The Complete Bifurcation Analysis of Boost DC-DC Converter

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Abstract. **This paper is concerned with the problem of full bifurcation analysis in piecewise-smooth systems with controllable switching. Models of this kind of systems are widely used in engineering practice, in particular in power electronic converters. In distinction to majority of known methods this paper presents novelty approach, allowing the complete bifurcation analysis. Exact analytical methods for the search of periodic regimes and their stability estimation are used along with various numerical computation techniques. Main results are illustrated on one of the simplest forms of switching converters – step-up (boost) converter, for which the complete one-parameter bifurcation diagrams and two-parameter diagrams are constructed. The results include – the detection of various types of rare attractors, smooth and non-smooth events and the investigation of different principles of birth of chaotic attractors.**

Keywords: **Bifurcations, border collisions, chaos, numerical continuation, power converters, rare attractors, subharmonics.**

I. INTRODUCTION

It has been shown in the several last decades that DC-DC converters, being time-varying nonlinear dynamical systems, are capable of exhibiting several periodic steady state responses as well as chaotic behavior [1]-[4]. The proper design of the DC-DC converters assumes that the dependences of all possible steady state responses on variation of different converter parameters are known in advance. Thus, it is possible to avoid the occurrence of dangerous properties during operation – like increased output voltage ripple, audible noise or even the damage of power switching elements. Therefore, a complete knowledge about the domains of subharmonic operation and chaos in the parameter space is of particular importance for the power electronics designers as it is necessary to choose parameter values in order to avoid undesirable and even dangerous regimes of operation.

The main widely used tool for nonlinear analysis that shows the equilibrium solutions as a function of one (or several) parameters is called a bifurcation diagram (map). In order to generate this diagram, the derivation of a discrete iterative map is needed. In literature, the bifurcation diagrams for this map are generally created with a brute force approach [5]. This approach could be implemented by means of trivial code and has the advantage of capturing most (but not all) of the long time dynamics, but has the disadvantage of not being able to capture some features playing an important role in the global analysis of the systems, such as unstable orbits and narrow stable regions (also called rare attractors [6]). Thus, the brute-force approach gives incomplete information about the possible types of nonlinear phenomena in the system.

In contrast to majority of known researches we use the improvement of innovative approach, allowing the complete bifurcation analysis, considering stable and unstable orbits, called Method of Complete Bifurcation Groups (MCBG)[6-9]. The MCBG allows finding some important unknown regular and chaotic attractors and new bifurcation groups in the model under consideration, exploring complex dynamics of systems with one and several degrees of freedom.

This paper contains the first (to the author's knowledge) application of MCBG for the analysis of nonlinear dynamics of one of the most widespread switching power DC-DC converters – boost converter under voltage mode control, operating in discontinuous current mode (DCM).

The objectives of this paper are two-fold. To designers of switch mode power supplies, some commonly seen, but often ignored nonlinear phenomena exhibited by switching regulators, are shown. To circuit system theorists, different kinds of transition to subharmonic and chaotic operating regimes are demonstrated. Nevertheless, the investigation provided in this paper focuses on the particular example – boost converter, general results regarding structural instability of the system and different types of bifurcations uncommon to smooth systems, hold for the wide class of switching converter circuits under different control strategies.

II. THE DISCRETE-TIME MODEL OF VOLTAGE MODE CONTROLLED BOOST CONVERTER OPERATING IN DCM

The schematic diagram of the conventional voltage mode controlled boost converter operating in discontinuous conduction mode with a feedback path comprising a comparator without latch is shown in the Fig.1.

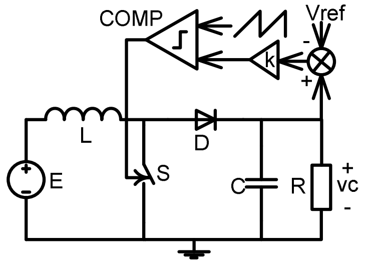

Fig. 1. Simplified schematics of voltage mode controlled boost converter.

