very long base interferometry (VLBI) measurements are being performed and the obtained results are being applied for the improvement of the geodetic reference system and creation of scientifically innovative global solutions. The best possibilities for development in all these fundamental spheres in the Baltic States are in Latvia.

- Evaluation of all available gravimetric data.

In Latvia, extensive measurements were carried out before the Second World War under the guidance of scientist Voldemārs Jungs. These measurements gave the only available gravimetric data in the Nordic countries until the regaining of Latvian independence and the updating of the database information in 1994. However, this information was secret during the entire Soviet period because of its strategic importance. This information was not freely available even to Soviet citizens, to say nothing of foreigners, even for scientific goals.

- Shortage of experts and support provided by colleagues abroad.

Thus, after the restoration of independence of the state, this subject which had been wrapped in secrecy was particularly fascinating for researchers, as it had been inaccessible up

to this time, but has now become available. There were no experts in Latvia. The interest in gravimetry was boosted by the support offered by foreign experts, in order to solve common scientific problems. Fundamental gravimetric works of great intransient importance were started as a result of cooperation with the Finnish Geodetic Institute and later also with Danish geodesists.

- Theoretical preparation and choice of gravimetric sites.

After the completion of the theoretical preparation stage, the author of this thesis started to select absolute gravimetric sites within the territory of the country. He visited many churches in Latvia, both in Kurzeme and in Latgale, aiming to install a gravimetric base site. This is necessary because the established gravimetric point should remain intact for a maximally long time for many future generations. A point suitable for lasting use was established in the Pope parish church in Ventspils district. The clock cellar owned by the Latvian University Astronomical Institute in the Botanic Garden was chosen as a special site. These three base points or, we may say figuratively – three whales – are the base of the entire further gravimetric network of Latvia. Both the establishment of the national gravimetric reference system and the further measurements are the direct merit and contribution of the Thesis author. .

- Purchase of a new gravimeter and densifying measurements of the network in the Baltic States.

The measurements of the densifying network were started in Latvia during the implementation of the Danish – Baltic sectoral programme, when the national gravimetric reference network was commenced in 1998 and when 9 gravimetric base points of the Baltic States were connected into a united chain. Thereafter, the relative gravimeter CG-3, the first gravimeter corresponding to contemporary requirements in the Baltic States, was purchased for our own needs. The establishment of the above-mentioned reference network, the densifying measurements of the network and the initial work with the new relative gravimeter is a contribution made by the author of the present Thesis in person as well.

- Digitisation of gravimetric data and arrangement in one joint reference system.

As the work of gravimetric surveying is very time-consuming, which the author, from his own experience, has established , then the quickest and only possibility of getting a precise geoid model was to use the geodetic data which had already been surveyed and are available, efficiently. The capture, adjustment and processing of the necessary data was an

important task for the author of this Thesis, to be able to start the calculations of the geoid model. Gravimetric data were available mostly only as paper map sheets of a scale of 1: 200 000, moreover, these data were depicted in different reference systems, and each map sheet had its own reference system. Digitisation of all the map sheets was performed by me in person, thus systematic errors caused by various performers were avoided and data heterogeneity was reduced to the lowest notch. Gravimetric information on 12 000 points was acquired as a result. A geodesist would have to work for 20 years, if he wanted to measure this number of points by gravimeter. This conclusion is based on the supposition that one person can measure 600 points with one gravimeter during a season on average. After the data conversion from paper form to digital form, data transformation to a unified reference system should still be performed.

- GRAVSOF software and programming.

I have performed the coordinate conversion and other standardised calculations by adaptation of solutions included in the GRAVSOF open code program package to the Latvian situation. Programs are compiled in the FORTRAN programming language which is a programming language known by the author of the Thesis. Thus I could confidently perform the necessary theoretical calculations. Due to the specific character of Latvian data, many non-standard conversions had to be performed, when the formulas for obtaining of a gravimetric value had to be created and checked by myself. Such special solutions were not available in the GRAVSOF package. In such cases, I have performed programming works unaided thus extending the possibilities for calculations and applications of the GRAVSOF program package.

- Scientific and practical evaluation of the digitised map data.

The captured data on more than 12 000 points were to be overhauled. I have analysed the values of the gravimetric points in detail and a significant work of verification was performed. These data are recorded into the common data base of NKG and are available for international circulation so that scientists not only in Latvia, but also in all Nordic countries can use them.

- Application of the global gravitational model and of the digital height model within the territory of Latvia.

The local gravimetric data acquired in the further course of work were used for the modelling of the geoid surface in the GRAVSOF programs. The global Earth gravitational field models which were determined during the satellite missions should be used mandatorily

by the application of summarized mean gravimetric values on the surface of the Earth. **The EGM96 global gravitational model** is used as a reference global gravitational field in the Thesis. In the modelling calculations, the difference of values of this global field and of values of the local gravimetric field is used because the average value of this difference is close to zero, and therefore we can get quicker mathematical solution in the calculations of the surface integral. Also the digital height model is required in the course of calculation of the geoid. I have got it independently from the web and have applied it in my work. The international abbreviation of the height model is GLOBE.

- The international maintenance of the gravimetric system.

Repeated gravimetric measurements are performed in the base points of the system on a regular basis in order to provide precise gravimetric and spatial data at any reference moment. The most recent repeated absolute gravimetric measurements were performed in Rīga, in December 2007. The aerial gravimetric measurements performed in the Baltic Sea in 1999 are regarded as a particular element in the control and linkage of gravimetric data. When they were performed, a specially equipped plane landed at Liepāja City airport, and the author of the Thesis calibrated the plane gravimeter in the gravimetric network of Latvia.

