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Effects of Non-smooth Phenomena on the Dynamics of DC-DC Converters

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***Abstract* – This paper provides the analysis of nonlinear phenomena in switch-mode power converters. In distinction to majority of known researches this paper presents novelty approach, allowing the complete bifurcation analysis, considering stable and various types of unstable behavior of nonlinear systems. Main results are illustrated on one of the most widely used switching converters – current controlled boost converter, for which the complete one-parametric bifurcation diagrams are constructed. The results include the detection of various types of rare attractors, smooth bifurcations and non-smooth phenomena, specific to piecewise linear dynamical systems.**

Keywords – Bifurcations, border collisions, chaos, numerical continuation, power converters, rare attractors, subharmonics.

I. Introduction

In the last few decades studies of complex phenomena in switching power converters have gained increasingly more attention from both the academic researchers and engineers. Various kinds of nonlinear phenomena, including bifurcations, subharmonic operation and chaos have been revealed [1] - [4]. These complex types of behavior, implying instability, are observed as one or more system parameters are changed, or in the presence of unavoidable noise even if all circuit parameters are fixed.

Thus it has been proved, that the profound study and analysis of nonlinear phenomena in power electronics are profitable and even mandatory, as studying bifurcation and chaos can help engineers to understand the change of behavior in power electronics when some parameters are varied. Furthermore, a complete knowledge about the domains of subharmonic operation and chaos in the parameter space is of particular importance for the power electronics engineers as they must choose the parameter values in order to obtain the desirable behavior. Moreover, the engineers will consciously avoid chaotic and subharmonic domains if the causes and circumstances of the occurrence of nonlinear phenomena are thoroughly understood.

In order to study the dynamics of the converter as one or more parameters are changed, the bifurcation diagrams are constructed. In general, the construction of bifurcation diagrams demands the derivation of discrete iterative maps (IM). The common method for construction of bifurcation diagrams, based on the IM is called a brute-force approach, and corresponding diagrams are referred as brute-force (or Monte Carlo) bifurcation diagrams [5].

The brute-force approach could be simply implemented and has the advantage of capturing most (but not all) of the long time dynamics, but has the disadvantage of not being able to capture unstable orbits (playing an important role in the global bifurcational analysis of any nonlinear system) and narrow stable regions (rare attractors [6], [7]). Thus the brute-force approach gives incomplete information about the possible types of nonlinear phenomena in the system under test.

In order to overcome the mentioned drawbacks and locate the regions of stable subharmonic operation and unstable periodic orbits (in many cases leading to small stable fragments) it is necessary to use the numerical path-following technique ([6]-[8]).

The key idea of numerical path-following is to detect all possible stable and unstable periodic orbits (up to specific order) for defined parameter values. The obtained results are used as starting points for the computing of sequence of points at small intervals along the solution curve.

In this paper it is shown how the mentioned path-following technique could be applied to the investigation of nonlinear phenomena in switch-mode DC-DC boost converter. The complete bifurcation diagrams are constructed using special software and results are verified by means of SIMULINK SymPowerSys model of the mentioned converter.

II. The Discrete-Time model of Current Mode Controlled Boost Converter

The schematic diagram of the conventional current controlled boost converter operating in continuous conductions mode (CCM - when the inductor current never falls to zero) with a feedback path comprising a comparator and a RS flip-flop is shown in the Fig.1.

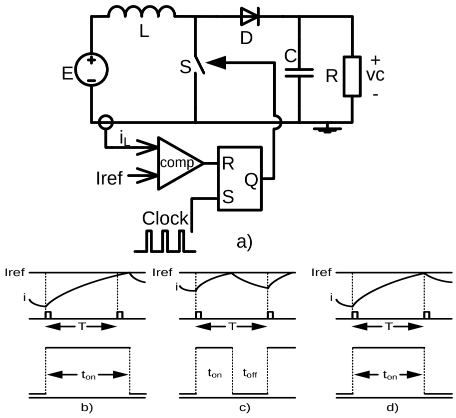

Fig. 1. (a) The current programmed boost converter. (b)-(d) Typical waveforms of inductor current and the control signal for the switch.