As is common to all power electronic systems, the boost switching regulator is a multi-structural system with its circuit topology varied according to the states of the switches. Normally the switch and the diode are turned *ON* and *OFF* in the complementary fashion. The inductor current in this case never reaches zero and the converter is operating in the continuous conduction mode (CCM). However, if the value of inductance L is relatively small or the switching period T is relatively long, the inductance current may reach zero value before the beginning of the next cycle. In this case there exist time interval during which both diode *D* and switch *S* are not conducting – this is referred to as discontinuous conduction mode (DCM). The critical value of *L*, which should not be exceeded in order to operate in DCM, could be defined as:

$$L_{crit} = D(1-D)^2 RT/2 \qquad (1)$$

Although the switched model is the most accurate description of the switch-mode DC-DC converter, it does not allow analysis to be performed in a conventional fashion. Thus, C.K. Tse [4] has derived the more desirable model for the boost converter operating in DCM – the unimodal (having one maximum or one minimum) iterative map, which provides the tool for the direct investigation of various types of smooth and non-smooth nonlinear phenomena:

$$v_{C,n+1}=(av_{C,n}(v_{C,n}-E)+E^2b(H(D-k(v_{C,n}-V_{REF})))^2)/(v_{C,n}-E) \qquad (2)$$
$$D=\sqrt{((1-a)(V_{REF}-E)V_{REF}/bE^2)},$$

where $a=1-T/R/C+T^2/2/C^2/R^2$, $b=T^2/2/L/C$, D – steady-state duty cycle; $H(.)$ – function, limiting the range of duty cycle between 0 and 1; V_{REF} – reference voltage; k – small signal gain; T – switching period.

As it could be seen, the discrete time model of the boost converter is the function that relates the capacitor voltage vector at one instant $(v_{(n+1)})$ to the vector at previous instant $(v_{(n)})$ (as the converter is operating in DCM, the inductor current takes zero value for a finite sub-interval of time and does not act as a state variable in this discrete – time model).

III. THE CONCEPTS OF MCBG

The Method of Complete Bifurcation Groups, developed at Riga Technical University in a group under guidance of professor M.V. Zakrzhevsky [6], includes the complex of approaches used for the analysis of dynamic systems, which implies the following procedures: for fixed parameters of the system – detection of all stable and unstable periodic orbits and bifurcation subgroups with unstable periodic infinitiums (UPI), on plane of states – construction of regimes' basins of attraction; for varied system parameters – constructing of one parameter bifurcation diagrams and bifurcation maps. The continuation along a solution branch of definite regime (not parameter) is of special importance in the MCBG. This approach allows finding new stable regimes that have not been known before, in broadly used dynamical models of strongly nonlinear oscillatory systems.

One-parameter bifurcation analysis within the MCBG includes following procedures:

- The construction of periodic skeleton: all stable and unstable periodic solutions at fixed value of bifurcation parameter are found, the number and types of bifurcation groups are detected;
- On the basis of obtained periodic skeleton, using the method of numerical continuation, the branches of bifurcation diagram, corresponding to each found group are constructed. In many cases, on the branches of diagram corresponding to unstable regimes, the greater or smaller stable fragments are found (rare attractors).
- The complete bifurcation diagram is composed, including stability estimation procedure, defining appropriate monodromy matrixes for all found regimes and determining their multipliers.
- The revision of the diagram allows detection of different types of RA and UPI (unstable periodic infinitiums – the area in which only unstable periodic orbits of definite bifurcation group exist [6]).

The described procedure will be applied to the model (2) and appropriate bifurcation diagrams for boost DC-DC converter

