I-CORE SANDWICH PANEL OPTIMIZATION EMPLOYING DIFFERENT PLATE MODELS.

I-SERDES TIPA DAUDZSLĀŅU PANEĻU OPTIMIZĀCIJA, IZMANTOJOT DAŽĀDU PLĀTŅU MODEĻUS

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Introduction

Validation between different plate models used to optimize I-core sandwich panels will be discussed in this paper. The effectiveness of determination the most suitable plate element will lead into development of the fast and reliable reanalysis tool since application of direct minimization algorithm and multiple finite element analysis is too expensive from a computational point of view. For this reason four different numerical solution approaches were chosen to investigate sandwich panel:

1) Analytic with trigonometrical functions.
2) Substitution of sandwich panel with an equivalent orthotropic material plate model.
3) Replacement of sandwich panel with an equivalent multi-layered laminate plate model.
4) Full-scale finite element model.

Optimization technique for sandwich panel has been developed employing the method of experiment design. This methodology is based on a collection of mathematical and statistical techniques that are useful for the modeling and analysis of problems in which a response of interest is influenced by several variables and the objective is to optimize this response. Optimization procedure based on experiment design is not only an effective tool for the optimum design of different systems and processes requiring computationally expensive analyses, but it also easily combines modeling and optimization stages in comparison with other approaches. Moreover, this approach is general in the sense that it permits the optimization of any systems or processes under arbitrary conditions with respect to any objective function (e.g. weight, durability, reliability, cost) taking into account all practical requirements. That is why the methodology examined is used so widely in different areas: aerospace engineering [1], mechanical engineering [2], electrical engineering [3], electrochemical engineering [4], civil engineering [5], material science [6], etc. All these applications can be related to optimization of different technological and operational processes or systems including the obtaining of materials and structures with predicted properties.

Finite element modeling

At present, optimization of steel I-core sandwich panels (Fig.1), with general dimensions of one meter, is considered for square plate, such an assumption will simplify parametric studies and a definition of the cost function. I-core sandwich constructions are made from steel with Young's moduli of E=210 GPa and poisson's ratio of u =0.3. All numerical calculations are carried out by the finite element code ANSYS [7], except analytical solution, which was directly incorporated into the optimization algorithm. Two different types of shell elements were used to describe behavior of the I-core sandwich shell. A 4–node Shell 63 element was used in calculations for a full-scale model as well as an equivalent orthotropic plate, while an 8–node Shell 99 element was used to describe an equivalent multi-layer I-core sandwich plate. Since sandwich plate is admitted as square plate, a span between I stiffeners is a constant 125 mm. It is assumed that the first and the last spans in an I-core panel have half the span dimensions than those in the middle. In this particular case the distance between I stiffeners *p* are taken based on previous experience from the sandwich panel optimization procedure [8]. Simply supported boundary conditions are applied at all outer edges with the uniformly distributed pressure load acting on top of the panel with a magnitude of 1MPa.

