Modelling the Behaviour of Stability of Production Systems in Economics

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Abstract. **Information technologies provide an opportunity to create, collect, save, process and efficiently use information in the processes under investigation. In the contemporary world, information technologies are most frequently linked with modelling, making use of computer technology and information networks. Modelling is considered to be an indirect investigation method for originals of objects used while researching the substitutes of the objects. The information image of the object (information model) may be used as the object substitute. In economic studies it is important to create a business model.**

Key words: **production systems, *s*tability, statistical modelling, optimization.**

I. INTRODUCTION

One of the constituent parts for successful application of production systems in economics (PSE) is optimization of functioning of all production and economic parameters of the given system. The aim of this research is modelling the behaviour of the criterion of stability of PSE.

In terms of stability of PSE, many authors consider the organisational and economic stability of PSE under changing parameters of external environment. Stable position (functioning) of PSE in the market is understood as the capacity of PSE to maintain or increase the tendency of positive growth, i.e., to maintain (increase) the sales volume of goods or services and the profit margin gained by sales of goods and services rendered under different changes of external and internal environment of PSE functioning.

Steady position of PSE in the market is understood as PSE ability to keep or increase the tendency of positive development, i.e., to keep the volume of the goods or services sold and the profit received from sales of the given goods and services in changing external and internal environment of PSE.

Modelling of behaviour of the criterion of PSE stability is based on establishing alternative strategies, which are introduced during definite moments of modelling time, thereby supporting stable PSE performance. Thus, the concept “criterion of stability” has been introduced. The primary goal of management of PSE stability is maintaining the criterion of stability with regard to the given interval of allowable values of the chosen criterion of stability. Uncertainty in PSE is understood as a situation when there is incomplete or no information at all about the possible conditions of the system itself and the environment, in which the system functions. Conditions of uncertainty can also be explained as various unexpected fluctuations of some factors of external and internal PSE environment. Steady performance of PSE is the ability of all PSE participants to perform a complete set of functions and also maintain (or even increase) the services to be rendered for a long period of time under the conditions of uncertainty. Ignoring the evaluation of the influence of variable factors during the research on the processes occurring in PSE frequently expands the “zone of risk”, thereby leading to mistakes in the real time situation that, in turn, may finally result in significant material losses.

The financial stability of PSE is the capability of all participants of PSE to implement their financial liabilities in full on the timeframe set by the contract.

Industrial stability of PSE designates the ability of the system to keep or increase volumes of production or services during all technological processes in case the conditions of functioning of the given system are changed.

The key problems presented in the paper are the following:

- development of integral criteria for estimation of the financial and industrial stability of PSE;
- stochastic modelling of stability of PSE during modelling time under the conditions of uncertainty;
- setting alternative strategies of PSE performance;

Introduction of an integral criterion of stability of PSE assumes the analysis of all industrial and financial processes in the system, i.e., the internal and external processes of PSE. The novelty of the given research is the algorithm of research of stability of industrial production systems under the conditions of uncertainty.

II. TRADITIONAL METHODS OF STATISTICAL MODELLING FOR INVESTIGATION OF ECONOMIC SYSTEMS

Using traditional methods of statistical modelling [1,2,3,4] for investigation of economic systems, it is possible to set the task of creating an efficient procedure for generating incidental parameter values constituting factors of an simulation model, to effectively use up-to-date information technologies, to ensure continuous control of the behaviour of the specific economic system that is being researched. The traditional scheme of simulation modelling is the generation of a mass of incidental parameter values featuring the changes of model factors (Fig. 1). The algorithm of generation of incidental continuous value X (see Fig. 1), having continuous distribution function F, can be described in the following steps:

Let us generate, within an interval (0.1), an evenly distributed incidental parameter u ~U(0.1). Let us calculate $X = F^{-1}(u)$. The value of $F^{-1}(u)$ will always be definite, since

$0 < u < 1$, but the area of defining the function F is the interval [0,1].

