

Application of Fractal Geometry to Foundry Ice Model Investigations

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Abstract: In this paper, during the freezing process of ice construction the laws of ice crystal formation and growth are considered with the help of the obtained foundry ice model. The growth of dendritic structures is described by means of fractal geometry, which gives the approach to the measurement of the dendritic surface area of ice model, as well as to the description of its crystallization front. It provides a possibility to investigate the processes of obtaining ice constructions, in general, and the ones, which are used in the foundry industry, in particular.

Keywords: Foundry ice model, fractal geometry, dendritic structure, crystallization.

I. INTRODUCTION

A characteristic feature of the modern technological use of science in production is involvement of environmentally secure materials, appropriate for the development of resource-constrained processes, into the research field. In the modern foundry technology, the formation of metal parts in sand molds gives 75–80 % of the casting tonnage. At the same time, the search for optimal materials for foundry models, which meet the above-mentioned criteria, continues. Here, it is necessary to take into account that the best precision of castings is achieved at moulding with one-time patterns.

In comparison with traditional one-time paraffin-stearic and foam plastic (melted and gasified) patterns, ice models give information about the pollution of the molding sand. Therefore, recently such models have become the subject of intensive researches of different scientists [1]. However, still the actual problem is the research of the polycrystal structure, as well as the process of obtaining the ice foundry models.

Evaluating the gradual development of ice technologies as one of the steps towards the industrial production at a new level of the environmental culture, we can note that, in our opinion, a boom of cryotechnologies, compared with the today's focus on nanotechnologies, is still expected.

II. ANALYSIS OF THE LATEST INVESTIGATIONS

At the beginning of the 20th century, Polish geophysicist prof. A. B. Dobrowolski proposed to call “cryology” the branch of science, which studies the ice in all its forms and developments [2]. At present, the term “cryotechnologies” is common only among physicians and specialists in refrigeration equipment. Ice, as a material for the production of industrial constructions, is still not widely used. Ice

structure under different conditions of water freezing determines the properties of ice products, which include, in particular, foundry ice models. Mathematical modelling of this structure formation is the example of involvement of cryology into modern environmental technological processes.

It is known that water has such physicochemical anomalies as high surface tension and viscosity, ability of supercooling to low temperatures, high melting heat of solid phase (ice) and others. These anomalies may be explained by hydrogen bonds and by water structure. They define the complex nature and dynamics of the crystal formation process during the process of obtaining the ice foundry model.

Thus, when the temperature of water decreases to the point of crystallization, a new network of hydrogen bonds between molecules H_2O appears that is more stable in space and time. The technical water goes into another aggregative state – the solid phase – at low negative temperatures, which depend on the mineralization. The process of crystallization traditionally begins in the discrete local areas – germs of crystals. Conditions, which promote growth of ice crystals in the water, are determined by the degree of supercooling, the presence of crystallization centres and the rate of the crystallization heat removal.

Some minimum crystal size corresponds to each state in the process of the crystal growth. Such size is called the critical, if the spontaneous crystal growth begins at that size. Crystals, which have sizes, smaller than the critical size for the given conditions, no longer grow but dissolve. According to the determination of the minimum perfect ice crystal [3], the critical size of the germ at water crystallization consists of 460-470 molecules of the volume 15.7 nm^3 . This corresponds to the sphere with the radius 1.56 nm.

It is experimentally determined that ice crystals grow mostly parallel to the mold base in cases, when overcooling increases away from its surface into the water column, and the base is only the place of the crystal origin, but not the source of cold. On the other hand, ice crystals grow mostly to the base under the condition of the heat transfer in this direction in case when overcooling into the stage of protocrystallization is limited by a sufficiently thin water layer, adjacent to the base.

After the formation of a primary permanent centre of crystallization, the crystal growth continues in all directions. For the study of ice crystal growth in water, the cluster model has been applied [4]. In this model, water is a conglomerate of large associates (clusters) of molecules, which arise and then again decompose. Water displacement during the formation

(crystallization) of the ice model results in the formation of the displacement front, which is very unstable.

From the mathematical point of view, the process of crystallization of the ice foundry model is analogous to the process of aggregation of particles, which is described by fractal geometry as the appearance of fractal clusters [5]. Thus, fractal modelling can be effective enough in studying the formation process for ice foundry models.

