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The Model of Nonstationary Rotor Magnetic Field Observer in the Induction Motor

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Abstract - This article is devoted to the questions, associated with observer construction for monitoring the values of rotor magnetic vector magnitude and angular deflection of induction motor oriented on bidimensional convolution on temporal and spatial actual parameters. The interrelation of induction motor breakdown torque and rotor characteristic time and transportation lag is shown. The system of rotor running stream observer on the basis of gage rotor position and stator current is put forward.

Key words - induction motor, distributed system, impulse admittance function, nonstationary component.

I. THEORETICAL FOUNDATION

Modern vector control systems of induction motor are typically constructed on the basis of the direct or indirect determination of rotor magnetic field [1]. Magnetic field direct control was used for the first time in [2], however in practice indirect methods are used. They are based on the measurements of voltage, current, position or rotor speed. Computational methods are based on the magnetic flow representation in the form of two-component vector, in which one component corresponds to the in-phase current component and the second to the quadrature current component of magnetic flow.

The disadvantage of vector representation is the fact, that in such systems the alloted character of rotor currents induced by stator currents is not taken into consideration. It is quintessence in dynamic conditions of speed-up and acceleration.

In-phase current component of rotor current vector is formed as the result of interaction between two simultaneous processes – attenuation of ring currents, induced by stator in rotor and the transportation process of these currents concerning stator current wave, in general, with variable speed. It is known that the transportation process with the variable speed can be described by the first-order partial differential equation [3].

Mathematical models of transitions, which take place in rotor of induction motor, are appropriate to construct on the terms of structure theory of distributed systems, described in [4]. In this theory the rotating rotor of the induction motor permeated by the stator magnetic field, can be presented as the linear distributed block with the input signal *w(x,t)* and output signal *Q(x,t),* where *w(x,t)* is the value of stator magnetomotive force, *Q(x,t)* is the current division over the rotor surface, x are the space points pertaining to the range space of the arc polar pitch D of stator, t is time. The output value of the block can be calculated as:

$$Q(x,t) = \int_{-\infty}^{t} \int_{D} G(x,t,\xi,\tau) w(\xi,\tau) d\xi d\tau . \qquad (1)$$

Generally the kernel of integral transformation depends on the four actual parameters- two time input and output parameters *τ, t,* and two dimensioned input and output parameters *ξ, x*. The function *G(x, t, ξ, τ)* is usually called impulsive admittance function or Green function. It is the output signal of the distributed block to the ordinary impulsive disturbance in the form of delta function. In some cases the expression (1) is called bi-dimensional convolution. In the lumped parameter systems the influence function looks the same, but doesn't have the dimensioned variables *ξ, x*. The calculation of the output signal in this case is usually called Duhamel integral.

Straight calculations using these formulas are more complicated than the usage of differential or operator equations, however they can be used in the cases when system differential equations have float factors. It can be shown that, particularly, in transient motions such as rotor speed-up or rotor deceleration this case takes place.

The values of influence function can be obtained by investigation of systems of partial differential equations, by numerical simulation, by the method of direct determination and other methods. Generally, for description of the rotor magneto-electric processes history first-order differential equation is applied. Its influence function is of the form of damped exponential. In special case of constant speed motion the first-order linear differential equation with fixed coefficients is used for consideration of distribute character of the process:

$$\begin{gathered} a\frac{\partial Q(x,t)}{\partial t} + b\frac{\partial Q(x,t)}{\partial x} + cQ(x,t) = w(x,t), \\ Q(x,0) = Q_0(x), \;\; Q(0,t) = g(t), \qquad (2) \\ x \ge 0, \;\; t \ge 0. \end{gathered}$$

It describes the process of the substance, energy and field transfer along the x axis with constant speed in the case of simultaneous state change under the influence of distributed disturbance f(x,t). According to [4], Green function of this system is:

$$G(x,\xi,t) = 1(x-\xi)\cdot$$
$$\cdot\exp\left[-\frac{c}{b}(x-\xi)\right]\cdot\delta[bt - a(x-\xi)]. \tag{3}$$

