IDENTIFICATION OF ELASTIC CONSTANTS OF CARBON/EPOXY COMPOSITE PLATES

OGLEKĻŠĶIEDRAS KOMPOZĪTMATERIĀLA ELASTĪGO ĪPAŠĪBU IDENTIFIKĀCIJA

Ruchevskis S.

Keywords: identification, laminated composites, experiment design, elastic constants

Introduction

The determination of stiffness parameters for complex materials such as fiber-reinforced composites is much more complicated that for isotropic materials. Conventional methods of determination of stiffness parameters of composite materials are based on the direct measurements of strain fields. Boundary effects, sample size dependencies and difficulties in obtaining homogeneous stress and strain fields are some of the serious problems. Because of this, indirect methods have recently received increasing attention. One of such indirect method is based on measurements of the dynamic behavior of composite plates and application of the numerical-experimental identification technique [1].

While the use of the polymer and fiber reinforced polymer materials was rapidly growing in the last years, it became apparent that the classical techniques for measuring the material properties did no longer satisfy the needs [2]. Many problems arose, due to the microscopically heterogeneous character and the high degree of anisotropy in these materials. Simultaneously due to the evolution of computer science, numerical techniques for solving problems in solid mechanics became more and more widespread [2]. During the last decade investigations to develop a new technique for material identification, the so-called mixed numerical-experimental technique, started [3].

Numerical-experimental identification methods mainly are used in structural applications [1-9]. For example, Mota Soares *et al.* [1] identified the elastic properties of laminated composites by using the experimental eigenfrequencies. The stiffness parameters were identified from measured natural frequencies of the laminated composite plate by direct minimization of the identification functional. Similar approach in order to identify the stiffness properties of the laminate composites was also used by other authors (Araujo *et al.* [10], Frederiksen [11]).

In the present study instead of direct minimization of the criterion it is proposed to use the method of planning of experiments. The main advantage of this method is significant reduction of the number of computations of the criterion. The response surface approximations are obtained using the information on the behaviour of a structure in the reference points of the experiment design. The finite element modeling is performed only in the reference points. The functional to be minimized describes the difference between the measured and numerically obtained parameters of the response of structure. By minimizing the functional the identification parameters are obtained. Previously this method was used in the solution of the optimal design problems of laminated composites and sandwich plates [12-14] and also for the identification of mechanical properties of laminates [15].

Parameters of identification and criterion

The numerical-experimental method proposed in the present investigation consists of the following stages. In the first stage the physical experiments have been performed. Also the parameters to be identified, the domain of search and criterion containing experimental data have been selected. In the second stage the finite element method has been used in order to model the frequency response of the structure. The results of the finite element solution of the eigenvalue problem have been employed as numerical experimental data. The finite element calculations have been performed in the reference points of the variables to be identified. The experiment design points have been determined using the method of design of experiments. In the third stage the numerical data obtained by the finite element solution in the reference points have been used in order to determine a simple functions using response surface method for calculation of the eigenfrequencies. In the fourth stage on the basis of the simple models and experimental data of the measured eigenfrequencies the identification of the material properties is performed. For this a corresponding functional is minimized using a conventional method of non-linear programming.

Parameters of identification

The proposed numerical-experimental approach is used for identification of the elastic properties of laminated composite plates. For this the experimental data of the measured eigenfrequencies have been used. It is assumed that the plate dimensions (see Fig. 1), plate mass and the layer stacking sequence is to be known. The plate is composed of transverse isotropic layers with principle material directions 1-2-3, where 1 is the fiber direction and 2,3 are the directions transverse to the fiber direction. In general the *i*th layer of the laminated plate is oriented at an arbitrary angle β_i. The angles of the layers assumed to be fixed.

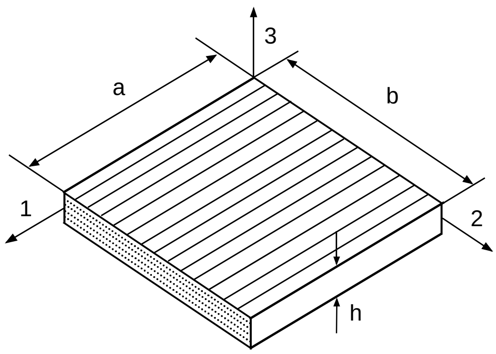


Fig.1. Unidirectional reinforced laminated composite plate.

The parameters to be identified are the elastic constants of a single layer in the laminated composite plate. These five parameters of a transverse isotropic layer are

E_1, $E_2 = E_3$	- two Young's modulus,
$G_{12} = G_{13}$	- shear modulus,
$\nu_{12} = \nu_{13}$	- Poisson's ratio,
G_{23}	- shear modulus in the plane of isotropy.

The vector of parameters $\boldsymbol{x}$ to be identified can be chosen in a different ways [15]. The major problem in parameter estimation is ill condition caused by unknown variables having substantially different order of magnitude. For example, Young's modulus and Poisson's ratio cannot be directly selected as components of vector $\boldsymbol{x}$ without proper scaling.

In this study it is proposed to use two different methods of scaling. In Frederiksen [18] the scaling by longitudinal modulus E_1 was employed and in addition a fixed scaling factor was chosen. Similar scaling and reparametrisation was employed in [15], where additional scaling by the first experimental frequency allows reducing the number of unknown variables from five to four. Thus, material parameters of single layer can be expressed in terms of dimensionless variables α_I [1]

$$\begin{aligned}
\alpha_2 &= 4 - 4(E_2 / E_1) \\
\alpha_3 &= 1 + (E_2 / E_1)(1 - 2\nu_{12}) - 4(G_{12} / E_1)\alpha_0 \\
\alpha_4 &= 1 + (E_2 / E_1)(1 + 6\nu_{12}) - 4(G_{12} / E_1)\alpha_0 \\
\alpha_5 &= 4(G_{23} + G_{12})\alpha_0 / E_1
\end{aligned} \tag{1}$$