III. GOAL OF THE INVESTIGATION

The goal of this work is applications of concepts and means of fractal geometry for investigations of polycrystal structures, as well as processes of ice models obtained in the modern foundry.

IV. PROBLEM STATEMENT AND RESULTS OF THE INVESTIGATION

Analysis of the latest investigations shows that foundry ice models are not investigated enough at present. Therefore in this paper there is stated the problem of using the concepts and possibilities of fractal geometry for identification of principles of the structure formation for environmentally safe ice models in foundry.

Fractal geometry allows describing different irregular and fragmentary forms during crystallization of the ice model, setting the family of figures, which are called fractals. The fractals are natural structures, consisting of particles, which in some sense are similar to the whole [5]. Thus, the crystallization front of the ice model may be considered a self-similar and compact, i.e. bounded and closed, set E of the Euclidean space R^n . Such a set may be presented as a finite union of its subsets E_n , which do not intersect each other:

$E = E_1 \cup \dots \cup E_n$. The simplest example of the self-similar fractal is the classical Cantor set [6].

Fractal structures, which appear during crystallization of ice models, contain elements of randomness regarding the subordination to the static laws of the crystallization process. Therefore, it is convenient to consider fractal structures, which appear during crystallization of ice foundry models, to be some fractal sets. An experimenter most often observes such sets as a recurring form of typical treelike configurations, or so called dendrites.

The law of the crystallization process of the ice model in the mold is the following: heat is evolved at the points of crystallization, and it is absorbed at the points of branching of a dendritic tree at the interface of liquid and solid phases.

In practice, the density of crystallization sources of the ice model determines the volumetric multidendritic nature of the crystal formation. It varies depending on the interface of liquid and solid phases, as well as on the intensity of thermal effects. Fractals in this process combine a wide class of objects, i.e. they form some fractal set, which may be considered a set of points in Euclidean space R^n .

At fixed thermodynamic conditions there is a spatial frame

of fractal clusters with a statistical distribution by size. The decrease in temperature, thus, leads to the increase in molecules in the cluster. Hence, according to the cluster model, ice-similar formations in water exist even before the formation of stable clusters.

Therefore, in the inter-dendritic space, floating ice crystals are intensively generated, which on a vertical plane create convection currents, directed downward. Simultaneously, convection currents of water, replaced by floating ice crystals, are directed upward. It is obvious that the wider the inter-dendritic space is, the higher the convection rate is. Here, the flow of ice crystals, which are deposited, heats up, but the counter flow of liquid is cooled. Therefore, the higher the convection rate is, i.e. the greater the area of deposition is, and the more intensive the growth of crystals is on the branches of dendrites.

After solidification, the ice model as a polycrystalline body is a mixture of periodic structures: microlayers of a coarse-grained crystal structure and spaces between them, filled by microlayers of a fine-grained structure, alternate. Such layers have the increased strength. The average size of crystals of fine-grained structures, as well as the difference in size between large and small crystals, is determined by technological additions such as surface-active materials and binders, which are usually introduced into liquid water composition of the ice model. Also, they are determined by freezing conditions, such as the rate and the degree of overcooling, etc.

To illustrate ice structures, a "classical" photograph taken by the famous Soviet mineralogist and crystallographer G. Lemmleyn [7] is provided in Fig.1. It shows outlines of conjoint crystals and water bubbles in places where melting began. M. Shaskolska gives this photograph when describing the dense polycrystalline mass of the ice cover of rivers and glaciers [7]. Such ice consists of separate crystals, but it is not always possible to discern them because they are small, transparent and accrete together. In spring, during the ice melting, one can see that the ice consists of columns, mostly hexahedral ones. Such columns are parallel to each other and perpendicular to the water surface; they resemble pencils in a pack and each such "pencil" is an ice crystal.

A dendrite ice structure of the foundry ice model is confirmed by our photographs of ice samples with a diameter of 100 mm and a thickness of 10 mm, which were frozen in the freezer at -15°C (Fig. 2).