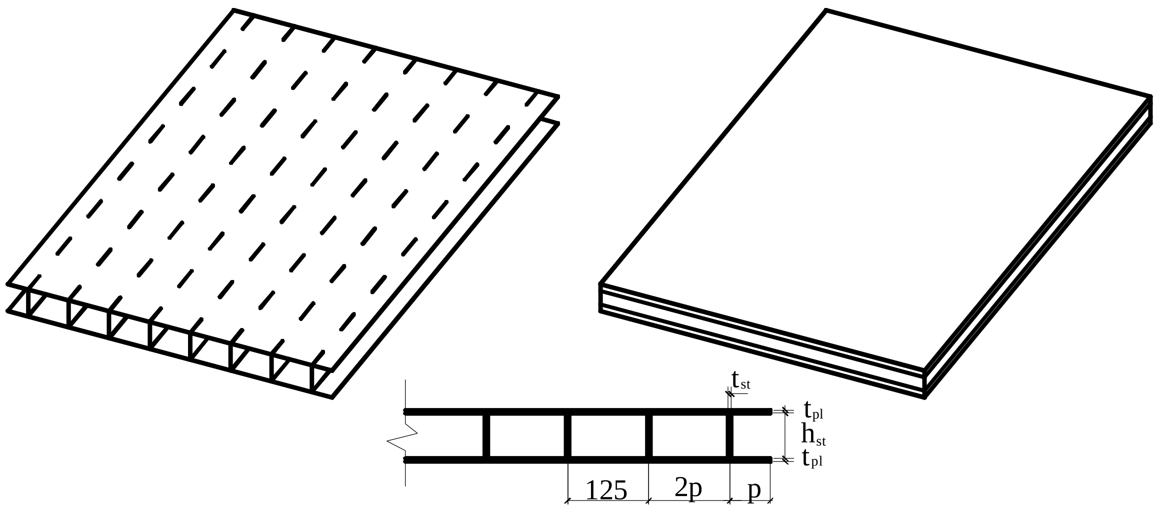

Figure 1. Sandwich panel construction.

Determination of the equivalent plate properties

To considerably decrease the dimension of the numerical problem, the I-core sandwich panel could be considered as an homogeneous orthotropic plate [9] with equivalent stiffness and thickness properties. These equivalent stiffnesses properties for an empty I-core sandwich panel are given in Table 2. Where t_{pl} is the thickness of the face and the bottom plates, t_{st} is the thickness of the stringer plates and h_{st} is the height of the stiffener as well as d being the distance between centroids of the face sheets with G as the shear modulus of the steel. To use ANSYS, homogenized orthotropic material properties should be introduced. These properties are defined from the following relations presented in Table 1.

Table 1. Homogenized orthotropic material properties.

$E_x = \frac{12D_x}{d^3}$	$G_{xy} = \frac{6D_{xy}}{d^3}$	$u_{xy} = u_f$
$E_y = \frac{12D_y}{d^3}$	$G_{xz} = \frac{S_x}{kd}$	$u_{yz} = \frac{D_y}{D_x} u_{xy}$
$E_z = \frac{12D_y}{d^3}$	$G_{yz} = \frac{S_y}{kd}$	$u_{xz} = \frac{D_y}{D_x} u_{xy}$

Where the shear correction factor k is equal to 5/6. In this case the equivalent thickness of the homogeneous plate is equal to the distance d between centroids of faces.

Table 2. Equivalent stiffness for empty I-core sandwich panel

Stiffness	Determination of stiffness	Additional terms
Longitudinal bending	$D_x = \frac{2t_{pl}^3 E}{12} + \frac{t_{pl} d^2 E}{2} + \frac{I_{wx} E}{2p}$	$I_{wx} = \frac{t_{st} h_{st}^3}{12}$ I_{wx} - moment of inertia of stiffener
Lateral bending	$D_y = \frac{2t_{pl}^3 E}{12} + \frac{t_{pl} d^2 E}{2}$	
Twisting	$D_{xy} = t_{pl} d^2 \frac{E}{2(1+u)}$	
Longitudinal shear	$S_x = G \times t_{st} \frac{\frac{d}{2p}\frac{d}{h_{st}}\frac{t_{pl}}{t_{st}} + \frac{1}{6}\left(\frac{h_{st}}{2p}\right)^2}{\frac{t_{pl}}{t_{st}} + \frac{1}{3}\frac{h_{st}}{2p}\frac{h_{st}}{d}}$	$I_{pl} = \frac{t_{pl}^3}{12}$ I_{pl} - moment of inertia of face
Lateral shear	$S_y = \frac{1}{\frac{1}{6EI_{pl}}\frac{1}{p}\left(p - \frac{t_{st}}{2}\right)^3 + \frac{1}{12EI_{st}} pd\left[\left(\frac{t_{pl}}{d}\right)^3 - 3\left(\frac{t_{pl}}{d}\right) + 2\right]}$	$I_{st} = \frac{t_{st}^3}{12}$ I_{st} - moment of inertia of stiffener

Besides as the authors [10,11] note, an equivalent multi-layer laminate theory approach is suitable to describe an equivalent sandwich plate. In the multi-layered laminate plate approach, two outer layers have the same material properties as in a real structure, but middle the layer uses reduced properties as determined and presented in Table 3.