The comparator compares the inductor current with a definite reference value I_{ref} to control the state of switching element *S*. Thus the circuit could be described by two different topologies, depending on whether the switch is open (*OFF*) of closed (*ON*). When the *S* is in the *ON* position, the diode *D* is reverse biased and practically non-conducting. Neglecting the resistance of inductor *L*, the current i_L rises almost linearly (see Fig. 1(b)) and the energy is stored in the magnetic field of the inductor. S is opened, when the inductor current reaches the I_{ref} level, then the voltage Ldi_L/dt is induced in the inductor in order to maintain the current flow. This forward biases the diode and the current decays also almost linearly, accompanied by the transfer of energy from the inductor to the capacitor *C*. The switch is closed each time the clock pulse with period *T* arrives.

Nevertheless the operation of boost converter could be described by the system of differential equations, it is now a common practice to use the discrete time models, which provide the tool for the direct construction of bifurcation diagrams as well as analytical investigation of various types of nonlinear phenomena.

The discrete time model of the boost converter is the function that relates the voltage and the current vector at one instant $(i_{(n+1)}, v_{(n+1)})$ to the vector at previous instant $(i_{(n)}, v_{(n)})$; the instances here are defined by the arrival of clock pulse. The boost converter is unusual in that the discrete-time model can be derived in closed form without approximations. Thus, the investigation is based on the precise discrete time model of the mentioned converter, provided by Banerjee [2]. It could be deduced that there are two ways in which the state can move from one clock instant to the next. A clock pulse may arrive before the inductor current reaches the I_{ref} (Fig.1(b)). In that case, the obtained discrete time model is the following:

$$i_{n+1} = I_{ref}\rho T_0/2\varsigma + i_n;$$
$$v_{n+1} = v_n e^{-\gamma}, \tag{1}$$

where the variables are normalized, using following definitions:

$$\varsigma = \sqrt{L/C}/2R;\ T_0 = T/\sqrt{LC};\ \rho = V_{in}/I_{ref}R;\ \omega = \sqrt{1-\varsigma^2};$$
$$\gamma = T/RC.$$

On the other hand if the inductor current reaches I_{ref} before the arrival of the next clock pulse (Fig.1(c)), the map would include the *ON* and *OFF* period. Thus the map is as follows:

$$i_{n+1} = (e^{-\varsigma\tau'_n}(C_1\cos(\omega\tau'_n) + C_2\sin(\omega\tau'_n)) + \rho)I_{ref}$$
$$v_{n+1} = (\rho - e^{-\varsigma\tau'_n}(K_1\cos(\omega\tau'_n) + K_2\sin(\omega\tau'_n)))I_{ref}R, \tag{2}$$

where

$$\tau_n = (2\varsigma/\rho)(1 - i_n/I_{ref});\ V_f = (v_n/I_{ref}R)e^{-2\varsigma\tau_n};$$
$$\tau'_n = T_0 - \tau_n;\ C_1 = 1-\rho;\ C_2 = (\rho - V_f)/2\varsigma\omega + C_1\varsigma/\omega;$$
$$K_1 = \rho - V_f;\ K_2 = (2\varsigma/\omega)((V_f - \rho)/2 - C_1).$$

The borderline between these two modes is given by the value I_{border} for which the inductor current reaches the reference value exactly at the arrival of the next clock pulse (Fig.1(d)):

$$I_{border} = I_{ref}(1 - \rho T_0/2\varsigma) \tag{3}$$

The model presented in this section is used in order to obtain the complete bifurcation diagrams and provide all necessary investigation of phenomena, observed in the current controlled DC-DC boost converter.

III. The Complete Bifurcation Analysis

The iterative mapping presented in the previous section can be used to obtain the evolution of state variables starting from any initial conditions. The discrete time model enables one to avoid numerical computation of the phase space orbit from the continuous time model. The map is therefore very useful in quick computation of the system behavior in the defined parameter range.