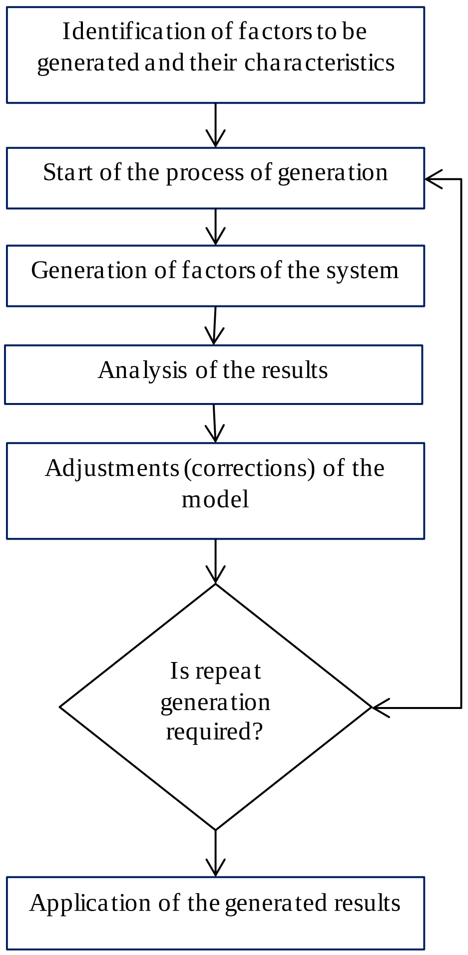

Fig. 1. Algorithm of repeated generation of incidental parameters

The figure below presents the essence of the algorithm graphically; here an incidental value may be assumed to be either positive or negative. This depends on the specific value of parameter u. In the figure, the value of parameter u_1 produces a negative incidental value X_1, but parameter u_2 yields a positive incidental value X_2 (see Fig. 2).

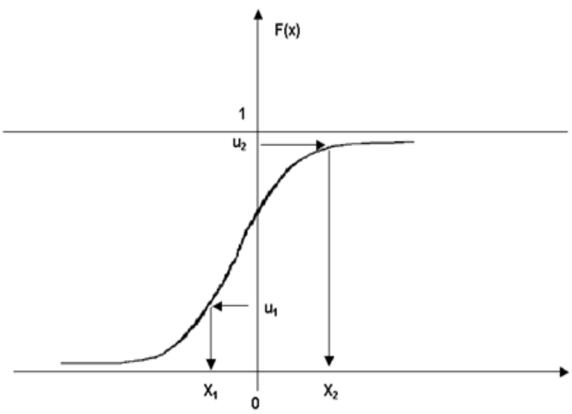

Fig. 2. Scheme of reverse transformation

The method of reverse transformation may also be used if value X is discrete. In this case the distribution is as follows:

$$F(x) = P\{X \le x\} = \sum_{x_i \le x} p(x_i), \qquad (1)$$

where $p(x_i)$ is probability $p(x_i) = P\{X = x_i\}$.

It is admitted that incidental parameter X may have only such values as x_1, x_2, ..., for which $x_1 < x_2 <$ .... Thus the algorithm of developing the values of incidental parameter X will have the following consequences:

Let us generate, within the interval (0.1), uniform distributed incidental parameter u ~ U(0. 1);

Let us establish the least positive round value i, for which $u < F(x_i)$, and assume that $X=x_i$.

Both options of the method of the reverse transformation for continuous and discrete values (at least formally) can be combined in one formula:

$$X = \min P\{x : F(x) \ge U\}, \qquad (2)$$

which is true also for mixed distributions (i.e., containing both continuous and discrete components). In contrast to common direct methods of generating incidental values (the method of the reverse transformation composition and implosion), for imitating the factors of the simulation model it is recommended to use the so-called indirect methods, namely, the acceptance-refusal method. This method may turn out to be suitable if due to certain reasons it is impossible to apply direct methods or if these methods are inefficient.

The “acceptance-refusal” principle is rather common. If the aforementioned algorithm is looked upon from a slightly different perspective, it is clear that it may be extended for generating incidental points in areas having higher dimensions – i.e. in multi-dimensional areas. A more popular method, which is used for stochastic modelling, is a copula method [3,4,5,6,7].