Table 3. Equivalent multi-layer laminate properties

	E_x	$E_y = E_z$	$G_{xy} = G_{yz}$	G_{xz}	v_{xy}	$v_{xz} = v_{yz}$
Top layer	E	E	G	G_t	u_t	u_t
Middle layer	$E \times \frac{t_{st}}{2p}$	0	0	$G \times \frac{t_{st}}{2p}$	$u_w \times \frac{t_{st}}{2p}$	0
Bottom layer	E	E	G	G_b	u_b	u_b

Due to unavoidable singularity in finite element calculations, all zero values have been replaced with the equivalent modulus value divided by a thousand. For comparison one particular example of equivalent properties is presented in Table 4.

Table 4. Comparison of the different equivalent material properties

Full-scale model properties	Homogenized orthotropic material properties [MPa]	Equivalent multi-layer laminate properties [MPa]
t_{pl} = 2.5 mm	E_x = 120100 MPa	E_x =7000 MPa
t_{st} = 2.5 mm	$E_y = E_z$ =114900 MPa	$E_y = E_z$ =7 MPa
h_{st} = 25 mm	G_{xy} = 44060 MPa	G_{xy} = 2692 MPa
2p = 125 mm	G_{xz} = 3377 MPa	G_{xz} = 2.6 MPa
d = 27.5 mm	G_{yz} = 20 MPa	G_{yz} = 2.6 MPa
h =30 mm	$v_{xz} = v_{yz}$ =0.2875	$v_{xz} = v_{yz}$ =0.006

Design of experiments

The design of experiments is formulated for a three-dimensional space (3 design parameters, see Table 5.). For global approximations, it is planned to use second order polynomial functions. It is well known that for such approximating functions the *D*-optimal experimental design is the most suitable [12]. Since we have discrete and continuous design variables, the combined Latin hypercube and *D*-optimal experimental design was chosen. The second order model contains $L=(k+1)(k+2)/2$ parameters. Therefore for $k=3$, the number of parameters is $L=10$. It is recommended that the minimal number of runs is about twice that number of coefficients L in the approximating function. For this reason we should carry out 20 numerical experiments.

Table 5. Boundaries of design parameters

Minimum bound	Design parameter	Maximum bound
1.5 mm	t_{pl}	4 mm
1.5 mm	t_{st}	4 mm
50 mm	h_{st}	100 mm

Since thicknesses of the top and the bottom panels, as well as stinger thickness from a manufacturing point of view could be produced with step 0.5 mm, these two variables have been discretized into 6 levels as presented in figure 2.

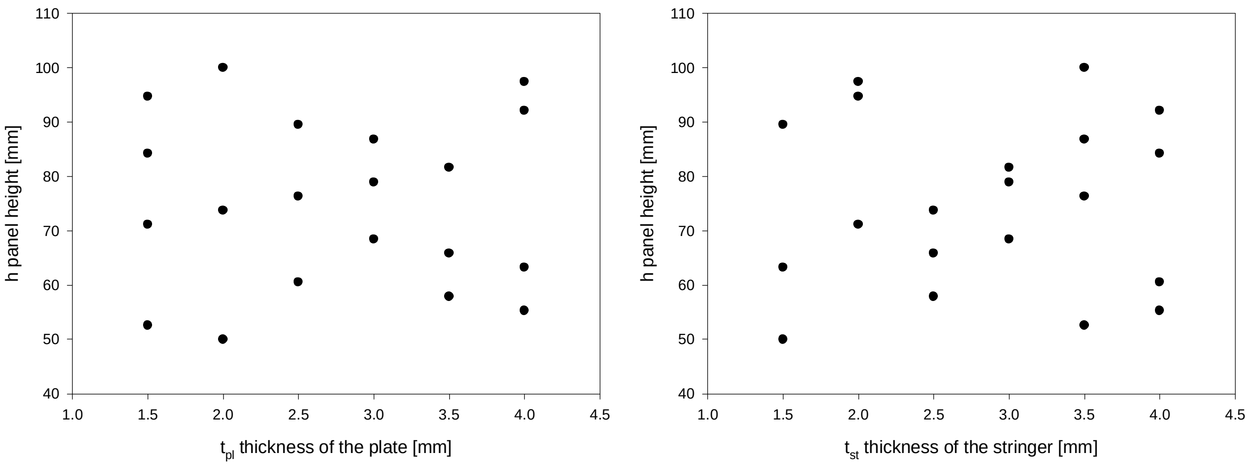

Figure 2. Experimental design in planes 1-3 and 2-3.