In order to determine boundaries of crystals, water for the samples was tinted with ink and other dyes. Among the various options of structures at freezing there was observed a local darkening of the sample with the displacement of the dye molecules by growing water crystals mainly to the centre (Fig. 2). Also, two dark spots were observed, which were pierced by dendritic needles (Fig. 3).

The intergrowth of ice crystals during the formation of ice foundry models occurs not as a result of the joining of separate molecules, but with some "jumps" through the joining of whole blocks of molecules with the size of 10^{-7} - 10^{-3} m.

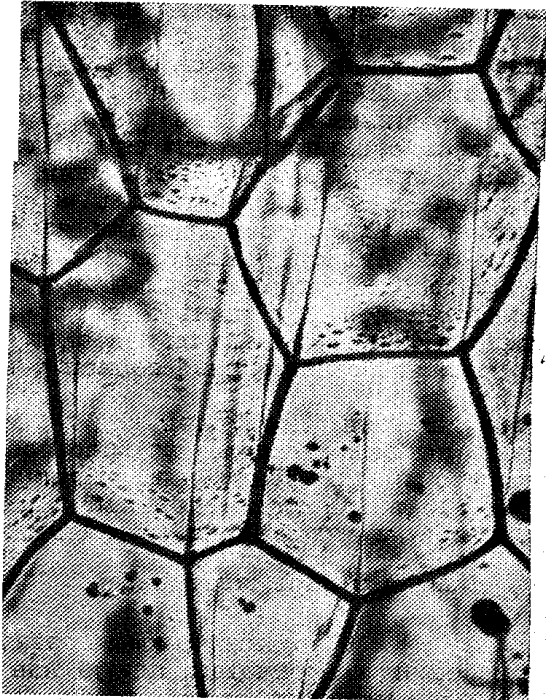


Fig. 1. The river ice under the microscope

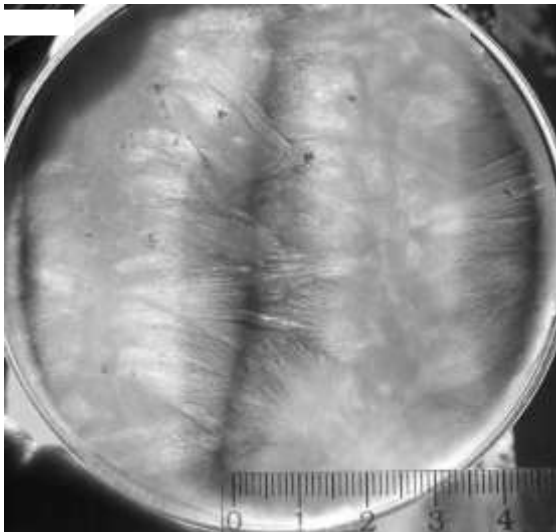


Fig. 2. The example 1 of the ice model with a thickness of 10 mm

It is natural that crystals of the dendritic layer of the ice model can not grow continuously, therefore the process of crystallization of the dendritic layer can be divided into four phases: the solid phase, the liquid-crystal phase with the predominance of the solid phase (the dendritic layer), the liquid-crystal phase with the predominance of the liquid phase (the adjacent layer with floating crystallization centres) and the liquid phase.

When the quantity of overcooling heat, eliminated through the dendrites, is much greater than the quantity of heat, absorbed by the dendritic space, the dendrite vertexes begin to expand and close up, forming a solid crystalline layer.

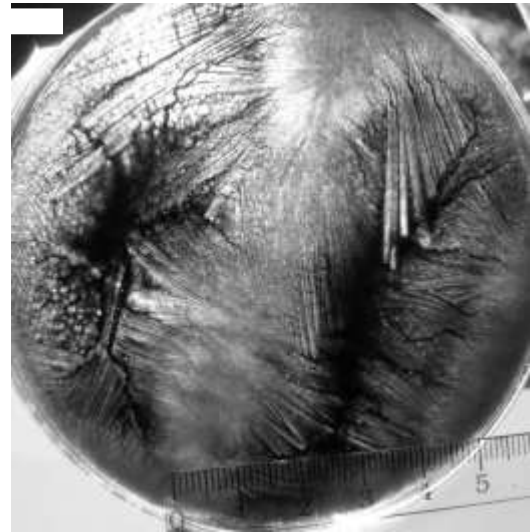


Fig. 3. The example 2 of the ice model with a thickness of 10 mm

During the further process of the dendritic layer crystallization, the difference of temperatures between the interface of liquid and solid phases and the surface at some distance inside the solid phase is minimal. Therefore, due to random inhomogeneities in some points on the interface of liquid and solid phases, the overcooling heat, emerged during the crystallization process, does not have time to eliminate. This leads to melting of microvolumes of the solid crystalline layer, and the dendritic tree of crystals begins to grow on it.