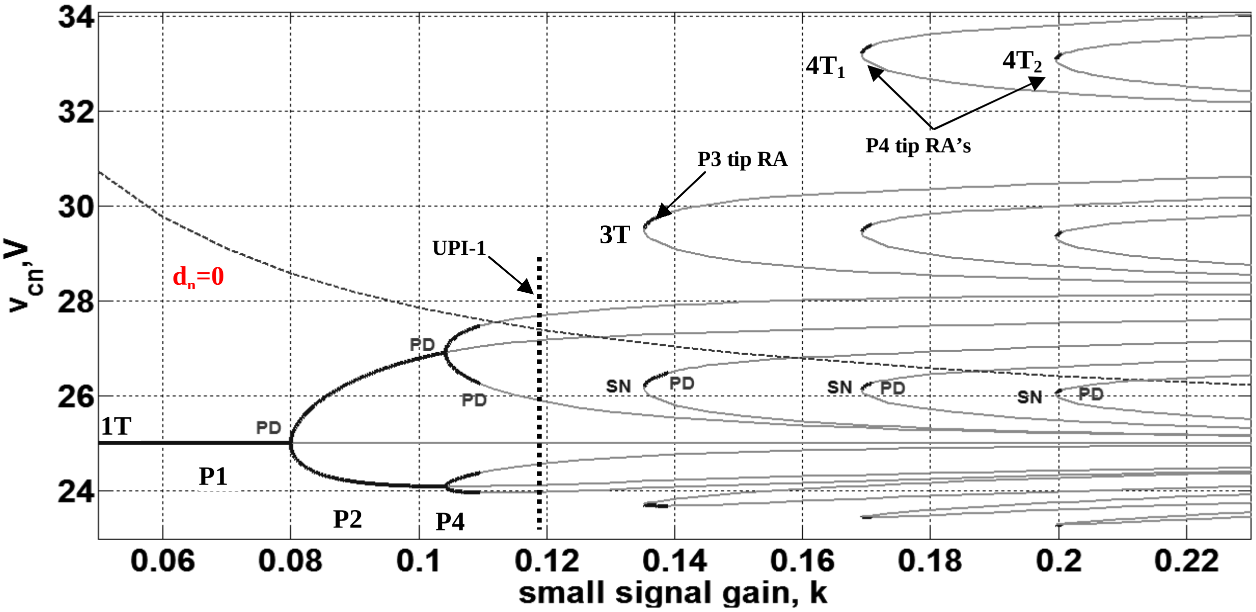

Fig. 2. Bifurcation diagram for boost converter with following parameters: $R=12.5(\Omega)$, $L=208(\mu H$, $C=222(\mu F)$, $T=333(\mu S)$, $Vref=25$, $E=16(V)$, $k=var$, depicting 1T, 3T and two 4T bifurcation groups. Solid bold dark lines represent stable orbits, solid light lines – unstable orbits and thick dashed lines represents border, when duty cycle $d_n=0$. PD- states for period-doubling bifurcation, SN- saddle-node bifurcation, RA- rare attractor, UPI- unstable periodic infinitium.

will be constructed in the following sections.

IV. THE COMPLETE ONE-PARAMETER BIFURCATION DIAGRAMS

The discrete-time model provided in section II can be used to obtain the evolution of state variables starting from any initial conditions, avoiding numerical computation of the phase space orbit from the continuous time model. The map is therefore very useful in quick computation of the system behavior in the defined parameter range.

As it has been already mentioned in the previous section, the first step in obtaining the complete one-parameter bifurcation diagram is the construction of periodic skeleton, when all stable and unstable periodic solutions (up to definite order) are found for fixed parameter values. The periodic skeleton for the boost converter was constructed for k=*0.23*. The points in periodic skeleton are characterized by their coordinates $(v_{(n)})$ and characteristic multipliers, defining the stability of found regimes. The fixed points from the periodic skeleton were used as starting points in the numerical path-following, constructing the branches of complete bifurcation diagrams.

Fig.2 represents the complete bifurcation diagram for parameter values defined in the appropriate caption of the figure. It should be noted that for the sake of simplicity this diagram represents just several bifurcation groups- 1T, 2T, T3 $4T_1$ and $4T_2$.

As it was already mentioned, the (2) represents the typical example of unimodal map. The common route to chaos for this kind of maps is the following:

- initially the bifurcation parameter is chosen in the region, where system exhibits stable period-1 operation;
- then the value of bifurcation parameter is increased and we observe that the system loses stability via period doubling bifurcation;
- if the value of parameter is further increased, the typical subharmonic cascade and chaotic mode of operation is observed.

The described sequence of events could be observed for the bifurcation group 1T. The bifurcation diagram shows the transition from P1 to P2 and P4 orbits through period doubling bifurcations. The period doubling cascade evolves and finally for $k=0.12$ the UPI-1 could be observed. As the diagram does not show any stable bifurcation groups coexisting with UPI-1, it could be concluded that at this point the boost converter is operating in chaotic (aperiodic regime).

The obtained diagram also shows one 3T and two 4T bifurcation groups. As it could be seen from Fig.2, all the mentioned groups include the so-called “tip type” RA (see for example [9]). The general features of these attractors are the following: on the one hand, it becomes unstable through period doubling bifurcation and, on the other hand, through the saddle-node bifurcation. The possibility of detection of these types of attractors is one of the main advantages of MCBG.

Fig.2 shows that no changes in the behaviour of the system could be observed as the branches of bifurcation diagram cross the upper boundary $d_n=0$. However it should be noted that there is the other boundary, depicting the situation when $d_n=1$, which for the parameter values defined in the caption of Fig.2, is situated much lower and is not shown in the diagram. In order to probe the effects of interaction of this borderline with bifurcation groups, we alter the input voltage of boost converter and on the basis of periodic skeleton for $k=0.4$, obtain another complete bifurcation diagram (see Fig.3).