Here

$$\alpha_0 = 1 - \nu_{12}^2 \frac{E_2}{E_1} \tag{2}$$

Now the parameters to be identified $\boldsymbol{x}$ are defined through non-dimensional quantities α_i

$$\boldsymbol{x}_{(1)} = [x_1, x_2, x_3, x_4] = [\alpha_2, \alpha_3, \alpha_4, \alpha_5] \tag{3}$$

Let the experimental eigenfrequencies be designated by $\overline{\omega}_1, \overline{\omega}_2, ..., \overline{\omega}_I$, where I is the number of measured eigenfrequencies $\bar{f}_i$ ($\overline{\omega}_i = 2\pi \bar{f}_i$). The value of I is typically taken between 7 and 15. The corresponding numerical eigenfrequencies f_i for the set of given material parameters α_i are represented by $\omega_1, \omega_2, ..., \omega_I$. Let us consider the scaling parameter C, which is chosen through the relation

$$C = \frac{\overline{\omega}_1^2}{\tilde{\omega}_1^2} \tag{4}$$

where $\tilde{\omega}_1 = C\omega_1 = 2\pi C f_1$ is the first numerical eigenfrequency calculated with the prior selected longitudinal Young's modulus E_1^0.

The second set of identification parameters is defined by the stiffness A_{ij}. The vector of parameters $\boldsymbol{x}_{(2)}$ to be identified is given by [15]

$$\boldsymbol{x}_{(2)} = [x_1, x_2, x_3, x_4, x_5] = [A_{11}, A_{22}, A_{12}, A_{44}, A_{66}] \tag{5}$$

Here

$$A_{11} = \frac{E_1}{1 - \nu_{12} E_2 / E_1} \qquad A_{22} = \frac{E_2}{1 - \nu_{12} E_2 / E_1} \qquad A_{12} = \frac{\nu_{12} E_2}{1 - \nu_{12} E_2 / E_1}$$

$$A_{44} = G_{23} \qquad A_{66} = G_{12} = G_{13}$$

These parameters practically are to be on compatible scales. However, in order to obtain the engineering constants from in addition a system of three non-linear equations should be solved. Identification procedure can be formulated either using the vector of unknown variables $\boldsymbol{x}_{(1)}$ or $\boldsymbol{x}_{(2)}$.

Identification functional and minimization problem

In the first case of formulation of the identification problem the vector $\boldsymbol{x}_{(1)}$ is employed and is defined by equation (3). The functional to be minimized is the deviations between the measured $\overline{\omega}$ and numerically calculated $\tilde{\omega}$ frequencies [1].

$$\Phi(\boldsymbol{x}_{(1)}) = \sum_{i=2}^{I} w_i^{(1)} \frac{\left(\overline{\omega}_i^2 - C\left[\tilde{\omega}_i(\boldsymbol{x}_{(1)})\right]^2\right)^2}{\overline{\omega}_i^4} \tag{6}$$

It is seen that criterion (6) is a non-linear function of the parameters of identification $\boldsymbol{x}_{(1)}$. The identification of the material properties $\boldsymbol{x}_{(1)}$ is performed on the basis of information obtained by the measurements of the *I* lowest frequencies. The identification problem is formulated as follows [1]

$$\min_{\boldsymbol{x}_{(1)}} \Phi(\boldsymbol{x}_{(1)}) \tag{7}$$

Subject to

$$g_1(\boldsymbol{x}) = \alpha_2 > 0 \quad \text{or} \quad E_1/E_2 > 1 \tag{8}$$

$$g_2(\boldsymbol{x}) = \frac{(8 - \alpha_2 - 3\alpha_3 - \alpha_4)}{16\left\{1 - \left[\dfrac{\alpha_4 - \alpha_3}{8 - 2\alpha_2}\right]^2 \left(\dfrac{4 - \alpha_2}{4}\right)\right\}} > 0 \quad \text{or} \quad G_{12}/E_1 > 0 \tag{9}$$

$$g_3(\boldsymbol{x}) = \frac{\left[2\alpha_5 - 1/2(8 - \alpha_2 - 3\alpha_3 - \alpha_4)\right]}{8\left\{1 - \left[\dfrac{\alpha_4 - \alpha_3}{8 - 2\alpha_2}\right]^2 \left(\dfrac{4 - \alpha_2}{4}\right)\right\}} > 0 \quad \text{or} \quad G_{23}/E_1 > 0 \tag{10}$$

$$g_4(\boldsymbol{x}) = -\left|\frac{(\alpha_4 - \alpha_3)}{(8 - 2\alpha_2)}\right| + \sqrt{\frac{4}{4 - \alpha_2}} > 0 \quad \text{or} \quad \sqrt{E_1/E_2} - |\nu_{12}| > 0 \tag{11}$$

$$\alpha_i^{\min} \le \alpha_i \le \alpha_i^{\max}; \quad i = 2,3,4,5 \tag{12}$$

Here $\alpha_i^{\min}$, $\alpha_i^{\max}$ are the lower and upper side constraints, respectively. The upper and the lower limits of the parameters of identification are chosen different for each numerical example of identification.

In the second formulation of the identification problem the vector $\boldsymbol{x}_{(2)}$ is employed and the functional to be minimized is as follows [15]

$$\Phi(\boldsymbol{x}_{(2)}) = \sum_{i=1}^{I} w_i^{(2)} \frac{\left(\overline{\omega}_i^2 - \left[\tilde{\omega}_i(\boldsymbol{x}_{(2)})\right]^2\right)^2}{\overline{\omega}_i^4} \tag{13}$$

Here $w_i^{(2)}$ are weights for identification functional using the second set of variables. Again constrains for upper and lower bound are used and in addition constrains for positive definiteness of elasticity matrix are employed

$$g_1(\boldsymbol{x}_{(2)}) = A_{11} > 0 \qquad g_2(\boldsymbol{x}_{(2)}) = A_{22} > 0 \qquad g_3(\boldsymbol{x}_{(2)}) = A_{11}A_{22} - A_{12}^2 > 0$$
$$g_4(\boldsymbol{x}_{(2)}) = A_{44} > 0 \qquad g_5(\boldsymbol{x}_{(2)}) = A_{66} > 0 \tag{14}$$

Experiment

Experiments have been performed on unidirectional carbon/epoxy laminate. Plates were tested for vibrations in order to measure eigenfrequencies and corresponding modes. Experiments were performed with free boundary conditions on all edges of the plate in order to exclude influence of boundary conditions to the results of identification.