III. Modelling the Behaviour of Criterion of Stability

Modelling the behaviour of criterion of stability of PSE is based on building a set of alternative strategies, the introduction of which at certain periods of time allow maintaining of PSE functioning in a stable position, i.e., the concept of “criterion of stability” is introduced [8, 9, 10]. Thereby, the main objective of managing stability of PSE is provision of criterion of stability to the given interval of permissible values of the chosen criterion of stability. Taking into account the infrastructure of PSE, as well as the character of relationship of internal and external factors of the functioning environment of PSE, the criterion of integral profit is considered to be the generalized (integral) criterion for stability, which may be presented as [11, 12, 13, 14]:

$$P(T_0, T, w) = \int_{T_0}^{T} R(t,w)dt - \int_{T_0}^{T} C(t,w)dt \ \geq \ K, \quad (3)$$

where $[T_0, T]$ – the period of modelling time of PSE functioning;

w – the incidental parameter (vector of incidental parameters) with the given distribution affecting the behaviour of PSE to be investigated

P (T_0, T, w) – the criterion of integral profit of PSE functioning in the period of modelling time of PSE functioning;

$\int_{T_0}^{T} R(t,w)dt$ – the integral criterion of profit gained from PSE functioning in the period of modelling time of PSE functioning;

$\int_{T_0}^{T} C(t,w)dt$ – the integral criterion of losses linked with the functioning of PSE in the period of modelling time of PSE functioning;

K – the given minimal value of integral criterion of profit due to PSE functioning in the period of modelling time of PSE functioning.

In this paper, the authors consider the stochastic modelling of stability of PSE during the modelling time t of PSE functioning. The task for stochastic modelling of PSE functioning is presented as follows:

$$\begin{aligned}
& P(T_0, T, w) = \int_{T_0}^{T} R(t,w)dt - \int_{T_0}^{T} C(t,w)dt \ \geq \ K; \\
& \sum_{t=T_0}^{T} \sum_{i=1}^{n} Q_{i,t} c_{i,j,t} \ \leq \ s_j; \ \ j = 1,2,\dots,n; \\
& p^{\min}{}_{i,t} \ \leq \ p_{i,t} \ \leq \ p^{\max}{}_{i,t}; \\
& Q^{\min}{}_{i,t} \ \leq \ Q_{i,t} \ \leq \ Q^{\max}{}_{i,t}; \\
& C_{i,j,t} \ \leq \ c_{i,j,t}; \\
& i = 1,2,\dots,n; \ \ t = T_o,\dots,T.
\end{aligned} \quad (4)$$

where $p_{i,t}$ – the value of the criterion of integral profit of PSE at the moment of modelling time t;

$p^{min}{}_{i,t}$ – the minimal value of the criterion of integral profit of PSE at the moment of modelling time t (determined by conditions of modelling);

$p^{max}{}_{i,t}$ – the maximum value of the criterion of integral profit of PSE at the moment of modelling time t;

$Q_{i,t}$ – the volume of sales (services of functioning) of PSE functioning implemented at the moment of modelling time t;

$Q^{min}{}_{i,t}$ – the minimal volume of sales (services) of PSE functioning implemented at the moment of modelling time t (determined by conditions of modelling);

$Q^{max}{}_{i,t}$ – the maximum volume of sales (services) of PSE functioning implemented at the moment of modelling time t (determined by conditions of modelling on the basis of information characterizing the real capacity of the market segment);

$C_{i,j,t}$ – the production (services of functioning) price per unit of PSE implemented at the moment of modelling time t;

$C_{i,j,t}$ – the maximum production (services of functioning) price per unit of PSE implemented at the moment of modelling time t (determined by conditions of modelling on the basis of information about the prices of similar production in the market);

n – a number of observations (control points) considered in the process of modelling;

t – the modelling time;

T_0 – the time for positiveness of the period of stability;

T – the time for completing the process of modelling.

IV. Modelling Process of Stability of Production System in Economics

Fulfilment of the conditions 3 and 4 provides ascertainment of values of integral profit P (criteria of stability of PSE) to the given boundaries during the period of modelling time t of PES functioning at given constraints. The dynamics of the process under investigation takes place during the period of modelling time $[T_0,T]$ (see Fig. 3).