The bifurcation diagram for new parameter values shows almost the same qualitative behavior of the system. However the most distinctive feature of the diagram shown in the Fig.3

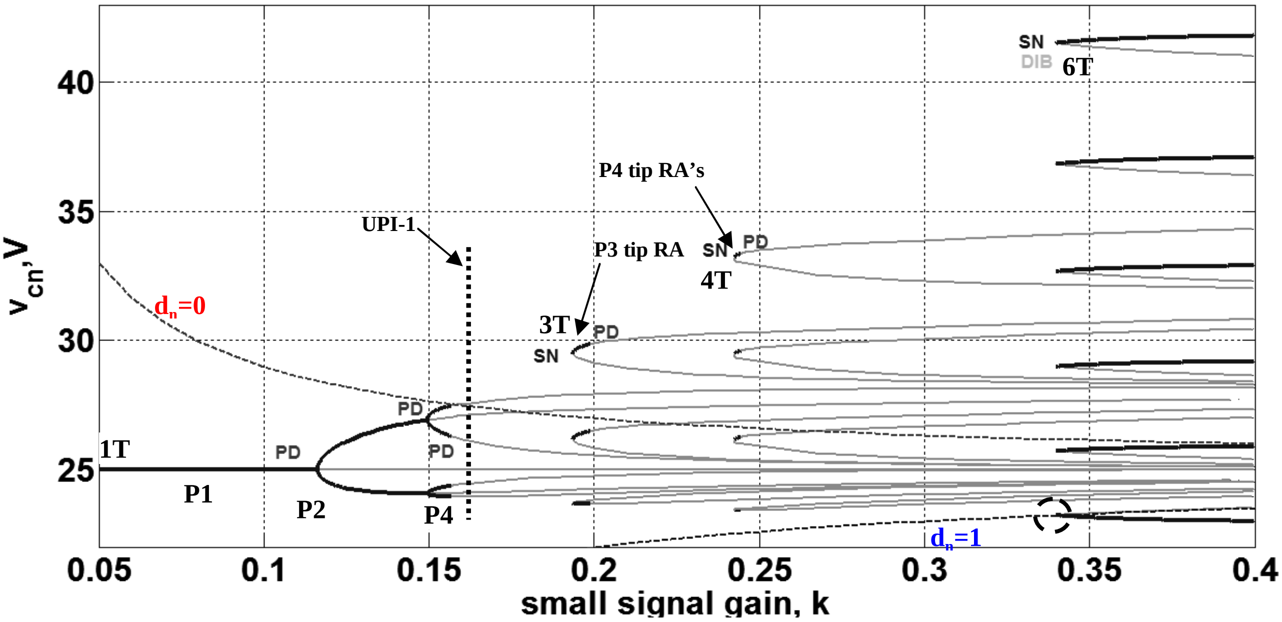

Fig. 3. Bifurcation diagram for boost converter with the following parameters: *R=12.5(Ω), L=208(μH, C=222(μF), T=333(μS), Vref=25, E=13.2(V), k=var*, depicting 1T, 3T, 4T and 6T bifurcation groups. Solid bold dark lines represent stable orbits, solid light lines – unstable orbits and thick dashed lines represent border, when duty cycle d_n=0. PD- states for period-doubling bifurcation, SN- saddle-node bifurcation, DIB-discontinuity induced bifurcation, RA- rare attractor.

is the appearance of 6T bifurcation group through non-smooth saddle-node bifurcation. The careful observation of the region of bifurcation (shown as dotted circle in the Fig.3) lets to conclude that the characteristic multiplier in this case does not smoothly cross the unit circle, but abruptly jumps over +1 value. This is the main feature of discontinuity induced bifurcation (DIB) [5], caused in this case by interaction of orbit with the boundary defined for $d_n=1$.

Based on the obtained bifurcation diagrams it is possible to conclude that:

- the boost type DC-DC converter, operating in DCM, may exhibit various types of subharmonic and chaotic modes of operation;
- RA of tip type are common in the dynamics of this converter;
- smooth bifurcations such as period doubling and saddle-node bifurcation, interplay with the non-smooth bifurcations, such as DIB, defining main qualitative changes in the dynamics of the system.