The investigation of the expression (3) shows that when the coefficient *c* is 0, the output value *Q(x,t)* is equal to the values *w(x,t)*, displaced in planar surface *x,t* along the direction *bt-a*. The occurrence of the absolute term leads to the fact that except the displace these values have the exponential damping proportional to the difference between input and output spatial values ξ, x. When left boundary data is zero *Q(x,0)=0* and under the condition $x - \xi < 0$ to all values *Q(x,t)=0*. However, it should be taken into account that left boundary data *Q(0,t)* is equal to the right boundary data

$$Q(0,t) = Q(l,t)\,. \tag{4}$$

It corresponds to the abridgement of the distributive block with ordinary back coupling. It can be shown that impulse response of the distributive block in this case becomes perpetual and the output signal of this block in addition to the transportation feature gains the feature of the cyclic permutation of *Q(x,t)*according to the spatial value x. Thereby, the calculation (1) in the given case can be reduced to the integrating of the values *w(x,t)* in the sense of temporal and spatial coordinates, besides the values, detained for the time τ are subjected to the cyclic permutation. If therewith the function *w(x,t)*can be presented as the multiplication of two functions, and each of these functions depends on one argument only, double integral can be calculated by the separate integrating in the sense of each argument.

Approximate solution of the integral equation (1) can be obtained by transition to the digital form by introduction the discrete functions in the sense of spatial and time arguments φ, t. If we separate the interspace *l* into the big enough number N of intervals with the size of $\Delta\varphi$, we can obtain the solution of the samples transposition problem. It is known that in discrete form there is so-called cyclic convolution [5], which provides the transposition of the discrete function values in one-dimensional version:

$$y(n\Delta\varphi) = \sum_{l=0}^{N-1} x(l\Delta\varphi)h((n-l)\Delta\varphi)\,. \tag{5}$$

In case, when $h(n\Delta\varphi)$ is represented as the array with N length and $h(n\Delta\varphi)\neq 0$ when $n = n_0$, the values of $y(n\Delta\varphi)$ are the cyclic convolution of $x(n\Delta\varphi)$ values to the n_0 samples. Therefore, $h(n\Delta\varphi)$ function in the form of unit impulse in n_0 point acts as delta function when samples transposition takes place.

On the other hand, there is an opportunity of using the discrete analog of common convolution (Duhamel integral) on the infinite time interval:

$$y(n\Delta t) = \sum_{l=0}^{\infty} x(l\Delta t)h((n-l)\Delta t)\,. \tag{6}$$

Predetermining $h(n\Delta\varphi)$ values correspondingly to the discrete influence function of the unidimensional dynamic element, it is possible to put in addition for calculating bidimentional system reaction to the actions with unspecified time and interspace behavior.

The equation similar to (2) can be constructed for stator winding of the induction motor as well. At that electrodynamic force of rotor and stator complicates significantly the estimation of the rotor influence function. The analytical model can be much-simplified supposing the stator winding power supply is performed from the current power supply (regenerating).

For performing calculations the following predicates will be accepted:

1. only one spatial value along rotor crosscut is taken into consideration;
2. the number of squirrel cage rotor conductions is fairly great;
3. the number of magnetic structure sides is two;
4. reactive and active rotor resistances are uniformly distributed along the surface;
5. the distribution of magnetomotive force from stator has sine-shaped form;
6. current density has exponential distribution along the rotor surface;
7. the magnetic system saturation and current displacement in rotor are missed.

For application (1) in order to calculate the transient response let's plot the influence function using equation (2). The form *w(x,t)* of the action in (1) may have any appearance, however in motor the magnetomotive force is determined by the winding distribution in stator slot and by the form of the power currents [6]. Let the form of the disturbing action concerning the running value of the stator current is given by:

$$F(\varphi,t) = F_0\sin(2\pi\varphi/l)\times 1(t)\,. \tag{7}$$

The differential equation of the induction rotor current system element is given by:

$$\frac{L_r}{R_r}\frac{\partial I_r(\varphi,t)}{\partial t} + \frac{L_r}{R_r}\omega_s(t)\frac{\partial I_r(\varphi,t)}{\partial\varphi} + I_r(\varphi,t) = F(\varphi,t)\,, \tag{8}$$

$$I_r(\varphi,0) = I_0(\varphi)\,,\ I_r(0,t) = I_r(l,t)\,,$$
$$0 < \varphi < l\,,\ t \geq 0\,. \tag{9}$$