Also, the mechanism of cluster growth does not exclude the possibility of joining separate molecules to the growing crystal face. However, this process is minor, especially at the significant water overcooling. At small overcooling, i.e. at the overcooling in the parts of Celsius degrees, the fractal cluster introduction is possible in practically full agreement of its orientation with the growing crystal orientation [3].

During freezing of the model, the process of water transition from the liquid state to the solid one occurs in the volume of the mold cavity, which is technologically given by the metal casting configuration. Therefore, in this case we do not have any crystallization front, but we have some crystallization area. This area includes the part of overcooled water, within which clusters with the corresponding quasilattice orientation can join the crystal surface. Moreover, this area contains the zone, closest to the interface inside the crystal, where the relaxation of structural crystal imperfection takes place.

Thus, there is some transition zone, which, responding to changes in external conditions, determines the structure and morphology of crystals of the ice foundry model. Here, the rate of the model crystal growth is determined by the rate of the substance (ice) introduction or the rate of the crystallization heat elimination. The direction of natural changes during freezing and the rate of their implementation are essentially only forms, in which the heat energy distribution is implemented.

If the axes of a rectangular Cartesian coordinate system present the time and the temperature, then the behaviour of the

freezing ice foundry model can be described as a movement of a point on this plane. Using the concept of the complex number $a+ib$, where a and b are real numbers, $i = \sqrt{-1}$, we can consider the temperature as an imaginary time. It means that some equations of usual dynamics are transformed into thermodynamic expressions when you change the time, expressed in real numbers, for pure imaginary numbers ib . Therefore, the water cooling process in the ice foundry model can be considered a thermodynamic process in time but along its imaginary axis.

Using the methods of thermodynamics of irreversible processes for the growing ice mass flow, we can write [8]:

$$q_s = \alpha_{11}(dP + \rho_s Q \frac{dT}{T}) ,$$

where α_{11} is the coefficient of the ice shear resistance, ρ_s is the density, Q is the heat of phase transitions, dT is the change in temperature, dP is the change in pressure.

Due to the large reaction rate, the form of ice model crystals is very sensitive to external conditions. The water environment change can cause a rapid aggregate change in its forms, which can grow or persist. Thus, any jut on the surface of the growing crystal is in more favourable conditions for growth. During the slow crystal growth, solid faces are formed with growth lines, parallel to the base. If the crystal basal plane is parallel to the growth surface, then separate "steps" in height about 0.4 - 0.6 mm are formed on the plane.

The fresh-formed ice of the foundry ice model is exposed to many microstructure transformations related to the recrystallization and completion of the crystal structure. In natural water there always are extraneous elements, which have a significant effect on the freezing and nature of crystallization during the process of obtaining the ice model.

Considering the process of the ice model crystallization in terms of fractal geometry, we underline that the concept of distance between points in space is central in determining the fractal dimension during the model crystal growth.

We consider the area of crystallization of the ice foundry model to be a compact set E . According to the Heine–Borel theorem, it is possible to cover the set E by a finite system of closed sets with diameters, not exceeding some real number $\delta > 0$, and for each δ one can determine the minimum number $N(\delta)$ of such sets [6]. Here, the function $N(\delta)$ has only natural values and increases with no limit at $\delta > 0$. We note that this function also depends on the metrics of the compact set E . Different metrics of the compact set E have, in general, different functions $N_E(\delta)$, hence the metric order of the compact set E is not a topological invariant.