V. THE COMPLETE TWO-PARAMETER BIFURCATION ANALYSIS

The switching power converters have two parameters, namely input voltage and load resistance that can vary continuously during the operation. The other parameters like inductance, capacitance are generally set at the design stage and are not assumed to be time dependent variables. The converter under study may undergo bifurcations during operation depending on various combinations of the continuously varying parameters keeping others unchanged. It is therefore necessary to study the bifurcations over parameter space of *E* and *R*.

The diagram, showing two parameters on axes, is called bifurcation map and is obtained by means of cell-to-cell mapping approach [10]. In the bifurcation map the asymptotic behaviour of the system is depicted with definite colour over a grid in the parameter space. Using such parameter space the designer can place the nominal operating point away from bifurcation boundaries, marking the transition to subharmonic or chaotic state.

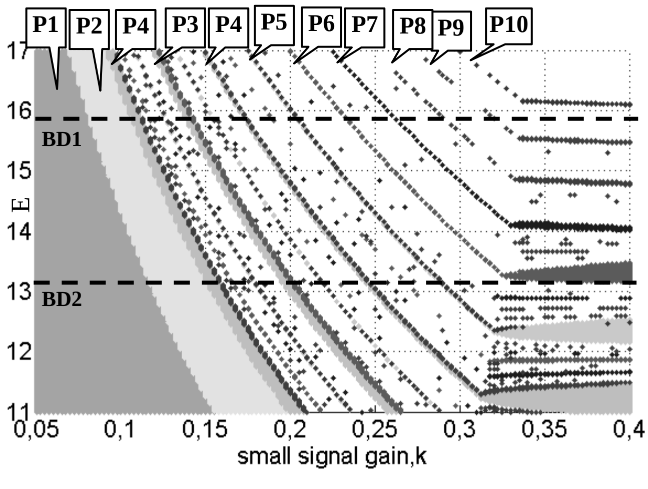

Fig. 4. Bifurcation map for boost converter with following parameters: *R=12.5(Ω), L=208(μH), C=222(μF), T=333(μS), Vref=25, E=var, k=var.* Each color in the map represents definite periodic regime. White color depicts chaotic (aperiodic) behavior. This map was computed by means of cell-to-cell mapping in the grid 700x700 points.

The bifurcation map was constructed for the boost converter in the *k*-*E* plane (see Fig.4).

The bifurcation map obtained in this study could be used to prove results from the previous sections. If we follow dashed horizontal lines in the Fig.4 in the direction of increment of *k*, we would obtain the information about existence of stable periodic orbits in the bifurcation diagrams in the Fig.2 (BD2) and Fig.3 (BD2).

VI. CONCLUSIONS AND FUTURE WORK

The application of a new approach for the global bifurcation analysis of strongly nonlinear dynamical systems to a widely used power electronic converter is under consideration. The main idea of the approach is a concept of complete bifurcation groups and periodic branch continuation along stable and unstable solutions. In this paper it is shown that the use of MCBG allows finding new nonlinear effects and unknown before periodic (rare attractors) and chaotic regimes in switching DC-DC converters.

The present study also shows that as the operation of converter should be robust against changes in parameters, there is a necessity of working out the bifurcation patterns in the parameter space at the design stage to place the nominal operating point away from boundaries marking different asymptotic behaviour. For this purpose one-parameter bifurcation diagrams and bifurcation maps should be obtained.

As it has already been mentioned, the Fig.4 shows the dependence of periodic and chaotic regimes on two parameters. However in this case it is not possible to detect the kind of bifurcation that leads to the loss of stability of selected regime. That is why it is proposed to use two-parameter bifurcation diagram, which could be constructed by means of numerical continuation of bifurcation points in complex parameter plane. The work on the MATLAB code, which could provide these possibilities, is still in progress.

The obtained diagrams allow the power electronics specialists choosing parameter values in order to stay away from undesirable and even dangerous regimes of operation. It should be noted that in practice all types of bifurcations are to be avoided, but it is also known that designing system far away from bifurcation boundaries may degrade the performance characteristics such as transient speed. Therefore, the design based on the complete bifurcation diagrams or maps, should have a significant impact on the practical methodologies taken to make design trade-offs and performance optimization.

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He has been involved in several research projects on examination of spread spectrum technique for switching converters, improvement of parameters of DC-DC converters operating in subharmonic and chaotic regimes, investigation of complex dynamics of switching systems described by discrete time models and systems of differential equations with discontinuous right hand sides.

