# Calculation of 3D Texture Parameters

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***Abstract* – The nature of the surface roughness is a result of mechanical processing, which can be described by random of two variables. 3D surface roughness parameters are determined by simple rough surface model. Two types of surface contacts were examined: plastic and elastic to calculate the contact resistance in welding and wear intensity. The Oscillating rolling machining influence on the surface state was determined and the model of surface simulation and the equation of surface texture height were offered.**

***Keywords* – surface texture parameters, contact area, contact criterion, intensity of wear, contact resistance**

## I. Introduction

At present surface roughness profile parameters are determined by standards. At the same time different exploitational properties of details are determined with the help of 3 dimensional image - microtopography.

In microtopography surface roughness is expressed by a random field *h (x, y)* of two variables *x, y* - Cartesian coordinates of a surface point.

Let's consider an irregular roughness, which is described by a normal homogeneous random field. Correlation function of the field is uninterrupted and has uninterrupted derivatives. This kind of roughness model lends itself to survey anisotropic surface as well.

## II. Initial Surface Parameters

To characterize the surface roughness according to the above mentioned model the following parameters are necessary (omitting the substantiation) [1]:

- *rms* deviation of the surface $\sigma$;
- average roughness steps in the direction of $x$ and $y$ – $Sm_1$, $Sm_2$ $(Sm_2>Sm_1)$;
- average steps between the tops of elevations in the same directions – $S_1$, $S_2$ $(S_2>S_1)$.

Surface structure parameter cross section is convenient for use:

$$\lambda = \mathrm{E}\{S_1\} / \mathrm{E}\{Sm_1\} \tag{1}$$

As well as anisotropy parameter of rough surface:

$$c = \mathrm{E}\{Sm_1\} / \mathrm{E}\{Sm_2\} \tag{2}$$

For normal field $0 \le \lambda \le 2(2/3)^{1/2}$ and $0 \le c \le 1$, if c=1 the field turns isotropic.

All the other microtopographic parameters are derived from these initial parameters.

## III. Microtopographic Parameters

Let's consider their calculation without proving the equations based on the integration of multidimensional distribution of a normal field.

*The mean arithmetical deviation of the surface roughness*:

$$Ra = \frac{1}{A}\iint_{\Omega} |h(x,y)|\,dxdy, \tag{3}$$

where $h(x, y)$ - deviation of random surface from the average;

$A$ - area of the region under investigation $\Omega$.

*Mathematical expectation of this parameter:*

$$\mathrm{E}\{Ra\} = (2/\pi)^{1/2}\sigma \tag{4}$$

*Surface top elevation height, mathematical expectation:*

$$\mathrm{E}\{h_p\} = 2\lambda\sigma / \sqrt{\pi} \tag{5}$$

*Highest elevation height, mathematical expectation* [2]:

$$\mathrm{E}\{H_{\max}\} = 2\sigma\sqrt{\ln\left[\mathrm{E}\{N_{01}\}\sqrt{c}\right]}, \tag{6}$$

where $\mathrm{E}\{N_{01}\}$ - number of zeroes in the direction of x axis within the confines of the route under consideration.

*Number of surface elevations.* The average is above the level $\gamma = u/\sigma$:

$$\mathrm{E}\{N_\gamma\} = k\gamma\exp(-\gamma^2/2), \tag{7}$$

where $k = \sqrt{2\pi}/(\mathrm{E}\{Sm_1\}\cdot\mathrm{E}\{Sm_1\})$. (8)

*Number of summits.* The average number of summits on a surface unit:

$$\mathrm{E}\{M_p\} = \frac{2\pi}{3\sqrt{3}}\cdot\frac{c}{\mathrm{E}^2\{S_1\}} \tag{9}$$

*Relative base area.* It's mean value on the level $\gamma = u/\sigma$

$$\mathrm{E}\{\eta_\gamma\} = 1 - \Phi(\gamma) \tag{10}$$

where $\Phi(\gamma) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\gamma}\exp(-t^2/2)dt$. (11)