The influence function can be determined for (7) if while steady conditions are attained $\omega_S = const$ at zero time the stator magnetic flow was out $F(\varphi,t)=0$. At this the initial condition is given by:

$$I_r(\varphi,0) = \sin(2\pi\varphi/l)\,. \tag{10}$$

The form of influence function (3) to the solution (8), (9), (10) in the graphic structure is shown in Fig 1.

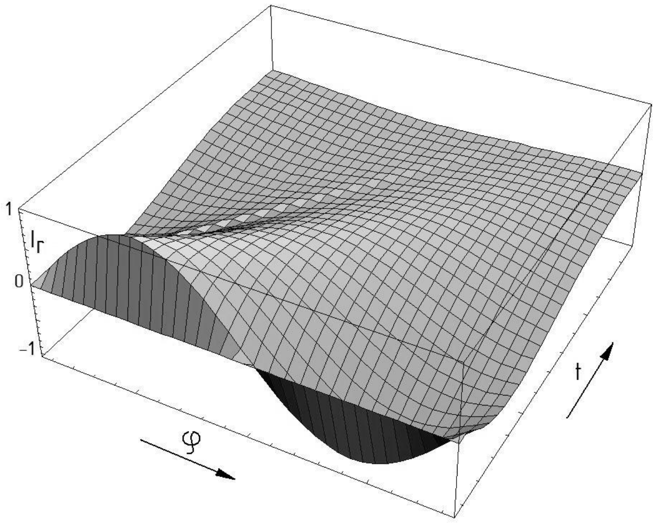

Fig. 1. The distribution of the induction currents of influence rotor function.

The transverse sections of the graph shown in Fig. 1 are sinusoidal waves. Their range decays according to exponential law. They have a phase change, proportional to the current time. In general the solution is the package of influence functions of ordinary exponential form broached along inclined lines, which are called “characteristics”, and the influence function itself is the multiplication of two functions. One of these functions depends on time and the other function depends on the angular deflection. The latter feature allows the performance of numerical evaluation of the transient response through the interplay of cyclic (according to the spatial value) and linear (according to the temporal value) convolution.

The system of the ring currents, occurring in rotor, under the affection of unit impulse of magnetomotive force with sine-shaped spatial distribution changes its position relatively to stator winding as rotor rotates (Fig. 2). The ring currents as Ir1, induced at zero-time, with their plane squared with the stator magnetic flow direction, do not generate the torque. At the same time to the currents as Ir2, that were induced earlier, due to the rotor rotation the plane occurred being turned through α toward the stator flow. The torque is being produced. The addition of all the common currents along the whole rotor surface, taking into consideration the position angle α toward the stator flow, determines instantaneous value of active and reactive rotor currents.

According to Fig. 2, the same element of the ring current while rotor rotates about stator can determine active and (or) reactive flow component. If the damping time of the influence function is short in comparison with the transportation time (ω1 curve), the reactive component has the advantage. When the motion seed increases (ω2 curve), the value of the active component increases as well. In case of the influence function value being relatively high for the given speed and α angle being more than π, the active component decreases as a result of the sinus sign reversal (ω3 curve). The behavior of the active and reactive components is defined by the ratio of rotor characteristic time TR and rotor currents transportation time TT on the arc length of polar pitch l.

The application of bidimensional convolution allows the constructing of the transient function to the stepwise variation of the rotor linear speed in the induction motor. The start value of rotor speed and rotor current are taken equal to zero, and the stator magnetic flow being stationary in bogey value (rotor is preliminary magnetized). The transient response in the given case corresponds to the regenerating of the induction motor. The bivariate distribution of the transient functions system of the induction rotor currents that takes place in the case of steady rotor rotation and the action of the stator currents magnetomotive force described in (7) is shown in Fig.3.