Natural frequencies of the test plates were measured using a scanning laser and a PC to store the data. The specimens were hung from one of their corners using a rubber band, simulating free boundary conditions along the plate's edges (see Fig. 2.). The plate was excited using a loudspeaker located behind it. The loudspeaker was connected to an amplifier and the frequency was varied using a frequency supplier. Response of the plate is measured by the scanning laser and stored on a PC. To enable a better scanning the specimens were painted in white. A typical test procedure is as follows: the plate is excited continuously and the laser measures its response while it is located on one fixed point. The data stored on the PC is used to perform a Fast Fourier Transform (FFT) technique yielding the experimental eigenfrequencies of the plate. Then the experimental results are compared with the predicted frequencies, which were calculated by the finite element code employing the initial guess values of elastic constants, for example, from static tests. Such preliminary finite element calculations are necessary to be sure that all experimental frequencies are recorded in the range. The mode shape associated with each natural frequency is then measured by scanning the specimen with the laser in lines parallel to its edges (the distance between two consecutive parallel lines can be adjusted by the user, according to the mode shape number, to yield the required accuracy), while being exited at one of its eigenfrequencies. The data is then stored on a PC. Using the MATLAB code the data is converted to yield the mode shape of the vibrating plate at one of its natural frequencies.

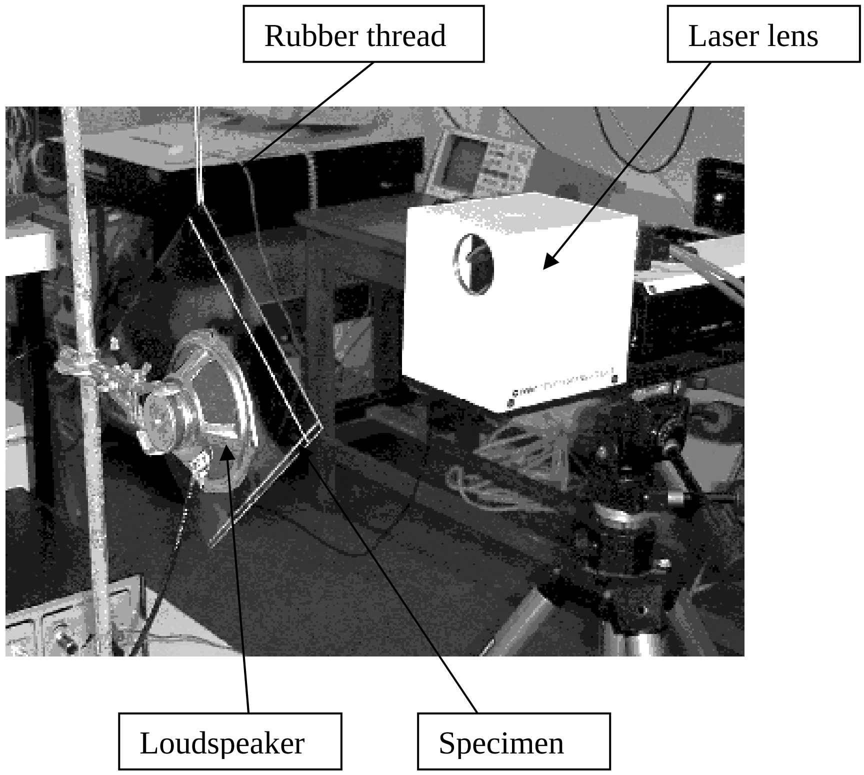


Fig.2.Vibration experiment of carbon/epoxy plates.

Method of Experiment Design

Let us consider a criterion for elaboration of plans of experiment, which is to be independent on the mathematical model of the object. Initial information for elaboration of the plan is the number of variables *n* and the number of experiments *k.* The main principles in the proposed approach are as follows [12]

1) the number of levels in the domain of experiments for each variable is equal to the number of the experiments and for each level only one experiment is performed;

2) the reference points (points of experiments) in the domain of experiments are distributed as regular as possible.

The plan of experiment is characterized by the matrix of the plan B_{ij}. The domain of the experiments is determined as $x_j \in \left[x_j^{min}; x_j^{max}\right]$. Thus, in this domain the reference points, where the experiments must be performed, are calculated by the expression

$$x_j^{(i)} = x_j^{min} + \frac{1}{k-1}\left(x_j^{max} - x_j^{min}\right)\left(B_{ij} - 1\right) \tag{15}$$

Here $i = 1, 2, ..., k$ and $j = 1, 2, ..., n$.

The details of this procedure and corresponding program PLANEX were described in [12].

Approximation of Response Surface

Information about the behaviour of the object can be obtained by the physical experiment or by the computer solution in the reference points. This information can be represented as a table of data, where the response function y(x) of the object is to be in relationship to the variables $x_1, x_2, ..., x_n$. The goal is by using the data of experiments (in our case data are obtained by the Finite Element solution in the reference points) to obtain the relation y(x) in the mathematical form or so called equation of regression. The details of this procedure and corresponding program RESINT were described in [12].