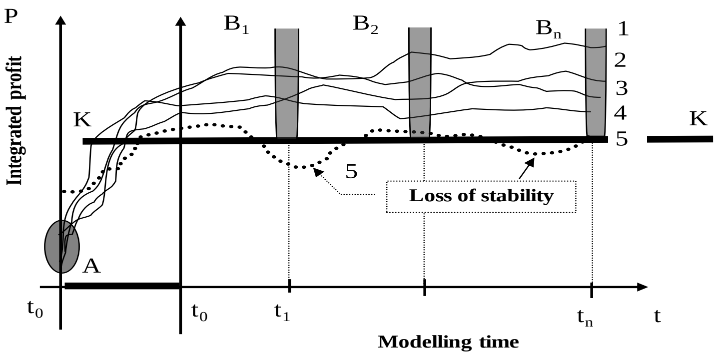

Fig. 3. Graphical representation of the use of algorithm of dynamic programming for ensuring stability of PSE

In Figure 3, a trajectory 5 shows violation of the condition of stability (condition 1). As the process of modelling of the stability of PSE is investigated dynamically, the necessity arises to use the methods of dynamic programming. The value of integral profit P (criterion of stability of PSE) is accepted as the target function. During the investigation, the dynamic state of PSE is modelled with variable values of parameters of PSE satisfying conditions in the system (2).

In Figure 3: B_n – a variety of permissible values of the criterion P after n periods of modelling time of PSE;
A – the interval (set) of values of parameters of PSE (Q, p, c,...) when the criterion of stability of PSE falls into an interval (set) B_n by optimal trajectories; B_1, B_2, …, B_n – the interval (set) through which the trajectory of values of criterion P must pass at the moments of time (control points) T_0+t_1, $T_0+t_1+t_2$, …, T. The task of managing the stability of PSE in the process shown in Figure 3 turns into a task of control of trajectory of the values of the criterion P and (if possible) introduction of constructive effects into the process of PSE development at the moments T_0+t_1, $T_0+t_1+t_2$, ...,T using the information about the possible optimal trajectories of the values of criterion P.

V. Graphical Representation of the Results of Statistical Modelling of Stability of the Production System in Economics

The process of statistical modelling may be implemented with the help of modelling programmes. Modelling of PSE stability was performed by changing the parameters characterizing the basic features of the financial reserve of PSE, as well as by changing the actual parameters of payments among participants of PSE. For modelling purposes the authors have used MS Excel software. The table illustrating the changes of the parameters to be modelled is shown in Fig. 4.

Using the results of statistical modelling of the PSE state makes it possible to determine the probability characteristics of its stability at the given moments of time, including the identification of risks linked with PSE functioning in the given periods of time. Figure 5 shows the results of statistical modelling of PSE transfer to a stable state (see Table 1).

	Income		Payment
Reserve	**10000**		
delay_min =	**7**	**delay_min =**	**3**
w1 =	**25**	**w2 =**	**20**
delay_max =	**32**	**delay_max =**	**23**

max_time_negat = **58**

Fig. 4. Graphical representation of changing parameters to be modelled of PSE stability by using the Monte Carlo methods

TABLE 1

FIRST TIME POSITIVENESS DEPENDING ON W_1 AND W_2

w	$f_1(w)$	w	$f_2(w)$
1	58	1	58
2	5	2	85
3	5	3	67
4	0	4	85
5	3	5	84
....			
97	85	97	2
98	85	98	6
99	83	99	7
100	85	100	3

Using the results of statistical modelling of the PSE state makes it possible to determine the probability characteristics of its stability at the given moments of time, including the identification of risks linked with PSE functioning in the given periods of time. Figure 5 shows the results of statistical modelling of PSE transfer to a stable state (see Table 1).

Graphical illustration of the results of statistical modelling of PSE stability by using Monte Carlo methods and changing parameters w_1 and w_2, is presented in Figures 5 and 6.

Figure 6 shows that a cubic polynomial describes the changes in PSE stability well enough when changing the basic parameters of the system to be modelled. In the case of the well-known distribution of incidental parameter w, it may be possible to use the parametric methods of modelling of incidental values. In the real life only rarely it is possible to gain a sufficiently good description of behaviour of incidental parameters. In this case non-parametric methods of modelling may be used by applying histograms, methods of local non-parametric regression etc. The authors have described in detail the non-parametric methods of modelling in other research papers (see references).