According to the obtained graphs, steady rotor currents division is sine-shaped at any moment, however, there is an offset of maximum sine value relative to the maximum stator current distribution towards the rotation, that obviously leads to the decelerating torque occurrence. The transient response time corresponds to the characteristic time of leakage currents of rotor winding.

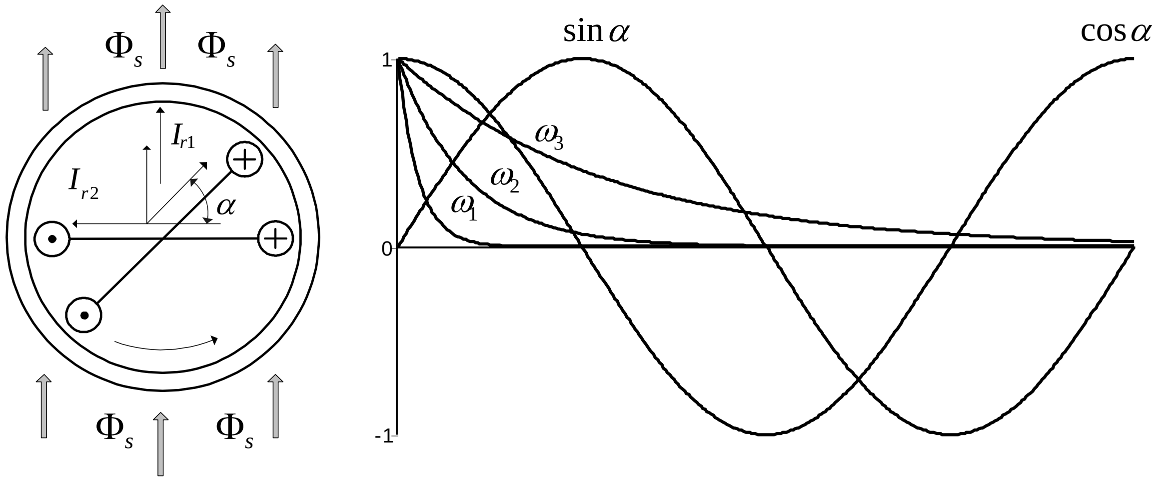

Fig.. 2. The production of torque in the induction motor rotor while induction currents are transported.

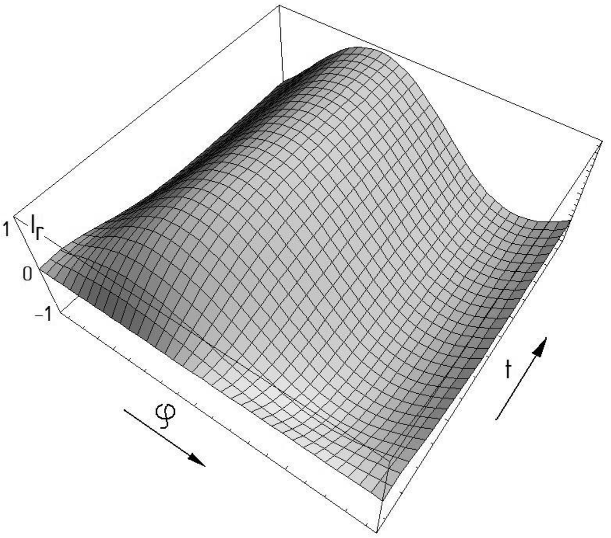

Fig. 3. The transient response of the variance of the induction currents distribution in the induction motor rotor under discontinuous variation of rotation speed.

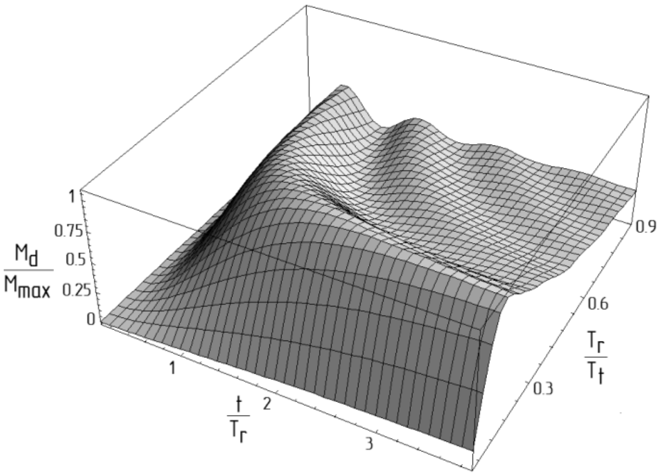

Fig. 4. The analytical array of transient response of the decelerating torque for different ratios of rotor characteristic time T_R and transportation time T_T.