Numerical analysis

A numerical analysis can be divided into two parts: the first approach is based on analytic formulas (1,2) evaluated by Zenkert [9,10] the second approach is connected with a finite element code ANSYS solution.

$$f_{\max} = W \times (\sin(\frac{p\frac{a}{2}}{a}) \times \sin(\frac{p\frac{a}{2}}{a}) \times \sin(\frac{p\frac{b}{2}}{b})) \quad (1)$$

$$W = \frac{16qb^4(1-v_{xy}v_{xz})}{p^6(D_x\left(\frac{b}{a}\right)^4 + 2(D_x v_{xz} + (1-v_{xy}v_{xz})D_{xy})\left(\frac{b}{a}\right)^2) + D_y)} + \frac{16qb^2}{p^4(S_x(\frac{b}{a})^2 + S_y)} \quad (2)$$

Where equivalent stiffness for empty I-core sandwich panel (presented in Table 2.) and q is uniformly distributed pressure load, a and b panel longitudinal and transverse edge length. Finite element modeled with mesh of 20 x 20 with 642 degrees of freedom was used to solve sandwich panel with equivalent orthotropic material properties. In comparison 1882 degrees of freedom where used to model an equivalent multi layered laminate material plate. In contrast a full-scale model was elaborated with 49476 degrees of freedom. In the full-scale model (presented in Figure 3.) local and global bending of the sandwich plate could be observed contrariwise to equivalent material properties of the multi-layered model and analytical solution.

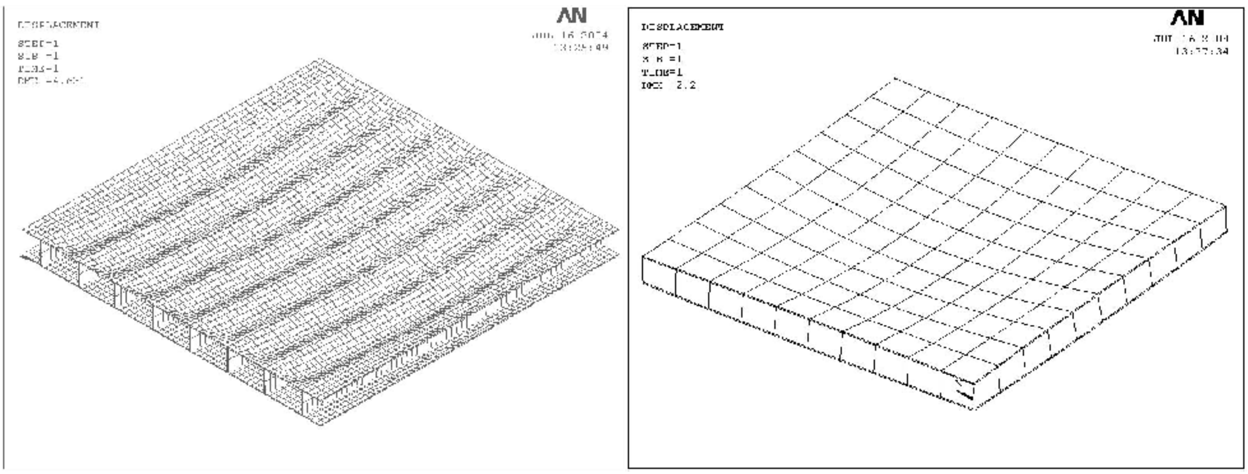

Figure 3. Full-scale sandwich plate bending (h=76;t_{pl}=3.5;t_{st}=2.5) and laminate plate bending.