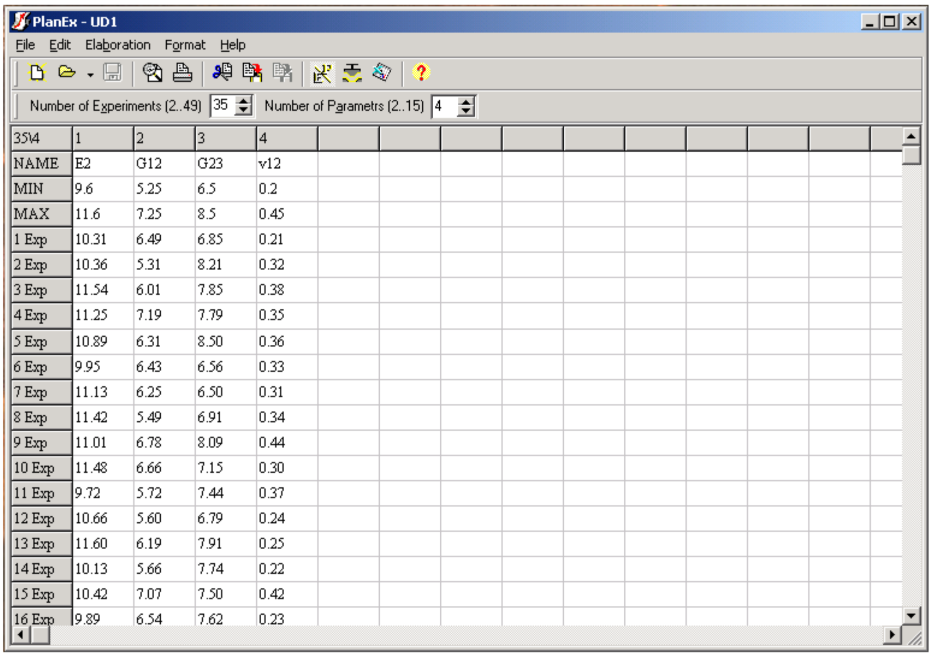
Software developed for the identification procedure

In the first stage of identification procedure the experiment design is selected. For this program PLANEX (see Fig. 3.) or software (see Fig. 4.) developed using *Compaq Visual Fortran* programming language can be used.

The second stage is Finite Element solution. A 22 x 22 regular mesh (968 Finite Elements) is considered in order to achieve appropriate accuracy for at least 20 first eigenvalues of the laminated plate with FFFF (all edges free) boundary conditions. A triangular Finite Element of laminated thick plate with a shear correction [17] is selected. Finite Element code and corresponding software was developed using *Compaq Visual Fortran* programming language (see Fig. 4.).

When Finite Element solution is done results are written into data files, which can be used to build a mode shape for corresponding eigenfrequency. Appropriate software using *Compaq Visual Fortran* is designed (see Fig. 5.).

In the third stage of identification procedure the data of the numerical simulations is used to determine the response surfaces. For this program RESINT is employed. The elimination diagram for the first frequency of the plate UD1 is shown in Fig. 6.



35\4	1	2	3	4
NAME	E2	G12	G23	v12
MIN	9.6	5.25	6.5	0.2
MAX	11.6	7.25	8.5	0.45
1 Exp	10.31	6.49	6.85	0.21
2 Exp	10.36	5.31	8.21	0.32
3 Exp	11.54	6.01	7.85	0.38
4 Exp	11.25	7.19	7.79	0.35
5 Exp	10.89	6.31	8.50	0.36
6 Exp	9.95	6.43	6.56	0.33
7 Exp	11.13	6.25	6.50	0.31
8 Exp	11.42	5.49	6.91	0.34
9 Exp	11.01	6.78	8.09	0.44
10 Exp	11.48	6.66	7.15	0.30
11 Exp	9.72	5.72	7.44	0.37
12 Exp	10.66	5.60	6.79	0.24
13 Exp	11.60	6.19	7.91	0.25
14 Exp	10.13	5.66	7.74	0.22
15 Exp	10.42	7.07	7.50	0.42
16 Exp	9.89	6.54	7.62	0.23

Fig. 3. Creation of experiment design by program PLANEX.

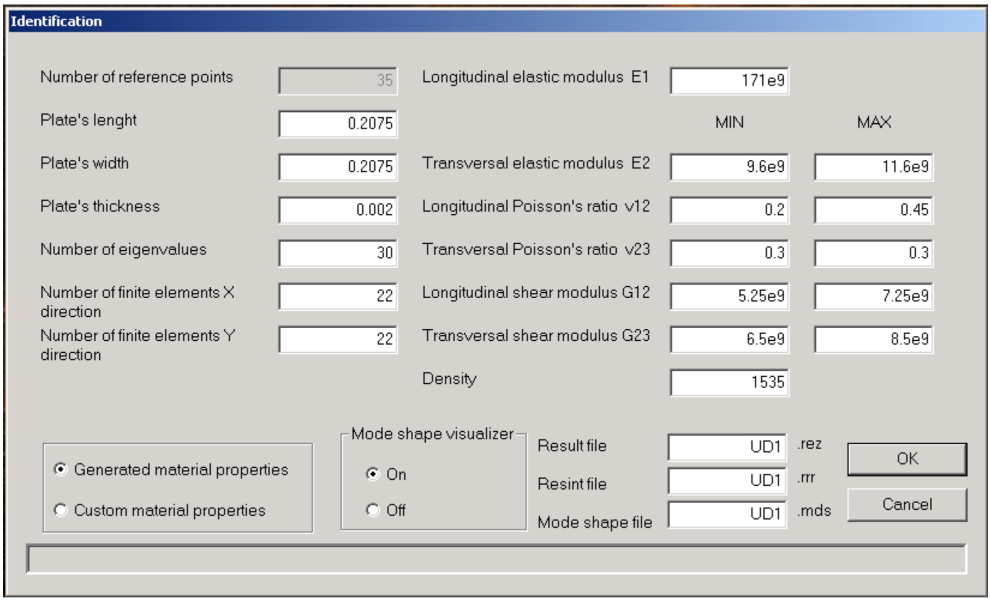


Fig. 4. "Input data table" of the software for identification procedure.

The last stage of identification procedure is minimization of the identification functional. It is performed using mathematical library of *Compaq Visual Fortran*.