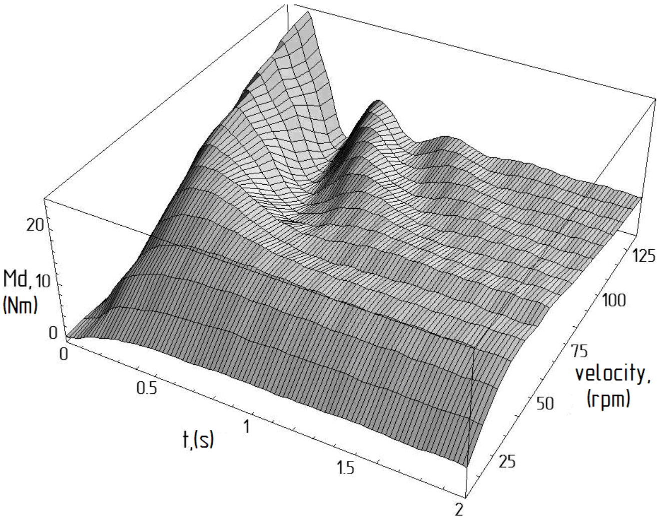

Fig. 5. The transient responses according to the decelerating torque for the АИР100L4У3 motor.

As the rotor influence function depends significantly on the rotor motion speed relative to the stator current wave, the transient response dynamic has the sophisticated behavior. The analytical array of variables of transient function according to the electromagnetic torque depending on the rotation speed is shown in Fig. 4.

The induction motor speed-load curve correspondence of the rotation frequency was examined on the laboratory bench, that consists of two АИР100L4У3 induction motors with the capacity of 4 kW, connected with each other by torque-measuring device of torsion type and by the frequency converter IntDrive. The direct current of 8 A was set up in the test motor stator winding. For the measurements of transient responses according to the torque the start of the second motor was performed on the given rotation frequency. The array of values of the transient responses is shown in Fig. 5.

The comparison of Fig. 4 and Fig. 5 shows that the given characteristic curves have affined behaviors. Greater increase of the animated range for the actual motor at the larger field sliding motion is determined, apparently, by the behavior of the IntDrive converter when operated at the given range.

It is known that the application of the term of the transient function in the operator form to the linear nonstationary systems is impossible, so the rotor field observer can not be constructed on the basis of standard dynamic elements. However the examination of the above-mentioned equations demonstrates that the range of effective control actions from the stator onto the rotor resides in some range of angles relatively to the instantaneous value stator current vector. At that the position of orthogonal rotor current component is defined by the value of bidimensional cyclic convolution of the difference of rotor rotational motion angular reference to the angle of the stator current vector on the time interval rotor influence function. As vector instantaneous value stator current can be defined with the help of lightning-current sensors and the rotor angular location is simply determined with the help of rotor position sensor, there is an opportunity to determine the direction and magnitude of rotor magnetic flow vector. From the formal point of view, for construction of the distributed system observer it is necessary to save the information about all the points inside the spatial and time coordinates. However, using the prerequisite for harmonicity of the space distribution rotor current, the problem of the observer construction can be traced to the vector element consisting of two aperiodic links, supplemented with two vector rotation blocks.

Induction current system that occurs in rotor under the unit impulse action of the magnetomotive force with the sine-shaped spatial distribution for any time moment can be described by the vector. Its magnitude is proportional to the magnetic flow value, produced by the whole current system from the given impulse. The phase angle varies in the course of time in accordance to the rotor position, stator current vector magnitude and phase. The addition of these vectors that occurred from the unit impulse on the time interval of current damping considering the spatial shift provides the full magnetic flow vector value, providing by the rotor currents.