- Further works for obtaining a geoid model of a new, higher standard of accuracy.

In order to reach a geoid accuracy of one centimetre, **systematic renewal of the national first-order levelling network** in an overall length of **3000 km** is being performed. The completion of works is planned for 2010. In the course of **works, they are being coordinated internationally** with Lithuanian and Estonian experts, as well as analysed in the work groups of the Nordic Geodetic Commission. Execution of the works is facilitated by the development of the global Earth gravitational field model, where the results of a new quality on principle are anticipated **after** the completion of the **GOCE mission**. I can maintain that the results of the new space mission will indirectly improve the accuracy of our national geodetic reference networks and of the geoid model. The work done for many years by geodesists also serves for the evaluation of the results of the most recent space missions. Thus all the experts involved in this technological process described in the Thesis and users of the calculated GPS-geoid benefit from it.

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## PARTICIPATION AT CONFERENCES

1. Participation with report „Latvijas ģeodēzisko atbalsta tīklu attīstība pēdējos 10 gados” (*„Development of Latvian geodetic reference networks during the last 10 years”*) at Latvian University 68th conference in the section for astronomy and geodesy in Raiņa boulevard 19, on 19 February 2010 (Work language of conference - Latvian)
2. Participation with report „Ģeoīda modeļa attīstība ģeodēzijas darbos” (*„Development of geoid model in geodetic works”*) at Latvian University 68th conference in the section for Geomatics (GIS and remote sensing) in Alberta street 10, on 2 February 2010 (Work language of conference - Latvian).
3. Participation with report „Relatīvie gravimetriskie pētījumi Pērnavas poligonā” (*„Relative gravimetric studies in the Parnu site”*) at Latvian University 67th conference in the section for Geomatics (GIS and remote sensing) in Alberta street 10, on 3 February 2009 (Work language of conference - Latvian).
4. Participation with report „Relatīvie gravimetriskie mērījumi ar CG-5” (*„Relative gravimetric measurements with CG-5”*) at Latvian University 67th conference in the section for astronomy and geodesy in Raiņa boulevard 19, on 12 February 2009 (Work language of conference - Latvian).
5. Participation with report „Latvijas absolūtā un relatīvā gravimetrisko tīklu attīstība” (*„Development of Latvian absolute and relative gravimetric networks”*) at Riga Technical University 48th international scientific conference in the subsection for Geomatics in Āzenes street 16, on 12 October 2007.
6. Participation with report „2006. gada Starptautiskais EUREF simpozijs Rīgā” (*„The International Symposium EUREF 2006 in Riga”*) at Latvian University 64th conference in the section for astronomy and geodesy in Raiņa boulevard 19, on 10 February 2006 (Work language of conference - Latvian).
7. Participation with report „Nivelējumu paaugstinājumu izmaiņu analīze līnijās Demene-Jēkabpils un Ainaži-Rīga” (*„Analysis of changes of levelled elevations within lines Demene-Jēkabpils un Ainaži-Rīga”*) at Latvian University 63th conference in the section for Geography (Geomatics) in Alberta street 10, on 28 January 2005 (Work language of conference - Latvian).

8. Participation with report „Geoid model for surveying in Latvia” at XXIV FIG (International Federation of Surveyors) Congress held in Sidney, Australia 11-16 April 2010.
9. Participation with report (on 10 September) at the 3rd Baltic-Swiss Geodetic Science Week held in Tallinn on 10 - 12 September 2008. Program can be found in Internet, see: <http://www.gece.ttu.ee/~artu/SWISS/>
10. Participation with report (on 20 June) at European annual geodetic symposium EUREF 2008 held 17 - 21 June 2008 in Brussels. See: <http://www.epncb.oma.be/EUREF2008/program.php>
11. Participation with report at the 7th International Conference *Environmental Engineering* held on 22-23 May 2008 in Vilnius (VGTU) in the 5th section – Technologies of Geodesy and Cadastre.
12. Participation with report at conference Baltic Survey 2008' held by LLU in Jelgava 8 - 9 May 2008. All 3 Baltic States and colleagues from Belarus were taking part.
13. Participation with report at the meeting of Work Group of the Nordic Geodetic Commission (NKG) on geodetic reference systems and positioning in Copenhagen on 15 - 16 January 2008.
14. Participation with report at the 12th Baltic military geospatial information conference in Riga in hotel *Reval Hotel Latvia* on 31 October 2007. All 3 Baltic States and participants from NATO countries were taking part.
15. Participation with report at the 7th session in the European annual geodetic symposium EUREF 2007 held on 6 - 9 June 2007 in London.
16. Participation with report at the meeting of Work Group of the Nordic Geodetic Commission (NKG) on geodetic reference systems and geodynamics in Tallinn on 27 - 29 March 2007.



**DOCTORAL THESIS**  
**IS PUT FORWARD AT THE RIGA TECHNICAL UNIVERSITY FOR**  
**OBTAINING OF DOCTOR'S DEGREE IN ENGINEERING**

The Doctoral thesis for obtaining of a doctor's degree in engineering will be defended publicly on 22 October 2010 at 2:15 p.m. in the Council Hall of the Faculty of Building and Civil Engineering of the Riga Technical University in Āzenes street 16.

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**CONFIRMATION**

I hereby confirm that I have elaborated the present Doctoral Thesis submitted for consideration at the Riga Technical University for the obtaining of a doctor's degree in engineering. The Doctoral Thesis has not been submitted for graduation at any other university.

Jānis Kaminskis .....(Signature)

Date: .....