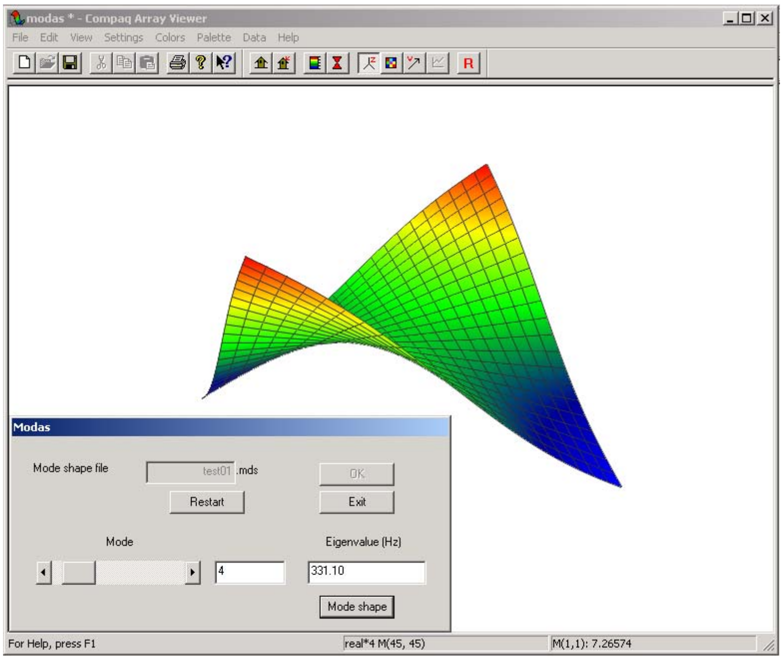


Fig. 5. “Mode shape visualizer” of the software for identification procedure.

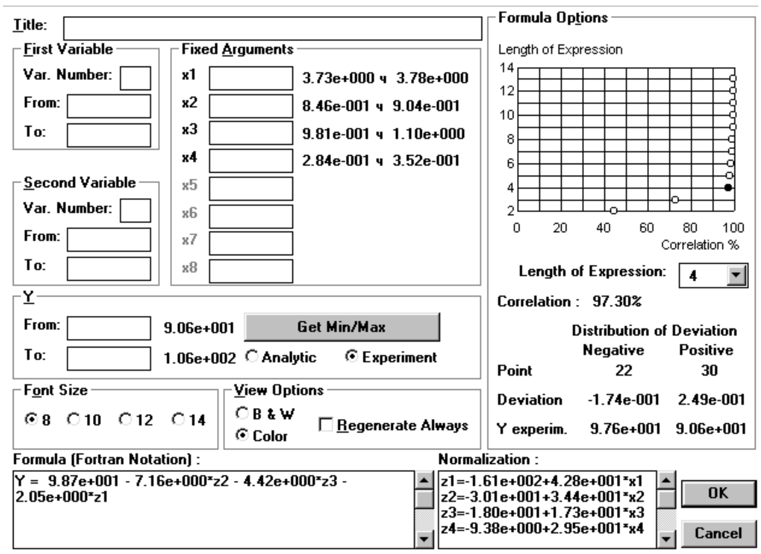


Fig. 6. Diagram of elimination generated by RESINT.

Identification examples

Identification of the elastic constants of unidirectional carbon/epoxy laminated plates from the measured eigenfrequencies was performed in two ways. In the first approach the identification was carried out by the first set of identification parameters (4) and for the second - the second set of identification parameters (5) was employed.

For the identification three carbon/epoxy plates were manufactured and tested. Fiber volume content of the present carbon/epoxy laminate is about 60%. Nominal thickness of the plates was h=2 mm. Geometric dimensions of all specimens are presented in *Table 1*.

Table 1. Geometric dimensions and density of plates for vibration tests

Plate No.	***a*, mm**	***b*, mm**	***h*, mm**	**ρ, kg/m^3**
UD1	207.5	207.5	2.000	1535
UD3	205	205	1.969	1598
UD5	205	205	1.960	1589

Experimental eigenfrequencies $f_i^{\exp}$ and eigenmodes are presented in *Table 2*. Since all frequencies experimentally were not observed, frequencies are ranged according to the finite element solution employing the initial guess values of elastic constants obtained from the static tests. In the second column corresponding vibration modes (m, n) are given, where m is number of nodal lines in the direction of axis 1 and n is number of nodal lines in the direction of axis 2. Note that axis 1 denotes a fiber direction and axis 2 denotes a transverse direction. It is seen that all frequencies were not observed in the experiment. For example, for the plate UD1 in the range of first 19 numerical frequencies only 12 experimental frequencies were measured.

Table 2. Experimental frequencies $f_i^{\exp}$ [Hz] and modes

Moda(*m*,*n*)	**UD1**	**UD3**	**UD5**
4 (1,1)	97	98.2	98
5 (2,0)	123.8	126	127
6 (2,1)	237.4	237	238
7 (3,0)	341	345.6	350
8 (3,1)	458	457	458
9 (0,2)	502.6	498	496
10 (1,2)	541.2	536.5	533
11 (2,2)	653	654	649
12 (4,0)	-	-	-
13 (4,1)	-	780	-
14 (3,2)	-	863	858
15 (5,0)	-	1125	-
16 (4,2)	1168	-	-
17 (5,1)	-	1242	-
18 (0,3)	1381	1368.5	1355
19 (1,3)	1413	1401.6	1391
20 (2,3)	1512	1506.5	1487
21 (5,0)	-	-	-
22 (3,3)	-	-	-

Determination of the elastic constants using the first set of identification parameters

The experiment design with four variables (n = 4) and 35 reference points (k = 35) was selected. The initial value of the Young's modulus E_1^0 [15] and the upper and lower limits of the identification parameters of corresponding plate are taken as follows