Thus, we can measure the area of the crystal surface of the crystallization zone for the ice foundry model as the value of the set of points in space, dividing the space into small cubes with the edge δ or into small spheres with the diameter δ [6]. Placing the centre of the small sphere into any point of the set, we get that all points on the distance $r < \frac{1}{2}\delta$ are covered by this sphere. Calculating the number of spheres, which cover

the set of points, we obtain a measure of the value of this set of points.

Let us consider the set of points that form the surface of the crystallization area for the foundry ice model. Obviously that the number of squares with the side δ , that are necessary to cover some surface, can be determined from the expression

$$N(\delta) = \frac{S_0}{\delta^2} \text{ at } \delta \rightarrow 0, \text{ where } S_0 \text{ is the area of the}$$

crystallization surface. Then the normal measure of the set of points, which form the surface of the crystallization area of the ice model, is the area S and we can write [6]:

$$S = N(\delta) \times \delta^2 \xrightarrow{\delta \rightarrow 0} S_0 .$$

Therefore, the number of squares, needed to cover the dendritic surface of the crystallization area of the ice foundry

model, is determined by the expression $N(\delta) = \frac{S_0}{\delta^2}$ at $\delta \rightarrow 0$.

It is necessary to note that some sets of points form curves, which can be twisted as much that the length of the curve goes to infinity. For example, the Peano curves fill up the whole plane. Analogously, there are surfaces, bent in such a complicated way that they fill up the whole space. The dendritic surface of the crystallization area of the ice model is similar to those complex surfaces. In future, for more precise study of such sets of points, it is expedient to generalize the above-mentioned measures of the set values for more complex surfaces.

V. CONCLUSIONS

In the fractal interpretation, the mechanism of the ice structure formation of the foundry ice model is considered in the paper. That can serve as a fragment of the theoretical foundations of the development of such model production technology. The basic concepts of fractal geometry, required to identify principles of the formation of the ice foundry model structure, are considered. The examples of ice models, which motivate the research of dendritic structures by the fractal geometry method, are presented. For the first time, it is proposed to use the concepts of fractal geometry for the original technology of the production of ice products.

Also, fractal geometry gives the approach to measure the area of the dendritic surface of the frozen ice model, as well as the front of its crystallization. This allows exploring in a new way the processes of the model solidification and melting. Understanding of the features of ice fractal structures, with the self-similarity of separate elements to the whole product, may explain the presence of coarse-grained and fine-grained crystalline microlayers.

The nature of the influence of such microlayers onto the construction strength, and the control of model parameters are the issues for further research.

Laws of ice crystal growth during freezing of the ice model and growth of dendritic structures have many similarities with the crystallization of metals. Therefore, processes of obtaining ice models, described in this paper, by analogy may be called "ice metallurgy" or "foundry of ice constructions".

Development and application of such cryotechnologies to obtain ice constructions may become a means of manufacturing application of a large array of interdisciplinary knowledge, including elements of physical and colloid chemistry, surface phenomena, thermodynamics of irreversible processes, etc. This will allow achieving the high-tech ways of production at a new environmental and energy-saving level.

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Vladimirs Kravčenko, Vladimirs Dorošenko, Olena Mul, Andrejs Shut. Fraktāļu ģeometrijas pielietojums lieta ledus modeļu pētīšanai