In order to show the main advantages of the complete bifurcation diagrams, the brute-force diagram based on the iterative mappings (1) and (2) is computed first (the reference current is chosen as the bifurcation parameter).

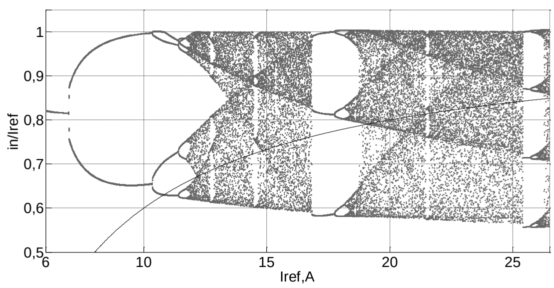

Fig. 2. The brute force bifurcation diagram. Parameters of the boost converter: *R=10(Ω); L=1.5(mH); C=20(μF); T=200(μS); Vin=30(V); Iref=var*. Black line represents the I_{border} (see(3)).

As it could be seen from the Fig.2., the obtained bifurcation diagram shows the main period-doubling route to chaos, as we increase the value of reference current. This diagram also depicts several periodic windows. However, this brute – force diagram is not capable of pointing out precisely the values of parameters at which different bifurcations occur and define the regions of existence of small periodic windows. Thus, in order to understand the dynamics of the system as the reference current is changed, the complete bifurcation diagrams are constructed and presented for various system parameters.

The first step in obtaining the complete bifurcation diagram is the construction of periodic skeleton [9] – all stable and unstable periodic solutions (up to definite order) are found for fixed parameter values. The points in periodic skeleton are characterized by their coordinates $(v_{(n)}, i_{(n)})$ and characteristic multipliers, defining the stability of found regimes. The fixed points from the periodic skeleton are used as starting points in the numerical path-following, constructing the branches of complete bifurcation diagrams.

Fig.3 represents the complete bifurcation diagram for parameter values defined in the appropriate caption of the figure.

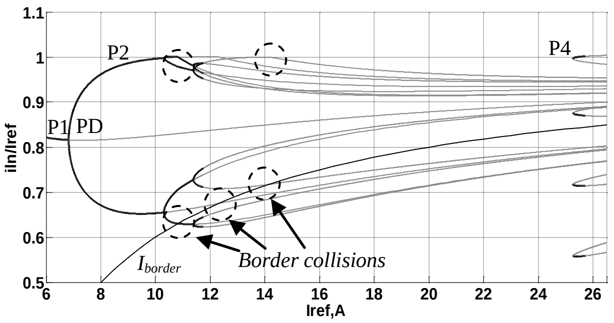

Fig. 3. Bifurcation diagram for normalized inductor current; depicting the period doubling branch, period-4 and period-8 regimes. Parameters of the boost converter: *R=10(Ω); L=1.5(mH); C=20(μF); T=200(μS); Vin=30(V); Iref=var*. Black lines represent stable orbits, grey lines – unstable orbits and thick black line represents the I_{border} (see(3)). It could be seen how the shape of branches changes as they cross the borderline (border collision points [5]).

We begin with relatively small values of I_{ref} =6(A), and the bifurcation diagram shows a stable period-1 operation. This is usual operation and the only practically acceptable operation for switch – mode power converters.

When the reference current is further increased, the period-1 operation is no longer possible and when the I_{ref} reaches the definite value (~6.9(A)), the period of operation doubles itself. The periodic operation with period 2T is clearly seen from the obtained complete bifurcation diagram. Moreover, the further increase of reference current, leads directly to the chaotic operation through the period-doubling cascades. It could be observed that as stable or unstable branches of the diagram collides with the border (physically it means, that the inductor current reaches the reference current exactly at the arrival of the CLOCK pulse (see Fig.1(d))), the branches change their shape, however this process does not effect the main dynamical route of the system.