Response surface technique

Approximations $\hat{F}$ of the finite element results F are performed by the second order polynomial functions

$$\hat{F}(x) = b_0 + \sum_{i=1}^{K} b_i x_i + \sum_{i=1}^{K}\sum_{k=i}^{K} b_{ik} x_i x_k \quad (3)$$

These approximations for all functions have been obtained using a conventional un-weighted least squares estimation with the elimination of three outlier runs for each function. The error of approximations is calculated by the following expressions

$$s = \sqrt{\frac{\sum_{i=1}^{N}\left(F(x^i) - \hat{F}(x^i)\right)^2}{N}}, \quad s_0 = \sqrt{\frac{\sum_{i=1}^{N}\left(F(x^i) - \hat{F}(x^i)\right)^2}{N-L}} \quad (4)$$

where s is a squared error, s_0 is a standard deviation, $F(x^i)$ is the value of original function in the sample point x^i of the experimental design, $\hat{F}(x^i)$ is the value of approximating function in the sample point x^i of the experimental design and N is the number of points used for approximation.

At the beginning, approximations have been obtained for all functions employing all sample points. The relative errors for all examined variants and variants where the three worst points were deleted due to cross validation are given in Table 6.

Table 6. Approximation relative errors

	Analytical solution	Homogeneous ortothropic plate	Multi-layer laminate plate	Full-scale FEM model
Without elimination	6.63%	8.96%	9.73%	7.41%
With elimination of 3 worst points	5.66%	4.88%	4.50%	4.00%

Parametric studies

Parametric studies are carried out additionally for designer convenience to investigate the influence of different design parameters on behavioral functions. This can be done by displaying contour plots 2D or 3D graphs of the approximating functions (surrogate models). The plate deflection dependencies over different design parameters are given in Figures 4-5.

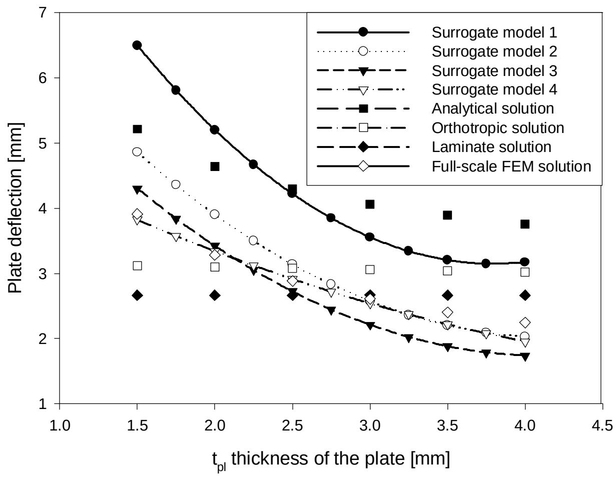

Figure 4. Plate deflection dependency over thickness of the sandwich panel.

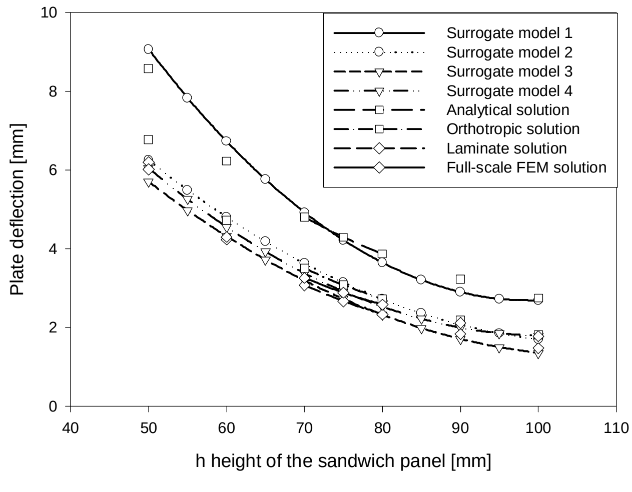

Figure 5. Plate deflection dependency over height of the sandwich panel.