*Elevation curvature.* Asymptotic dependence of the main elevation curvatures above γ level:

$$\mathrm{E}\{k_1\} = 2\pi^2\sigma\gamma/\mathrm{E}^2\{Sm_1\};\ \mathrm{E}\{k_2\} = 2\pi^2\sigma\gamma/\mathrm{E}^2\{Sm_2\} \tag{12}$$

*Surface gradients.* Mathematical expectation

$$\mathrm{E}\{V_n\} = \pi^{3/2}\sigma(1+c^2)/\mathrm{E}\{Sm_2\} \tag{13}$$

*Surface area.* At sharp angles of elevation side inclines (less than 30°):

$$\mathrm{E}\{S\} = 1+\pi^2\sigma^2(1+c^2)/\mathrm{E}^2\{Sm_1\} \tag{14}$$

*Elevation volume* of surface roughness above the level $\gamma = u/\sigma$:

$$\mathrm{E}\{V_\gamma\} = \sigma A\left\{\frac{1}{\sqrt{2\pi}}e^{-\gamma^2/2} - \gamma[1-\Phi(\gamma)]\right\}, \tag{15}$$

where $A$ - nominal area of surface region under consideration.

The given expressions of some microtopographic parameter mean values show that for the given model they are rather easily expressed by initial values. Dispersions have been elaborated for these parameters necessary for the solution of metrological problems for the choice of the measurement regime.

## IV. Methods of Experimental Microtopographic Regularities Determination

Surface roughness parameters can be obtained by sectional methods. For a general profile anisotropic surface determination five sections are sufficient with a definite orientation from x axis. E.g. $\sigma$, $n(0)$, $m$ are determined by section $\varphi=0°$ and $n(0)$ and m are determined by $\varphi=45°$, $90°$ and $m$ by $\varphi=30°$ and $135°$ ($n(0)$ and $m$ are the numbers of zeroes and maxims for a unit of route length). Microtopographical parameters can by calculated on the basis of the initial parameters drawn from the sections. Directional anisotropy rough surfaces contain their main information across three sections with the angles 0°, 30° and 60° to $X$ axis. Isotropic surfaces contain all their information in one randomly orientated section.

For a detailed microtopography study a measuring system has been elaborated, which consists of a two-coordinate table, a counter and a connecting block integrating this system with a computer into a common mathematical softwear circuit.

Such a system ensures 3D recording of roughness and surface micromap in terms of isoline tops and the numeric values of tops and steps as well as microelevation shape parameters.

## V. Methodical Aspects

The number of essential measurements for microtopographical parameter determination is much smaller when compared with the corresponding profile parameter measurements to achieve the same accuracy.

It is due to the fact that relatively larger amount of information is derived from the surface area element rather than from the route profile. So, for example, five times more measurements are necessary for profile determination on the basis of parameter *Ra* than by microtopographical evaluation. Parameter $\eta_\gamma$ gives even greater difference. *Ra* is the most constant parameter in terms of reliability and precision if compared with other parameters.

It can be compared with other parameters as follows:

$$n_{Ra} : n_{\eta_\gamma} : n_{H_{\max}} = 1:8:16, (\gamma = 1{,}0) \tag{16}$$

where $n_p$ - compulsory number of measurements for parameter $p$ to ensure the given precision.

## VI. Connection with Exploitational Characteristics

The elaborated rough surface microtopographical parameters lend themselves to creating methods for the calculation of various exploitation characteristics.

Let's consider some characteristics connected with rough surface contact interaction. The dependencies given below are simplified for engineers' practical usage [3].