To probe the results we alter some circuit parameters in order to observe how the period – doubling and the border collision interact with each other. In varying circuit parameters, there are several rules that need to be observed. First, as we wish to keep the operation in CCM which is defining condition in this study, the inductance and load resistance should be kept within a certain range but the *C* value moreover may be varied over a wide range, without affecting the operating mode.

The complete bifurcation diagram for another parameter values is shown in the Fig.4.

Based on the obtained bifurcation diagrams it is possible to summarize observations as follows:

- The boost converter looses stability via period - doubling bifurcation and may (or may not (see Fig.4)) continue to double its period. This is an indication of the possible occurrence of a standard period-doubling cascade observed in smooth systems.
- The collision with the border I_{border} may lead to the change of the shape of stable or unstable branches of bifurcation diagram (as it is shown in the Fig.3). In this case the occurrence of chaos is caused by period-doubling cascade.

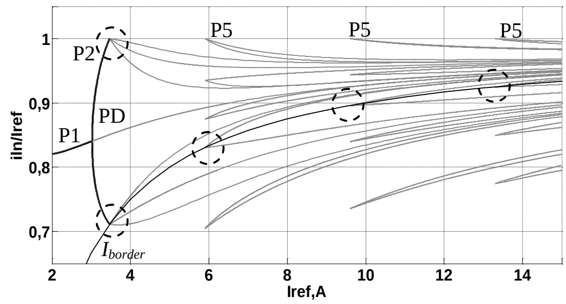

Fig. 4. Bifurcation diagram for normalized inductor current; depicting the period doubling branch, period-4 and period-5 regimes. Parameters of the boost converter: *R=40(Ω); L=1.5(mH); C=10(μF); T=50(μS); Vin=30(V); Iref=var*. Black lines represent stable orbits, grey lines – unstable orbits and thick black line represents the I_{border} . It could be seen that the period-doubling route to chaos is abruptly interrupted by border collision - no stable periodic orbits appear and the robust chaotic operation is observed.

- The border collision may lead to the abrupt interruption of the period – doubling cascade and after this the system directly exhibits chaos.

IV. Simulation of Boost converter

This section provides the verification of found phenomena in current controlled boost converter by means of SIMULINK software, as the analysis presented in the previous section was provided on the basis of discrete time model (not including any parasitic parameters of real elements) and numerical computations.

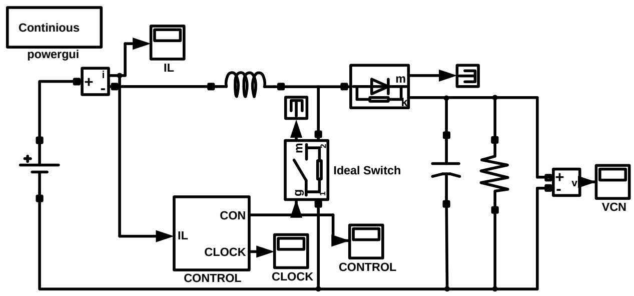

Fig. 5. The PSB schematic diagram of the boost converter

The model of the boost converter (see Fig.5) was created using the software package called Power System Blockset (PSB), which offers a competitive alternative for circuit diagram based simulation and evaluation of power electronic applications. PSB libraries provide a variety of models for power electronic devices and control blocks, allowing fast simulation model development. Since PSB is compatible with the Matlab/Simulink, the control system implemented using standard Simulink blocks can be easily integrated in the same circuit diagram. In addition variable-step integration algorithms used in the Matlab/Simulink environment allow very accurate simulation of nonlinear, stiff and discontinuous dynamic systems.

Thus the PSB allowed creating the simulation model of the boost DC-DC converter, based on the precise circuit, composed from the real element models, including the most distinctive parasitic parameters of inductors, capacitors and switching elements.

In order to verify previously obtained results, fixed parameters, shown in the caption of the Fig. 3, were used in the PSB model and, changing the value of I_{ref} ,waveforms of inductor current and equivalent phase portraits are obtained for different regimes, predicted by means of complete bifurcation diagram in the Fig.3. Corresponding regimes are shown in the Fig.6.