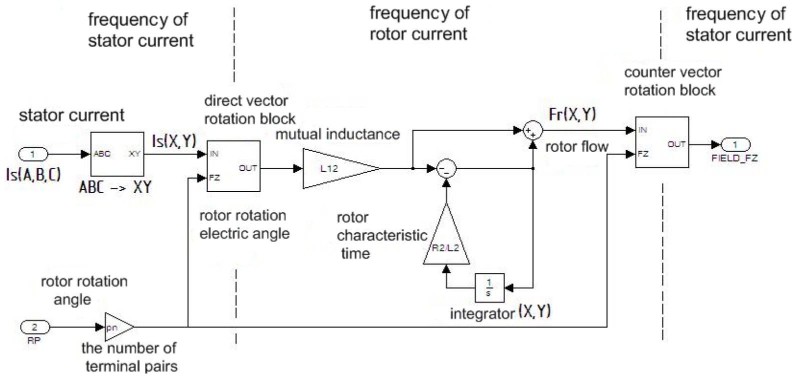

Fig. 6. The block scheme of the observer with diamond-knurled connections.

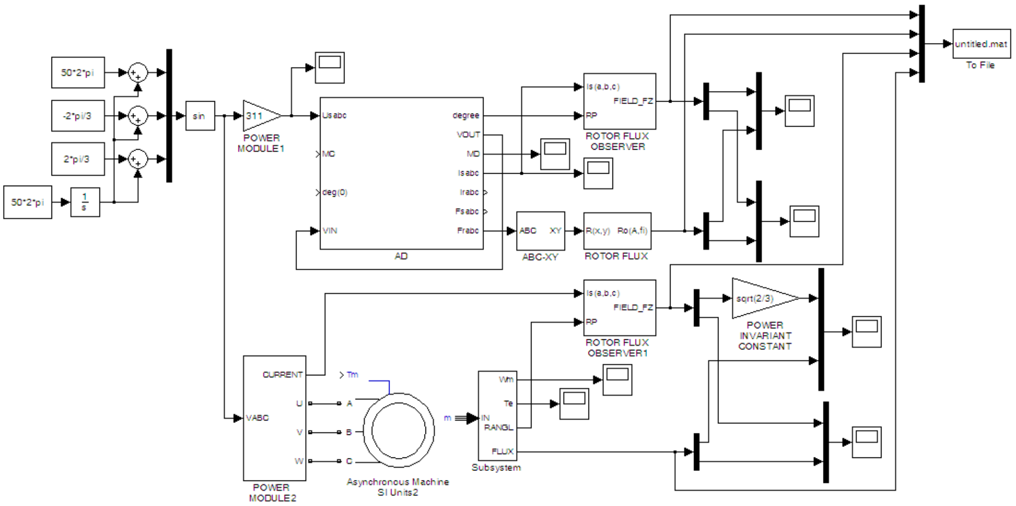

Fig. 7. The model for comparison the rotor flow observer functioning and the physical rotor flow.

The values of the difference of the rotation angles between the instantaneous values of stator current vector and the last values of stator current vector and the rotor rotation angle in the time interval $(-\infty -$ t) are necessary for calculating. The observer performs the addition of these vectors in the infinite time interval. However, only those values are significant that belong to the time interval taking priority of 3-4 characteristic rotor time. Instead of shift of the array of spatial variable in this case the rotation of two-component vector with the following convolution in each component with the influence function element corresponding to the rotor characteristic time is used. The vector gained by the integrating in this function, rotates towards the stator current vector, thus to its reduction to the coordinate frame that is stable toward the stator current, the block of counter-rotation (Fig. 6).

The initiated observer model was tested by simulation of the induction motor direct start in software system Matlab. The testing was carried out on two types of induction motor models- the user model and the standard model from Matlab packet (Fig. 7). The active X_R and reactive Y_R vectors components, calculated by the observer model, were the subject for the comparison. The graphs of the transient responses are shown in Fig.8.