***Contact area.*** Depending upon the properties of the contacting surfaces the contact area can be formed due to plastic or elastic deformation of elevations. Let's consider rough and smooth surface interaction. With predominantly plastic contact of irregularities the mean area contact value is determined from the solution of two equations:

$$\begin{aligned} \eta_{pl} &= 1-\Phi(\gamma) \\ q_{pl} &= k_q^{pl} H_\mu (Ra/Sm_1)^{n-2} F_2(\gamma, n), (\gamma \ge 1{,}0) \end{aligned} \tag{17}$$

where $\eta = A_r / A_a$ relative contact area, determined in relation to real $A_r$ and nominal $A_a$ area;
$q = P/A_a$ mean pressure on the nominal area;
$H_\mu$ - surface layer microhardness;
$k_q^{pl}$ - table constant on the basis of roughness anisotropy;
$n$ - surface layer solidification coefficient;
$F(y,n)$ - table function of parameters $\boldsymbol{y}$ and n.

In the case of utmost inclined surfaces when $n$= 2.0 we get:

$$\eta_{pl} = q/H_\mu \tag{18}$$

With predominantly elastic elevation contact the following equation is used for contact area evaluation:

$$\begin{aligned} \eta_{el} &= k_\eta[1-\Phi(\gamma)] \\ q_{el} &= k_q^{el}\frac{Ra}{Sm\theta}F_1(\gamma) \end{aligned} \tag{19}$$

where $k_\eta$, $k_q^{el}$ - table coefficients dependent on roughness anisotropy;
$F_1(\gamma)$ - table function;
$\theta = \dfrac{1-\mu_1^2}{\pi E_1} + \dfrac{1-\mu_1^2}{\pi E_2}$ - material stiffness constant of the contacting bodies (E- Young's module, μ - Poisson coefficient.

Contact area calculations $\eta_{pl}$ and $\eta_{el}$ are made according to the scheme $q \to \gamma \to \eta$: In the case of known load and surface characteristics the intermediate value $\gamma$ (relative level of deformation $\gamma = u/\sigma$**)** is calculated and then $\eta$.

***Contact criterion:*** $KK = Sm_1 H_\mu /(Ra \cdot E)$ determines the choice of one or the other equation. If KK<3.0 plastic contact conditions are satisfied, if KK>5.0 – the conditions of elastic contact. The structure of the given equation remains valid in the case of contact of two rough surfaces.

***Contact surface deformation***. Its calculation is based on the difference of two levels $\gamma$ and $\gamma_0$ elaborated for two loads causing deformation:

$$a = (\gamma_0 - \gamma)\sigma \qquad (20)$$

where $\gamma$, $\gamma_0$ are determined from the lowest contact interaction $\gamma > \gamma_0$.

Parameter $a$ is important for detail contact measurements.

***Intensity of wear***. On upper layer fatigue destruction (concept of Kragelski) the wear intensity (relationship of linear wear and the friction path) is:

$$I_h = k_l \left[Ra/(Sm_2\sigma_0)\right]^t q/\theta^{t-1} \qquad (21)$$

where $\sigma_0, t$ - material fatigue curve parameters;
$k_l$ - anisotropic roughness table.

***Contact resistance at point welding***. For small thickness (less than 100 um) sheet welding surface roughness impact must be taken into consideration. Resistance between the welded details can be evaluated by the equation:

$$R = k_{Ra} \frac{\rho}{(dq\theta)^2} \cdot \frac{Ra^2}{Sm_1} \qquad (22)$$

where $d$ - electrode diameter;
$q$ - contact pressure;
$p$ - inherent electric resistance of detail material;
$k_{Ra}$- surface anisotropy dependent table coefficient.

The given examples prove that exploitation properties are determined by equations of similar structure. Apart from special parameters they depend on roughness anisotropy and both height and step parameter interrelation. These parameters are technically racy to control, and ways can be shown for securing the desired exploitation properties.