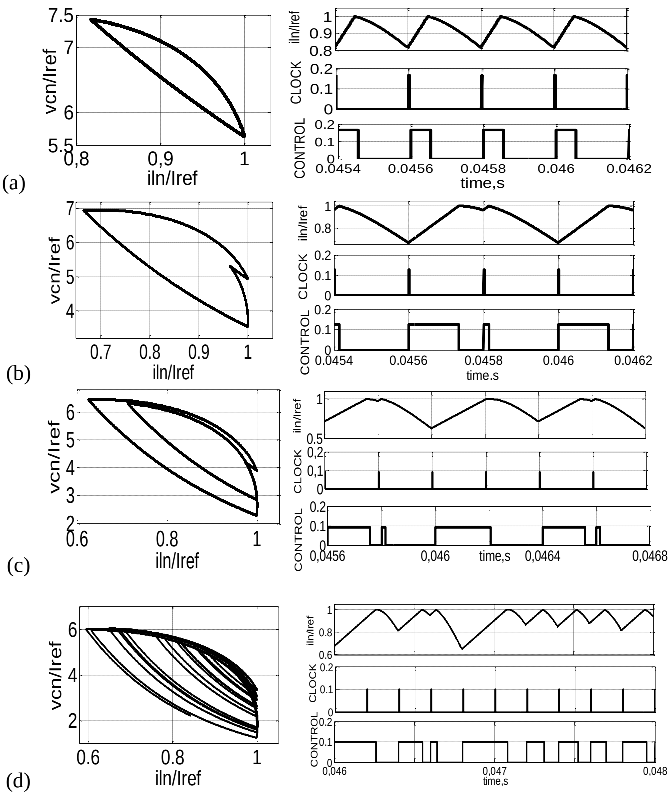

Fig. 6. Phase portraits and waveforms of inductor current, switching signal with corresponding CLOCK pulses for different periodic regimes. Parameters of the boost converter: *R=10(Ω); L=1.5(mH); C=20(μF); T=200(μS); Vin=30(V).* (a) Period-1 operation for $I_{ref}=6(A)$; (b) Period-2 operation for $I_{ref}=8(A)$; (c) Period-4 operation for $I_{ref}=11(A)$; (d) Chaotic operation for $I_{ref}=16(A)$. The values of I_{ref} correspond to those in the Fig.3.

The numerical results obtained by means of simulations in SIMULINK prove that there exists a period doubling route to chaos, as the value of reference current is incremented. The values at which the period – doublings appear and the regions of existence of these regimes exactly correspond to those, seen in the complete bifurcation diagram in the Fig.3. Thus it could be concluded, that the results obtained on the basis of discrete-time model using numerical path – following technique are very precise and the addition of real element nonidealities does not affect the global dynamics of the current controlled boost converter drastically, causing just insignificant shifts of the bifurcation points.

V. Conclusions

This paper shows the possibilities of applicability of novel approach for the global nonlinear analysis of dynamical systems, based on numerical path-following, to the widely used circuits – switch–mode DC-DC converters. Main results are illustrated on the current controlled boost converter, for which the complete one-parametric bifurcation diagrams are constructed for various parameters and the possible interaction of common period-doubling cascades and non-smooth border-collision phenomena are shown. The discrete time model is used instead of usual systems of differential equations, as this model enables one to avoid numerical computation of the phase space orbit from the continuous time model, thus being useful in quick computation of the system behavior in the defined parameter range. The numerically obtained results are verified by means of SIMULINK environment.

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He has been involved in several research projects on examination of spread spectrum technique for switching converters, improvement of parameters of DC-DC converters operating in subharmonic and chaotic regimes, investigation of complex dynamics of switching systems described by discrete-time models.

His research interests include spread spectrum techniques applied to switching converters, nonlinear control of power electronic converters, nonlinear dynamics of switching circuits and non-smooth phenomena, application of path-following technique to the investigation of global dynamics of systems with different kind of nonlinearities.