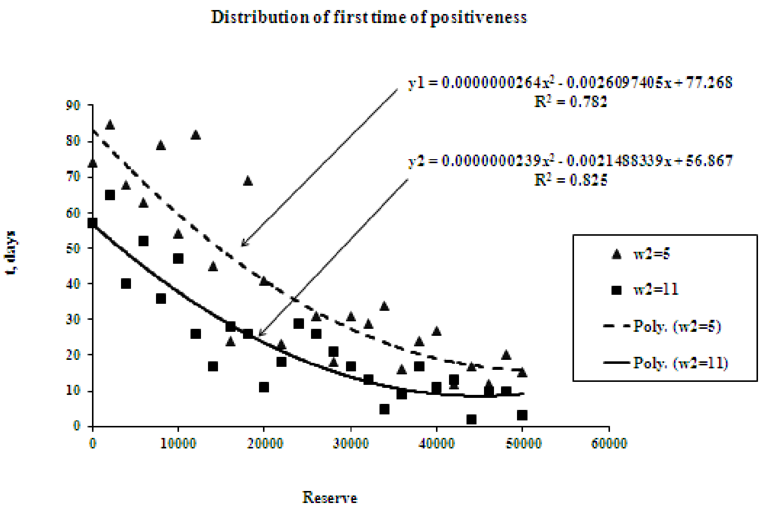

Fig. 5. Graphical representation of the results of statistical modelling of PSE stability by using Monte Carlo methods (Distribution of first time of positiveness)

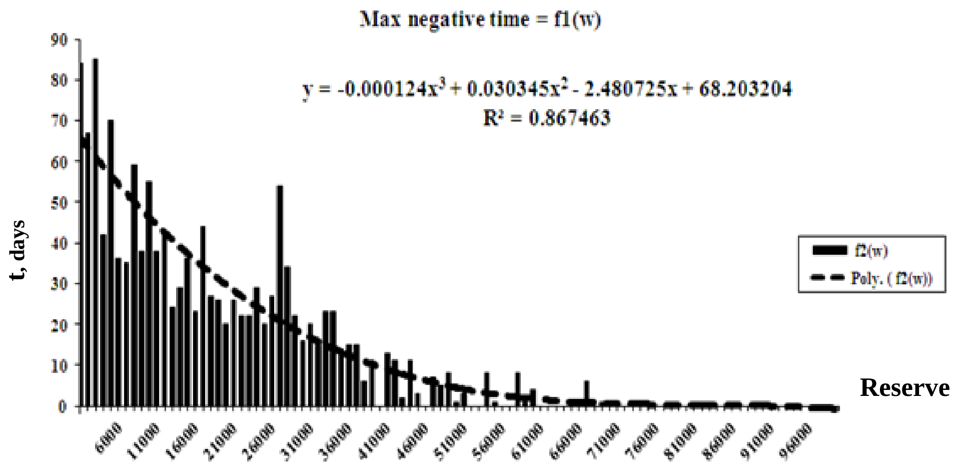

Fig. 6. Graphical representation of the results of statistical modelling of PSE stability by using Monte Carlo methods (polynomial approximation of PSE stability)

VI. Conclusions

The application of modelling is connected with the fact that frequently it is not possible to provide a definite description of the behaviour of the economic system being investigated. When investigating the dynamic behaviour of the economic system, i.e. by making definite changes of parameters of the system under investigation, we frequently observe the existence of incidental factors affecting the character of the behaviour of the system. In addition, it should not be forgotten that the very character of the research also brings its incidental elements into the research process. By using statistical dynamic programming, as well as the Monte Carlo method for modelling it has become possible:

1) to model PSE stability under conditions of uncertainty;

2) to model the “risk zones”, in which the stability of PSE has been distorted;

3) to identify the amount of the financial reserves required for PSE stability in the “risk zones”;

4) to manage functioning of PSE using the integral criterion of PSE stability.

The method of modelling allows developing (simulating) different scenarios of functioning of the investigated economic systems. The modelling method developed by the authors may be used for tackling a wide range of economic problems (design and analysis of industrial systems, stock management, balancing of production capacities, allocation of investment funds, optimization of investment funds, optimization of flows of cargoes and services etc.).