## VII. Surfaces with Regular Roughness [4]

It is characteristic of the surfaces of regular micro structure that all surface's micro asperities have approximately equivalent height and form, and their arrangement on the surface is regular. To obtain surfaces of regular micro structure, surface processing methods of plastic deformation are mostly utilized - oscillating rolling and rotation impactive processing. Hardened steel balls or diamond tips are used as instruments, which in working process in feed direction have low amplitude peripheral oscillations. As a result curved grooves are formed on the surface sine. Regular surface micro structure is formed in groove's covering process. For a more extensive and complete research of exploitation of surfaces with regular micro structure it is important to have their mathematical description. M. Longe - Higgins [1] advised the principle for modelling sea waves, i. e., to describe regular micro structure surface with the sum of many cosine waves (Fig.1), which are spreading in different directions as a base for manufacturing of mathematical model was taken.

$$h(x, y) = \frac{A}{n}\sum_{i=1}^{n} \cos(\omega_i x + v_i y) \qquad (23)$$

where $A$ - summary amplitude of cosine wave;
$\omega_i$, $v_i$ - i-th angular frequency of cosine wave for corresponding coordinates $x$ and $y$;
$n$ - number of cosine waves.

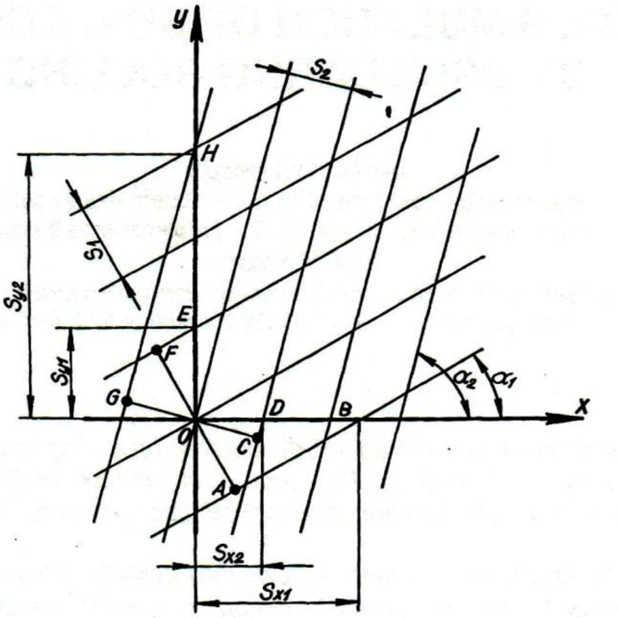


Fig.1. Formation scheme of regular micro structure of the surface

Summarizing two waves forms micro structure with micro asperities in tetragonal form, but summarizing three waves - with micro asperities in hexagonal form.

## VIII. Mathematical Description of Surfaces

In the process of mathematical description of real regular micro structure surfaces for calculation of wave amplitude and angular frequency as a parameter of the micro structure it is important to choose such surface describing parameters that are easy to measure with universal control surface roughness measuring instruments .

From this opinion, as surface measure parameters has been accepted:

$S_i$ - step between micro asperities in wave direction,
$\alpha_i$ - angle of micro asperities locality,
$R_{max}$ - maximum height of micro asperities.

Replacing in Eq.23 angular frequencies with cosine length of waves in the coordinate axis x and y direction, the equation of surface with tetragonal wave form is:

$$h(x, y) = \frac{A}{2}\left[\cos\left(\frac{2\pi}{Sx_1}x + \frac{2\pi}{Sy_1}y\right) + \cos\left(\frac{2\pi}{Sx_2}x + \frac{2\pi}{Sy_2}y\right)\right] \qquad (24)$$

Using triangles AOB, COD, EOF, GOH as shown in Fig.1, it is possible to find relations between wavelengths $Sx_1$ $Sx_2$, $Sy_1$, $Sy_2$ and parameters describing surface's micro structure's $S_1$, $S_2$, $\alpha_1$ and $\alpha_2$:

$$Sx_1 = \frac{S_1}{\sin\alpha_1}; Sx_2 = \frac{S_2}{\sin\alpha_2}; Sy_1 = \frac{S_1}{\cos\alpha_1}; Sy_2 = \frac{S_2}{\cos\alpha_2} \quad (25)$$