UD1	E_1, E_2, G_{13}, G_{23} [GPa] ν_{12}	
	Min	*Max*
E_1^0	171	
E_2	9.6	11.6
$G_{12} = G_{13}$	5.25	7.25
G_{23}	6.5	8.5
ν_{12}	0.2	0.45

UD3	E_1, E_2, G_{13}, G_{23} [GPa] ν_{12}	
	Min	*Max*
E_1^0	170	
E_2	10.1	11.1
$G_{12} = G_{13}$	5.5	6.6
G_{23}	5.3	6.5
ν_{12}	0.2	0.45

UD5	E_1, E_2, G_{13}, G_{23} [GPa] ν_{12}	
	Min	*Max*
E_1^0	170	
E_2	10.6	11.6
$G_{12} = G_{13}$	5.8	6.8
G_{23}	4.0	5.2
ν_{12}	0.2	0.45

The lower and upper bounds of identification parameters are recalculated by equation (1). Using the expression (15) the values of all four identification parameters are calculated in the 35 reference points. In each reference point the finite element solution of the eigenvalue problem for the first 30 frequencies was obtained.

The data of the numerical simulations were used to determine the response surfaces. For this the RESINT program [12] was employed. The elimination diagram for the first frequency of the plate UD1 is shown in Fig. 6. The simple model for the first frequency of the plate UD1 is given by the expression with four terms

$$y_1(\mathbf{x}) = 98.7 - 2.05z_1 - 7.16z_2 - 4.42z_3 \tag{16}$$

$$z_1 = -161 + 42.8\mathbf{x}_1 \quad z_2 = -30.1 + 34.4\mathbf{x}_2 \quad z_3 = -18 + 17.3\mathbf{x}_3$$

Similar models as expression (16) have been obtained also for the other frequencies. For the identification only the "best" experimental eigenfrequencies have been selected.

Minimization of the functional (6) subject to constraints (7-11) was performed using mathematical library of *Visual Fortran*. Results of the identification are given in *Table 3*.

Table 3. Mechanical properties obtained by the identification (1st set)

	UD1	UD3	UD5
E_1 [GPa]	171.05	172.88	171.37
E_2 [GPa]	10.44	10.95	11.2
$G_{12} = G_{13}$ [GPa]	6.07	6.35	6.35
G_{23} [GPa]	7.71*	5.82	4.57
ν_{12}	0.477*	0.337	0.233

It is of interest to compare the experimental eigenfrequencies with the calculated frequencies (see *Table 4*), where the elastic properties obtained by the identification are used. Residuals were calculated by the expression

$$\Delta_i = \frac{f_i^{FEM}(\boldsymbol{x}^*) - f_i^{\exp}}{f_i^{\exp}} \times 100 \qquad (17)$$

It is seen that differences between the experimental and numerical frequencies using elastic properties obtained by the identification are very small. Mostly residuals do not exceed 1%.

Table 4. Flexural frequencies f_i [Hz] and residuals Δ_i (%) for plates UD1, UD3, UD5 (1st set)

Moda (*m*,*n*)	**UD1**			**UD3**			**UD5**		
	Exp	FEM	Δ_i (%)	Exp	FEM	Δ_i (%)	Exp	FEM	Δ_i (%)
4 (1,1)	97	97.14	-0.144	98.2	98.18	0.020	98	98	0.000
5 (2,0)	123.8	124.2	-0.323	126	125.7	0.238	127	126.9	0.079
6 (2,1)	237.4	234.4	1.264	237	236.9	0.042	238	236.9	0.462
7 (3,0)	341	342.2	-0.352	345.6	346.3	-0.203	350	349.5	0.143
8 (3,1)	458	454	0.873	457	458.9	-0.416	458	460	-0.437
9 (0,2)	502.6	502.6	0.000	498	499.8	-0.361	496	496.1	-0.020
10 (1,2)	541.2	539.4	0.333	536.5	537.4	-0.168	533	533.3	-0.056
11 (2,2)	653	650	0.459	654	651.2	0.428	649	648.4	0.092
12 (4,0)	-	-	-	-	-	-	-	-	-
13 (4,1)	-	-	-	780	787.2	-0.923	-	-	-
14 (3,2)	-	-	-	863	862.6	0.046	858	857.2	0.093
15 (5,0)	-	-	-	1125	1126	-0.089	-	-	-
16 (4,2)	1168	1167	0.086	-	-	-	-	-	-
17 (5,1)	-	-	-	1242	1228	1.127	-	-	-
18 (0,3)	1381	1377	0.290	1368.5	1370	-0.110	1355	1359	-0.295
19 (1,3)	1413	1408	0.354	1401.6	1402	-0.029	1391	1390	0.072
20 (2,3)	1512	1503	0.595	1506.5	1499	0.498	1487	1487	0.000

Determination of the elastic constants using the second set of identification parameters

The experiment design with five variables (n = 5) and 36 reference points (k = 36) was selected. The upper and lower bounds of the identification parameters are taken as follows and are recalculated by equation (5). Using the expression (15) the values of all five identification parameters are calculated in the 36 reference points. In each reference point the finite element solution of the eigenvalue problem for the first 30 frequencies was obtained.

UD1	E_1, E_2, G_{13}, G_{23} [GPa] ν_{12}	
	Min	*Max*
E_1^0 [GPa]	168	174
E_2 [GPa]	9.5	11.5
$G_{12} = G_{13}$ [GPa]	5.2	7.2
G_{23} [GPa]	4	8
ν_{12}	0.2	0.45

UD5	E_1, E_2, G_{13}, G_{23} [GPa] ν_{12}	
	Min	*Max*
E_1^0 [GPa]	168	174
E_2 [GPa]	9.5	11.5
$G_{12} = G_{13}$ [GPa]	5.2	7.2
G_{23} [GPa]	4	8
ν_{12}	0.2	0.45

Employing procedure described in previous paragraph simple models for eigen-frequencies are obtained and minimization functional has been solved. From the results obtained by minimization (*Table 5*) elastic properties of plates has to be derived from equation (5) using non-linear programming.