The result compliance for the user model was absolute. For the standard motor model, the modules of the model flows and the observer were different in sqrt(2/3) times. The given coefficient is explained by the necessity of using normalizing constants for providing invariability in capability of two and three-phase motor models. The rotating direction of the model rotor flow and observer were opposite, but the angular deflection was the same at any time. The values of the active and reactive components of the models rotor current were marked with the solid graph, values, gained with the help of the observer were marked with bubbles and triangles.

II. ESTIMATION OF RESULTS

In summary, the application of induction motor rotor flow for the construction of the effective controlling systems can be achieved by the usage of stator current sensors and rotor position sensors. As in transmissions, designed for high-accuracy application, generally, position sensors on motor shaft are used, the solution of the problems of motor torque control is simplified significantly.

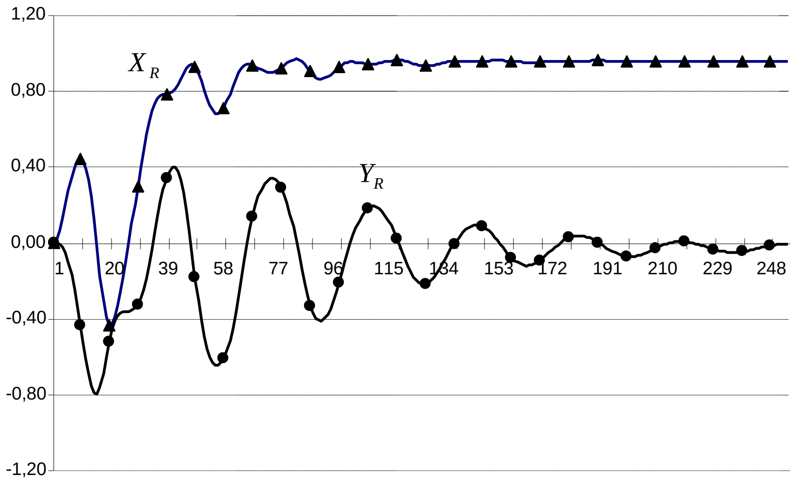

Fig. 8. The variance of active and reactive components of the rotor magnetic flow while simulating the transient response of the induction motor direct start.

The application of such observer provides the opportunity of the induction motor torque control without the use of transformation into the fixed coordinates system and back to the system, rotating with the frequency of the stator field.

In spite of the fact that in the number of projects it is mentioned that the application of the induction motor rotor flow torque control can lead to the characteristics similar to those of the direct-current motor. The occurrence of instability of influence functions leads to the change of the dynamic characteristics while transient response takes place in the motor even under the condition of the linearity of the system in general. The compensation of these changes can be carried out by the use of relay regulators. The alternative for the use of on-off modes can be the application of the adaptive regulation taking into account the instability dynamic of induction motor.

III. CONCLUSIONS

It is shown that the occurrence of the torsion torque in the induction motor is defined by the distance-velocity lag damping ring currents in rotor relatively to the present position of the stator magnetic flow.

Dirak induction motor rotor response considerably depends on the transportation speed, changing from the aperiodical to the fluctuating one. The value of critical moment for speed-torque characteristic of the motor corresponds to the value 0,15 T_R / T_T.

The simulation model of the induction rotor magnetic flow observer is initiated, based on the stator current sensors and rotor position sensors. It takes into consideration the instability of the measured object.

REFERENCES

[1] Asynchronous drives with vector control./Rudakov V.V., Stolyarov I.M., Dartay V.A.-L.:Energoatomizdat, Leningrad department, 1987.
[2] Blaschke F. Das Prinzip der Feldorientierung die Grundlage fuer die Transvektor-Regelung von Drehfeldmaschinen // Siemens Zeitschrift.-1971.-№45.-H10.
[3] Voronov A.A. The foundations of the theory of automated operation: Special linear and non-linear systems.-2 publication. M.: Energoatomizdat,1981.
[4] Butkovskii A.G.The structural theory of the distributed systems.-“Science”,M.:1977.
[5] Digital signal processing: Reference book/L.M. Goldenberg, B.D. Matyushkin, M.N. Polyak.-M.:Radio and coupling, 1985.
[6] Shreiner R.T. The mathematic simulation of alternating-current drives with semiconductor frequency converters. Ekaterinburg. YRO RAN, 2000.

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