Where with the equation of describing micro asperities surfaces of regular tetragonal form there is a successive subsequent:

$$h(x,y) = \frac{R_{\max}}{2}\left[\begin{array}{l}\cos\left(\frac{2\pi\sin\alpha_1}{S_1}x + \frac{2\pi\cos\alpha_1}{S_1}y\right)\\ +\cos\left(\frac{2\pi\sin\alpha_2}{S_2}x + \frac{2\pi\cos\alpha_2}{S_2}y\right)\end{array}\right] \quad (26)$$

To prove the suitability of the mathematical model for description of real surfaces, micro topographical maps of real surfaces were drawn and parameters $S_1$, $S_2$, $\alpha_1$, and $\alpha_2$ of surface micro structure on the surface three-dimensional drawing equipment were established. On the same equipment, using analogue computation method, surface micro topographical maps were drawn following Eq. 26 with parameters $S_1$, $S_2$, $\alpha_1$, and $\alpha_2$, $R_{max}$ corresponding to real surfaces.

To check – up theoretical full – fashioned micro asperities form and micro asperities form of real surface, profilograms of real surface on the plane, where the height of surface's micro asperities is maximal were taken. On the same plane (Fig.2, line OM) the following equation:

$$h(x) = \frac{R_{\max}}{2}\left\{\begin{array}{l}\cos\left[\frac{2\pi}{S_1}(\sin\alpha_1 + k\cos\alpha_1)x\right]\\ +\cos\left[\frac{2\pi}{S_2}(\sin\alpha_2 + k\cos\alpha_2)x\right]\end{array}\right\} \quad (27)$$

was calculated profile of full - fashioned surface.

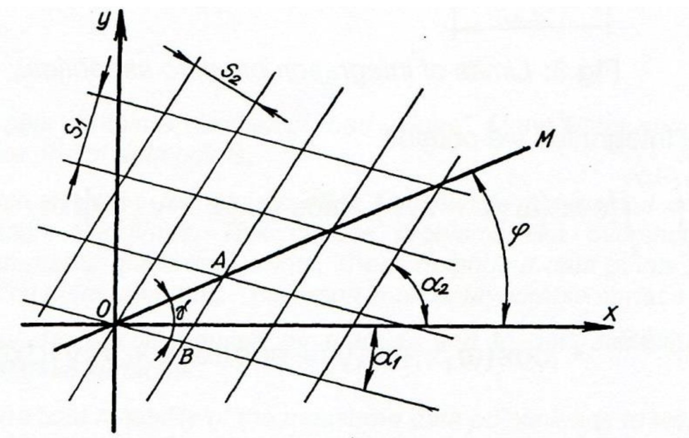


Fig.2. The scheme of evaluation of micro asperities form

For comparison of average deviation $R_a$ of real and full - fashioned regular micro structure surface expression of $R_a$ to estimate the height of micro asperities along the surface was found.

$$R_a = \frac{A}{2LxLy}\int_0^{Lx}\int_0^{Ly}\left|h(x,y)\right|dxdy \quad (28)$$

Noticing that cosine function is symmetrical, it is possible to perform integration in the limits of quarters of cosine wave period. Thus $h(x, y) = |h(x, y)|$ and

$$R_a = \frac{8A}{2LxLy}\int_0^{Lx/4}\int_0^{Ly/4}\left[\cos(\omega_1 x + v_1 y) + \cos(\omega_2 x + v_2 y)\right]dxdy$$

Resolving integrals and installing values of angular frequencies with parameters of surface micro structure:

$$R_a = \frac{8R_{\max}}{b_1 Lx_2}\left[\frac{S_1 S_2}{4\pi\sin(\alpha_1+\alpha_2)}\right]$$
$$\downarrow \quad (29)$$
$$R_a = \frac{8R_{\max}S_1S_2\cos\alpha_1\sin(\alpha_1+\alpha_2)}{4\pi S_1S_2\cos\alpha_1\sin(\alpha_1+\alpha_2)} = \frac{2R_{\max}}{\pi}$$

## IX. Conclusions

Examination of the equations shows that they have the following common features, the formulas contain:

1) one surface roughness height parameter *Ra;*

2) two spacing parameters $Sm_1$, $Sm_2$.