Table 5. Identification parameters obtained by the minimization (2nd set)

	UD1	**UD5**
$x_1 = A_{11}$ [GPa]	172.5	171.7
$x_2 = A_{22}$ [GPa]	10.72	11.24
$x_3 = A_{12}$ [GPa]	4.658*	2.725
$x_4 = A_{44}$ [GPa]	7.458*	4.521*
$x_5 = A_{66}$ [GPa]	6.250	6.362

Since some discrepancies have been found in the identification of the Poisson's ratio ν_{12} and the transverse shear modulus G_{23} it is proposed to use set of accepted Poisson's ratio ν_{12}. Identified elastic constants are presented in *Table 6*.

Table 6. Mechanical properties obtained by the identification (2nd set)

UD1	ν_{12}			
	0.30	0.32	0.34	0.35
E_1 [GPa]	171.54	171.40	171.26	171.19
E_2 [GPa]	10.66	10.65	10.64	10.64
G_{23} [GPa]	7.458*	7.458*	7.458*	7.458*
$G_{12} = G_{13}$ [GPa]	6.250	6.250	6.250	6.250
ν_{12}	0.30	0.32	0.34	0.35
UD5	ν_{12}			
	0.30	0.32	0.34	0.35
E_1 [GPa]	170.69	170.55	170.40	170.32
E_2 [GPa]	11.17	11.16	11.15	11.15
G_{23} [GPa]	4.521*	4.521*	4.521*	4.521*
$G_{12} = G_{13}$ [GPa]	6.362	6.362	6.362	6.362
ν_{12}	0.30	0.32	0.34	0.35

For the comparison of the experimental eigenfrequencies with the calculated frequencies (see *Table 7*) Poisson's ratio ν_{12} value is 0.34. Residuals were calculated by the expression (17).

Table 7. Flexural frequencies f_i [Hz] and residuals Δ_i (%) for plates UD1, UD5 (2nd set)

Moda (*m,n*)	**UD1**			**UD4**		
	Exp	FEM	Δ_i (%)	Exp	FEM	Δ_i (%)
4 (1,1)	97	98.4	1.44	97.5	97.88	0.39
5 (2,0)	123.8	125.4	1.29	125	125.4	0.32
6 (2,1)	237.4	237.4	0.00	236	236.3	0.13
7 (3,0)	341	345.4	1.29	344	345.4	0.41
8 (3,1)	458	459.1	0.24	457.5	457.9	0.09
9 (0,2)	502.6	503.0	0.08	497	498.7	0.34
10 (1,2)	541.2	540.7	0.09	537.5	536.4	0.20
11 (2,2)	653	654.2	0.18	650	649.9	0.02
12 (4,0)	-	682.5	-	682.5	682.5	0.00
13 (4,1)	-	787.0	-	784	785.9	0.24
14 (3,2)	-	866.1	-	866	861.3	0.54
15 (5,0)	-	1124	-	1124.5	1124	0.04
16 (4,2)	1168	1178	0.86	-	1174	-
17 (5,1)	-	1227	-	-	1226	-
18 (0,3)	1381	1378	0.22	1365.5	1367	0.11
19 (1,3)	1413	1410	0.21	1402.5	1399	0.25
20 (2,3)	1512	1508	0.26	1504	1496	0.53

Conclusions

The present method of identification based on planning of experiments can predict the major elastic properties of the laminated composite plate specimens. Some discrepancies have been found in the identification of the transverse shear modulus G_{23}, which is not very sensitive to the eigenfrequencies of the thin plates. This parameter must be identified from the experiment on the thick plates.

The main advantage of the method of identification proposed in the present paper is significant reduction of the computational efforts in order to calculate the numerical frequencies, which are presented in the functional to be minimized. Another advantage of the identification method used is that the elastic constants are determined only from vibration test using a plate sample. The proposed method can be used as a non-destructive method in order to determine the elastic constants of unidirectional laminated plates. For corresponding procedure user-friendly software was developed using *Compaq Visual Fortran.* Every stage of identification procedure can be done step by step using simple windows. During identification procedure it is possible to evaluate mode shapes and corresponding eigen frequencies exploiting Mode Shape Visualizer made employing *Compaq Visual Fortran.* Proposed software allows to employ the identification method in order to determine the elastic constants of laminated plates in user convenient way.