In these figures the approximation functions (surrogate models 1-4) of four different approaches are plotted compared with values obtained with the direct solution of a particular case.

Optimization results

The present weight minimization problem (5) is solved using the random search method. Optimum results for the sandwich construction modeled by different plate elements are presented in Table 7. In this particular optimization example, deflection constrained up to 1/400 from panel length and *h* height of the sandwich panel could exceed 75 mm.

$$G = (t_{pl}ab + (h - 2t_{pl})8at_{st})0.00000785 \qquad G \rightarrow \min \qquad (5)$$

Table 7. Optimum solution

Optimum solution	Analytic solution	Homogeneous orthotropic plate	Multi-layer laminate plate	Full-scale FEM model
Weight [kg]	71.52	61.06	53.27	56.82
t_{pl} thickness of the plate [mm]	3.5	3.5	3	2.5
t_{st} thickness of the stringer [mm]	4	1.5	1.5	4
h height of the sandwich panel [mm]	72.99	71.90	71.53	74.94

Conclusions

The I-core sandwich optimization problem has been formulated and methodology based on experimental design and response surface technique has been developed for the minimization of the construction weight. To describe the behavior of sandwich construction under bending loads, the finite element method together with the analytical solution has been applied in the sample points of experimental design. Approximations of the original functions for behavioral constraints have been obtained using second order polynomials. The minimization problem has been solved for the real construction by a method of random search employing the approximating functions instead of original functions. Parametric studies and sensitivity analysis has been carried out additionally for designer convenience. Based on the collected results equivalent material properties could not be used to determinate I-core stringer thickness, plate thicknesses and plate height. These are parameters that could be used to optimize sandwich construction.

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Kalniņš K., Skuķis E. I-core sandwich panel optimization employing different plate models.
Validation between different plate elements used to optimize I-core sandwich panels will be discussed in this present paper. The effectiveness of determining the most suitable plate element will lead into development of the fast and reliable reanalysis tool since application of a direct minimization algorithm and multiple finite element analysis is too expensive from a computational point of view. For this reason four different numerical solution approaches were chosen to investigate an I-core sandwich panel. Surrogate models were elaborated using response surface methodology. Based on these response function approximations, parametric studies were performed. Design optimization were performed using the random search method.

Kalniņš K., Skuķis E. I-serdes tipa daudzslāņu paneļu optimizācija, izmantojot dažādu plātņu modeļus
Dotajā darbā tiek salīdzināta efektivitāte dažādu plātņu aprēķina modeļiem, tos izmantojot I-serdes tipa daudzslāņu paneļu optimizācijā. Nosakot vispiemērotāko plātnes aprēķina modeli nodrošinātu efektīvu ātrās simulācijas procedūras izstrādi, jo tieša minimizācijas algoritma pielietošana un daudzkārtēji galīgo elementu aprēķini ir pārāk laikietilpīgi no skaitļošanas viedokļa. Šī iemesla dēļ četras dažādas skaitliskas metodes tika izvēlētas, lai pētītu parametru ietekmi I-serdes daudzslāņu panelī. Surogātie modeļi plātnei tika izstrādāti ar virsmas atbildes metodoloģiju. Izmantojot iegūtos rezultātus parametru ietekmes analīze tika veikta. Svara minimizācijai nejaušas meklēšanas metode tika izmantota.

К. Калниньш, Э. Скукис. Оптимизация I–профиля сэндвич панели, с использованием различных моделей пластин.
Сравнение результатов между разными моделами решений пластин, которые используются в I–профиля сэндвич панелей, были рассмотрены в данной работе. Эффективность определения наиболее подходящей эквивалентной характеристики пластины, ведет к быстрому и надежному развитию инструмента расчета и анализа т.к. применение прямого оптимизационного алгоритма и многократного расчета конечного элемента, получается слишком дорого с точки зрения вычислений. В данном случае, для изучения I–профиля сэндвич панелей было выбрано 4 численных решения. Суррогатная модель была разработана по методу частотного отклика. Основываясь на функции частотного оклика, были изучены параметры влияния. Для оптимизации модели был выбран случайный метод поиска.