Radot jaunas metāla liešanas tehnoloģijas, zinātnieki savos pētījumos iekļauj jaunus, videi draudzīgus materiālus. Liešanas process būs optimāls resursu saglabāšanai, kad smilšu formas materiāls maksimālu skaitu reižu atkārtoti kontaktē ar liešanas izkausēto metālu. Šim kritērijam visvairāk atbilst vakuuma forma no sausām smiltīm bez saistvielas, rūpnieciski pārbaudīts virziens smilšu formēšanai ar atkārtotu smilšu ciklu un minimālām reģenerācijas izmaksām. Tomēr rodas jautājums par šo formu liešanas modeļu optimāliem materiāliem. Lielāko lējuma precizitāti dod viengabala smilšu formas modeļi, kas izlieti vienu reizi. Salīdzinājumā ar tradicionāliem šāda veida parafīna – stearīna un putuplasta modeļiem mazāko piesārņojumu smilšu veidnēs var dot ledus liešanas modeļi, kuru pētīšana veikta šajā darbā (Дорошенко В.С. Предпосылки создания технологии литья по ледяным моделям в вакуумируемых формах // Металл и литье Украины. – 2009. – № 5. – С. 27–30). Ledus kā rūpnieciski ražotu konstrukciju materiāls pagaidām nav radis plašu pielietojumu. Ledus struktūru formēšanas matemātiskā modelēšana dažādos ūdens sasaldēšanas apstākļos izraisa interesi kā dabu sargājošas kriotehnoloģijas iesaistīšana mūsdienu tehnoloģiskajos procesos. Ir zināms, ka ūdenim piemīt fizikāli – ķīmiskas anomālijas, kas izraisa kristalizācijas sarežģītu raksturu un dinamiku, iegūstot lieto ledus modeli. Kristalizācijas process sākas diskrētos lokālos apgabalos – kristāla embrijos. Nosacījumi, kas veicina ledus kristālu palielināšanos ūdenī tiek noteikti ar pārsaldēšanas pakāpi, kodola kristalizāciju un kristalizācijas siltuma zuduma ātrumu. Katram kristāla augšanas stāvoklim atbilst noteikts minimālais kristāla izmērs, ko sauc par kritisko, no kura sākas spontāna augšana. Autori aplūkojuši kristāla veidošanās un augšanas likumsakarības, sasaldstot ledus konstrukcijām lietā ledus modeļa veidošanas procesā. Dendrītisko struktūru pieaugums aprakstīts, izmantojot fraktālo ģeometriju, kas dod iespēju izmērīt salstošā ledus modeļa dendrītiskās virsmas laukumu, kā arī tā kristalizācijas fronti. Tas atklāj plašas iespējas pa jaunam pētīt ledus konstrukciju, kuras izmanto liešanas ražošanā, iegūšanas procesus.

Владимир Кравченко, Владимир Дорошенко, Елена Муль, Андрей Шут. Применение фрактальной геометрии для исследований литейных ледяных моделей

Создавая новые технологии литейного производства металлических деталей, ученые-литейщики вовлекают в свои исследования новые экологически безопасные материалы. Литейный процесс будет оптимальным по ресурсосбережению тогда, когда материалы песчаной формы смогут максимальное количество раз повторно контактировать с расплавленным металлом отливки. Этому критерию наиболее соответствует вакуумируемая форма из сухого песка без связующего, промышленно проверенное направление песчаной формовки с повторным циклированием песка и с минимальными затратами на его регенерацию. Однако возникает вопрос об оптимальных материалах литейных моделей для этой формы. Наибольшую точность отливки дают одноразовые модели в неразъемной песчаной форме. По сравнению с такого рода традиционными парафиностеариновыми и пенопластовыми моделями наименьшее загрязнение формовочного песка могут дать ледяные литейные модели, исследование которых и проводится в данной работе (Дорошенко В.С. Предпосылки создания технологии литья по ледяным моделям в вакуумируемых формах // Металл и литье Украины. – 2009. – № 5. – С. 27–30). Лед как материал для изготовления промышленных конструкций пока не нашел широкого применения. Математическое моделирование формирования структуры льда в различных условиях замораживания воды вызывает интерес в свете вовлечения природоохранных криотехнологий в современные технологические процессы. Известно, что вода обладает физико-химическими аномалиями, определяющими сложный характер и динамику процесса кристаллообразования при получении литейной ледяной модели. Процесс кристаллизации начинается в дискретных локальных областях - зародышах кристаллов. Условия, благоприятствующие росту кристаллов льда в воде, определяются степенью переохлаждения, наличием ядер кристаллизации и скоростью удаления теплоты кристаллизации. Каждому состоянию в процессе роста кристалла соответствует определенный минимальный размер кристалла, называемый критическим, при котором начинается его спонтанный рост. Авторами рассмотрены закономерности образования и роста кристаллов льда при замерзании ледяной конструкции на примере получения литейной ледяной модели. Рост дендритных структур описан при помощи фрактальной геометрии, которая дает подход к измерению площади дендритной поверхности замерзающей ледяной модели, а также описанию фронта ее кристаллизации. Это открывает широкие возможности по-новому исследовать процессы получения ледяных конструкций, в частности тех, которые используются в литейном производстве.