His research interests include spread spectrum techniques applied to switching converters, nonlinear control of power electronic converters, nonlinear dynamics of switching circuits and non-smooth phenomena, application of path-following technique to the investigation of global dynamics of systems with different kind of nonlinearities.

Dmitrijs Pikulins. Paaugstinošā tipa DC-DC pārveidotāja pilnā bifurkāciju analīze.
Dotais raksts tiek veltīts pilnās bifurkāciju analīzes pielietošanas iespējām gabaliem lineārām sistēmām ar periodisko pārslēgšanos. Šādi modeļi tiek plaši pielietoti inženieru praksē, piemēram, impulsu sprieguma pārveidotāju dinamikas aprakstam. Atšķirībā no plaši pielietojamām metodēm, dotais raksts piedāvā inovatīvo pieeju – Pilnu Bifurkāciju Grupu Metodi – kura ļauj veikt nelineāro sistēmu pilnu bifurkāciju analīzi. Galvenie rezultāti tiek parādīti par piemēru ņemot vienu no vienkāršākiem impulsu sprieguma pārveidotājiem – ar spriegumu vadāmu paaugstinošā tipa (boost) pārveidotāju, kurš darbojas pārtrauktas strāvas režīmā. Dotā tipa ierīcei, balstoties uz viendimensionālo diskrētā laika modeli, tiek sastādītas pilnas vienparametriskās bifurkāciju diagrammas kondensatora spriegumam un bifurkāciju kartes. Pētījumu gaitā tika konstatēts, ka boost pārveidotajos ir novērojamas gludās perioda dubultošanas un seglu – mezglpunkta bifurkācijas, kuras ir raksturīgas arī nepārtrauktam sistēmām. Darbā ir parādīt, ka novērota negludo bifurkāciju parādīšanās ir saistīta ar vadības signāla darbaizpildes koeficienta piesātinājumu. Darbā tiek parādīts, ka Pilno Bifurkāciju Grupu Metodes pielietošana impulsu sprieguma pārveidotāju nelineārās dinamikas analīzei ļauj novērtēt to stabilitāti, noteikt rēto atraktoru, gludo un negludo bifurkāciju parādīšanas nosacījumus, kā arī izpētīt haotiskās darbības rašanas cēloņus. Iegūtās pilnās bifurkāciju diagrammas var tikt izmantotas sprieguma pārveidotāju projektēšanas procesā, ļaujot elektronikas speciālistam izvēlēties shēmas parametru diapazonu (ņemot vērā arī attiecīgās pielaides) kurā ierīce darbosies vēlamajā periodiskā vai aperiodiskā režīmā.

Дмитрий Пикулин. Полный бифуркационный анализ повышающего DC-DC преобразователя.
Данная статья посвящена описанию применения полного бифуркационного анализа для изучения нелинейной динамики кусочно-гладких систем с периодическими переключениями. Модели такого рода широко используются в инженерной практике, например, для описания динамики импульсных преобразователей напряжения. В отличие от обычных методов нелинейной динамики, данная статья предлагает новый подход – Метод Полных Бифуркационных Групп – позволяющий проводить полный бифуркационный анализ нелинейных систем. Основные результаты данного подхода демонстрируются на одном из простейших преобразователей напряжения – повышающего типа с управлением по напряжению, работающего в режиме прерывистых токов дросселя. Для данного устройства на основе его дискретной модели были построены полные однопараметричиские диаграммы для напряжения выходного конденсатора и бифуркационные карты. В ходе исследования было определено, что в импульсных преобразователях повышающего типа наблюдаются гладкие бифуркации удвоения периода и седловые бифуркации, характерные и большинству гладких нелинейных систем. В работе показано, что наблюдаемое возникновение негладких бифуркаций в данных преобразователях связано различными видами насыщения коэффициента заполнения управляющего сигнала. Исследование показывает, что применение Метода Полных Бифуркационных Групп в анализе нелинейной динамики импульсных преобразователей напряжения позволяет оценить их устойчивость в заданном диапазоне параметров, определить условия возникновения редких аттракторов, гладких и негладких бифуркаций, а также изучить причины возникновения хаотических режимов колебаний. Построенные полные бифуркационные диаграммы могут быть использованы в процессе проектирования преобразователей напряжения, позволяя специалистам силовой электроники осмыслено выбирать диапазон параметров (принимая во внимание допустимые отклонения) в котором устройство будет работать в заданном периодическом или апериодическом режиме.