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Vladimirs Jansons, Vitālijs Jurēnoks. Ražošanas sistēmu stabilitātes uzvedības modelēšana ekonomikā
Darbā tiek izskatīta iespēja ražošanas-ekonomiskās sistēmas stabilitātes riska un nenoteiktības apstākļos novērtējumam, izmantot statistiskās modelēšanas metodi Monte-Karlo un dinamisko programmēšanu. Ražošanas-ekonomiskās sistēmas stabilitātes modelēšana paredz alternatīvo stratēģiju izveide kuras, noteiktos laika periodos, ļauj stabilizēt un optimizēt pētāmās sistēmas darbību. Tas nozīmē, ka sistēmu izmantošanas efektivitātes raksturošanai tiek ievests un izmantots sistēmu darbības stabilitātes kritērijs. Izmantojot statistisko modelēšanu, var noteikt sistēmas trajektoriju kopu, kuru izmantojot var aprēķināt optimālās trajektorijas un riska zonas. Tas dod iespēju, izmantojot dinamisko programmēšanu, kontrolēt ražošanas procesu starpposmos un ievest nepieciešamās korekcijas ražošanas procesā. Tāda ražošanas procesa statistiskā kontrole, ar noteikto ticamības pakāpi, ļauj nodrošināt ražošanas procesa stabilitāti un samazināt ražošanas procesa izmaksas. Rodas iespēja ražošanas-ekonomisko sistēmu pētīšanā noteikt sistēmu darbības stohastiskos rādītājus katram darbības laika periodam. Modelēšanas izmantošana ražošanas sistēmu stabilitātes uzvedības pētīšanai ļauj:
- noteikt pētāmās ražošanas-ekonomiskās sistēmas dinamiskās uzvedības raksturu;
- noteikt būtiskos faktorus, kuri tieši ietekmē pētāmās ražošanas sistēmas darbības stāvokli reālajā modelēšanas laikā;
- paplašināt sistēmas uzvedības parametru diapazonu, saglabājot sistēmas stabilitāti;
- izstrādāt un praktiskajos piemēros parādīt efektīvākās modelēšanas metodes, kas varētu būt izmantotas ražošanas-ekonomiskās sistēmas pētīšanai riska un nenoteiktības apstākļos. Rakstā tiek apskatītas sistēmas gadījuma parametru modelēšanas metodes, tajā skaitā arī inversā transformācijas metodes izmantošana gadījuma lielumu masīva ģenerācijai. Autori izstrādāja lineāru stohastisku modelēšanas algoritmu izmantojot Monte-Karlo un Belmana metodes. Ražošanas sistēmu stabilitātes uzvedības modelēšana un pētīšana varētu būt praktiski realizēta pēc autoru izstrādātā un darbā aprakstītā algoritma izmantojot MS Excel statistiskās funkcijas. Statistiskās modelēšanas ar Monte-Karlo metodi tiek realizētas darbā izmantojot MS Excel 2010 programmu. Praktiskajā piemērā autori ilustrē optimālās trajektorijas un aprēķina risku zonu noteikšanas iespējamību.

Владимир Янсон, Виталий Юренок. Моделирование поведения стабильности производственных систем в экономике
В работе рассматривается возможность использования методов статистического моделирования – статистического метода Монте-Карло в совокупности с методом динамического программирования для оценки устойчивости функционирования производственно-экономических систем в условиях нестабильности. Моделирование поведения критерия устойчивости производственно-экономических систем базируется на том, что формируется совокупность альтернативных стратегий, введение которых, в определенные моменты времени, позволяет поддерживать деятельность производственно-экономических систем в устойчивом состоянии, т.е. вводится понятие „критерий устойчивости”. При этом основной задачей управления устойчивостью производственно-экономических систем является обеспечение принадлежности критерия устойчивости заданному интервалу допустимых значений выбранного критерия устойчивости. Используя результаты статистического моделирования состояния производственно-экономических систем, можно определить ее вероятностные характеристики для любого момента времени. Появляется возможность исследования зон устойчивости и рисков, связанных с деятельностью производственно-экономических систем, в заданные периоды времени. Процесс статистического моделирования Монте-Карло может быть осуществлен с помощью имеющихся программ моделирования. В работе для статистического моделирования использовалась программа MS Excel 2010. Использование методов моделирования для исследования поведения экономических систем в условиях риска и неопределенности дает возможность:
- определить характер динамического поведения экономических систем;
- выявить факторы, влияющие на изменение состояния экономической системы в режиме ее реального функционирования;
- расширить диапазон возможного изменения параметров системы, при которых сохраняется устойчивость исследуемой системы;
- разработать и проиллюстрировать на практических примерах наиболее эффективные методы моделирования и исследования экономических систем в условиях риска и неопределенности.