These parameters are relatively convenient for measurement and easy to be ensured in the engineering practise.

There are several methods to obtain the necessary 3D surface roughness parameters, such as special equipment for 3D measurement and the method of indirect measurements.

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**Jānis Rudzītis, Juris Krizbergs, Anita Avišane, Guntis Spriņģis, Māris Kumermanis, Jānis Lungevičs. 3D Tekstūras parametru aprēķināšana**
Dotajā darbā ir apskatīts raupju virsmu kontakts, konkrētāk, pārbaudītās virsmas bija mehāniski apstrādātas ar abrazīviem instrumentiem. Virsmas raupjums ir mehāniskās apstrādes rezultāts, tas var būt raksturots ar gadījuma lauka diviem mainīgajiem. Uzmanība tika vērsta trīsdimensiju (3D) virsmas raupjuma parametriem, kuru statistikas īpašības tiek noteiktas ar vienkāršu raupjas virsmas modeli: anizotrops gadījuma lauks veidojas, matemātiski iztiepjot izotropu gadījuma lauku. Uzmanība tika pievērsta virsmu kontaktam: raupjas un gludas virsmas mijiedarbībai un divu raupju virsmu kontaktam. Tika pārbaudīti negludumu kontakta divi veidi: plastiskais un elastīgais. Tika izstrādāts kontakta tipa kritērijs. Kontakta teorija tika izmantota kontakta pretestības aprēķināšanai metināšanā un nodiluma intensitātes noteikšanai. Iegūtie vienādojumi norādīja uz to, ka tiem visiem ir viens virsmas raupjuma augstuma parametrs un divi soļu parametri.
Šajā rakstā tika piedāvāta virsmas modelēšanas metode, izmantojot standarta virsmas raupjuma parametru attiecības ar viļņu augstumu, frekvenci un fāzi. Sakarā ar to tika izveidots vienādojums virsmas tekstūras augstuma noteikšanai pēc vibrovelmēšanas.

**Янис Рудзитис, Юрис Кризбергс, Анита Авишане, Гунтис Спрингис, Марис Кумерманис, Янис Лунгевичс. Расчёт параметров 3Д текстуры.**
В данной работе рассмотрен контакт шероховатых поверхностей, конкретнее, проверенные поверхности были механически обработаны абразивным инструментом. Шероховатость поверхности является результатом механической обработки, она может быть охарактеризована двумя переменными случайного поля. Особое внимание было уделено параметрам 3-х дименсионных (3D) шероховатых поверхностей, статистические свойства которых определяются с помощью простой модели шероховатой поверхности: анизотропное случайное поле образуется при математическом растягивании изотропного случайного поля. Также было уделено внимание контакту поверхностей: взаимодействию шероховатой и гладкой поверхности и контакту двух шероховатых поверхностей. Были проверены два вида контактов неровностей: пластичный и эластичный. Был разработан критерий для определения типа контактов. Контактная теория была разработана для определения контактного сопротивления в сварке и интенсивности износа. Полученные уравнения указали на то, что в каждом из них есть один параметр высоты шероховатости поверхности и два шаговых параметра.
В данной статье был предложен метод моделирования поверхности, используя отношения стандартных параметров шероховатости поверхности с длиной волны, частотой и фазой. В связи с этим, было выведено уравнение для определения высоты текстуры поверхности после вибропрокатывания.