References

1. Mota Soares C. M., Moreira de Freitas M. and Araújo, A. L. Identification of Material Properties of Composite Plate Specimens // In: Composite Structures, 25, 1993, pp. 277-285.
2. Sol H., De Visscher J., and De Wilde W. P. Identification of the viscoelastic material properties of orthotropic plates using a mixed numerical/experimental technique // In: Computational Methods and Experimental Measurements VI, Vol. 2: Stress Analysis, Eds. C.A. Brebbia and G.M. Carlomagno, Elsevier Applied Science, London - New York, 1993, pp. 131-142.
3. Sol H. Identification of anisotropic plate rigidities using free vibration data // In: PhD Thesis, Free University of Brussels, 1986.
4. Bolognini L., Riccio F., and Bettianli F. A modal technique for the identification of stiffness and mass parameters in large structures // In: Computational Methods and Experimental Measurements VI, Vol. 2: Stress Analysis, Eds. C.A. Brebbia and G.M. Carlomagno, Elsevier Applied Science, London - New York, 1993, pp. 337-352.
5. Frederiksen P. S. Identification of Temperature Dependence for Orthotropic Material Moduli // In: Mechanics of Materials, 13, 1992, pp. 79-90.
6. De Wilde W.P. Identification of the Rigidities of Composite Systems by Mixed Numerical/Experimental Methods // In: Numerical Identification of Composites, Eds. A. Vautrin and H. Sol, Elsevier Applied Science, London - New York, 1991, pp. 1-15.
7. Grediac M. and Vautrin A. Mechanical Characterization of Anisotropic Plates: Experiments and Results // In: Eur. J. Mech., A/Solids, 12 (6), 1993, pp. 819-838.
8. Link M. and Zou Z. A Two Step Procedure to Identify Physical Parameters of Composite Material Structures Using Vibration Test Data // In: Proc. Int. Conf. on Vibration Engineering, ICVE 94, Beijing, International Academic Publishers, Beijing, 1994, pp. 217-224.
9. Moussu F. and Nivoit M. Determination of Elastic Constants of Orthotropic Plates by a Modal Analysis Method of Superposition // In: J. Sound and Vibrations, 165, 1993, pp. 149-163.
10. Araujo A.L., Soares C.M.M, de Freitas M.J.M. Characterization of material parameters of composite plate specimens using optimization an experimental vibration data // In: Composites Part B, 27B, 1996, pp.185-191.
11. Frederiksen P.S. Experimental procedure and results for the identification of elastic constants of thick plates // In: J. Comps. Mater. 31 (4), 1997, pp. 360-382.
12. Rikards R. Elaboration of Optimal Design Models for Objects from Data of Experiments // In: Optimal Design with Advanced Materials. Proceedings of the IUTAM Symposium, Lungby, Denmark, 18 - 20 August, 1992, ed. P. Pedersen, Elsevier Science Publishers, 1993, pp. 148-162.
13. Rikards R., Chate A. and Bäcklund J. Optimal Design of Sandwich Plates Based on Planning of Experiments // In: Proceedings 1st World Congress of Structural and Multidisciplinary Optimization, 28 May - 2 June 1995, Goslar, Germany, Eds. N. Olhoff and G. I. Rozvany, Pergamon, 1995, pp. 569-574.
14. Rikards R., Chate A., Optimal Design of Sandwich and Laminated Composite Plates Based on Planing of Experiments // In: Structural Optimization, 10, (1), 1995, pp. 46-53.
15. Rikards R., Chate A., Gailis G., Identification of elastic properties of laminates based on experiment design // In: J. Solids and Struct. 38, 2001, pp. 5095-5115.
16. Rikards R., Chate A. Vibration and damping analysis of laminated composite and sandwich shells // In: Mech. Compos. Mater. Struct. 25, 1997, pp. 209-232.

17. Altenbach H., Altenbach J., Rikards R. Einfurung in die mechanik der laminat und sandwichtragwerke // In: Deutscher Verlag fur Grundstoffindustrie, Stuttgart, 1996.
18. Frederiksen P. S. Numerical studies for the identification of elastic constants of thick orthotropic laminates // In: Eur. J. Mech., A/Solids, 16, 1997, pp. 117-140.
19. Technical Report. POSICOSS. Task 3.2. Identification of realized material properties, May 5, 2002.
20. Bathe K. J. Finite element procedures in engineering analysis // In: Prentice-Hall, Englewood Cliffs.
21. Ruchevskis S., Identification of elastic properties of carbon/epoxy laminates // In: Proceedings - Junior Euromat 2002, Lausane, Switzerland.

Sandris Ruchevskis
Institute of Materials and Structures, Riga Technical University
Kalku 1, Riga, LV - 1658
M.Sc.Ing
sandris_r@bf.rtu.lv

Ručevskis S. Oglekļšķiedras kompozītmateriāla elastīgo īpašību identifikācija
Rakstā tiek apskatīta metode kompozīto materiālu mehānisko īpašību noteikšanai. Lai spētu nodrošināt konstrukcijas augsta standarta drošību, materiālu īpašībām ir jābūt precīzi noteiktām. Darbā tiek aprakstīta skaitliskā eksperimenta plānošanas identifikācijas metode kompozīto materiālu elastīgo īpašību noteikšanai. Metode ir balstīta uz kompozīto materiālu plātņu dinamiskajiem eksperimentiem un Galīgo Elementu metodes aprēķiniem. Šajā darbā tiešās minimizācijas metode elastīgo īpašību noteikšanai ir aizstāta ar skaitliskā eksperimenta plānošanas metodi. Galvenā šīs metodes priekšrocība ir ievērojama nepieciešamo aprēķinu samazināšana. Galīgo Elementu metodes aprēķini tiek veikti tikai skaitliskā eksperimenta plānošanas metodes iegūtajos punktos. Minimizācijas funkcionāls apraksta starpību starp eksperimentāli iegūtajiem un skaitliski aprēķinātajiem parametriem. Materiāla elastīgās īpašības tiek iegūtas minimizējot šo funkcionāli.

Ruchevskis S. Identification of elastic constants of carbon/epoxy composites plates
This paper deals with the method of the determination of mechanical properties of composite materials. In order to ensure high reliability of the structure the actual mechanical properties of the material must be accurately predicted. In this work it is proposed to use the numerical-experimental identification method. The method is based on measurements of the dynamic behavior of composite plates and Finite Element simulations. In the present study instead of direct minimization of the criterion it is proposed to use the method of planning of experiments. The main advantage of this method is significant reduction of the number of computations of the criterion. The finite element modeling is performed only in the reference points. The functional to be minimized describes the difference between the measured and numerically obtained parameters of the response of structure. By minimizing the functional the identification parameters are obtained.

Ручевскис С. Идентификация механических свойств углеродного композита.
В данной работе рассматривается метод определения механических характеристик композитного материала. Для того чтобы обеспечить высокую надежность конструкции, соответствующую стандартам, механические свойства материала должны быть определены наиболее точно. В работе описывается приминение экспериментально – числового метода идентификации. Метод основан на динамическом эксперименте пластины из композитного материала и приминением метода конечных элементов. Вместо прямой минимизации критерия в работе используется метод планирования экспериментов. Главное достоинство данного метода – значителное сокращение количества вычислений критерия. Расчёты методом конечных элементов производятся только для точек полученных методом планирования экспериментов. Функционал минимизаций описывается разницей между эксперименталными данными и данными, полученными путём вычислений. Идентификацонные параметры находятся путём минимизации функционала.