Statistics of Roughness Peak Height of Friction Surface

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***Abstract* – One of the most essential parameters in determining deformation of the friction surface is the height of surface roughness. This article studies and compares three different formulas for determination of the mathematical expectation value of surface peak height for the roughnesses being above the determined deformation level γ. The article treats the P.R. Nayak formula and two most well-known probability distribution laws: the normal distribution (Gauss) law and Rayleigh law. The work clearly shows and demonstrates graphically the ME values of surface peak height at different γ values, for all three formulas.**

***Keywords* – deformation, friction surface, mathematical expectation value of surface peak height, roughness height.**

I. Introduction

Rough surface contact researches are related to roughness deformations [1, 2]. One of the most essential parameters affecting roughness deformation and being used in the solution of contact tasks is the height of surface roughness. The height of the rough surface roughness is counted from the midline (in the case of profile) or from the middle plane (in case of 3D surface), which means that maximum values of random process or random function are determined.

The roughness deformation diagram is given in Fig. 1, showing the relation between deformation a_i and the height of the deformed peak $h_p(u)$, measured from the level u.

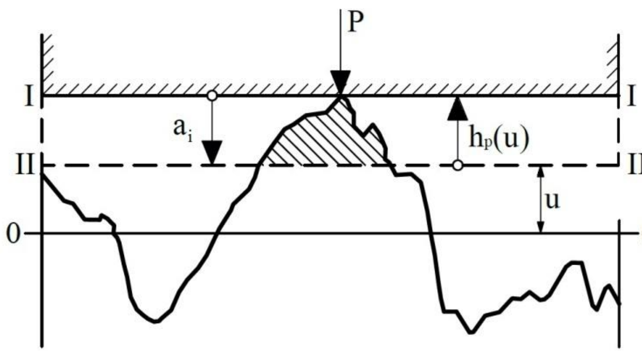

Fig. 1. Diagram of the contact of a perfect plane and the rough surface

The deformation diagram given in the figure relates to the case, in which we will consider the following contact of an ideal plane and the rough surface: under the effect of applied force P the ideal plane shifts from position I – I to position II – II, where it reaches a balance between the external force and resistance of micro-roughness deformation. In this position the distance between the ideal plane and middle plane of the rough surface 0 – 0 is equal to *u*.

In contact theory it is often assumed [3, 4, 5] that the roughness deformation level of surface roughness stops above the middle plane 0 - 0 (Fig. 1) on the standardised deformation level γ ($\gamma = u/\sigma$, where σ is the random field standard deviation).

In this article we will consider the ME value of height of all rough peak tops and also the determination of ME value of surface roughness height for the roughnesses being above the determined deformation level.

II. Mathematical Expectation Value Of Peak Height

To determine contact deformations of friction surfaces it is essential to determine the ME value of surface peak height. The surface roughness height distribution law 3D for irregular type surface (mathematically – for a normal random field) has been studied in the work by P.R. Nayak [6].

P.R. Nayak in his research on rough surface random processes determines the ME value of surface peak height according to the formula:

$$E_1\{\xi_p\} = \int_{-\infty}^{\infty} \xi_p \cdot f_1(\xi_p) \cdot d\xi_p \ , \tag{1}$$

where:

$f_1(\xi_p)$ – density of probability distribution of peak height of rough surface;

$\xi_p = \dfrac{h_p}{\sigma}$ – standardised value of peak height;

σ – standard deviation of random field.

From P.R. Nayak's research it is known that the density of probability distribution of surface peak height $f_1(\xi_p)$ can be found by dividing the number of peaks situated above level γ by the total number of peaks:

$$f_{1\gamma}(\xi_p) = \frac{E\{n_p(\gamma)\}}{E\{M_p(\gamma)\}} \ , \tag{2}$$

where:

$E\{n_p(\gamma)\}$ – ME value of the number of rough peaks on level [γ, γ+dγ];

$E\{M_p(\gamma)\}$ – ME value of the number of all surface peaks above level γ .

Whereas the ME $E\{n_p(\gamma)\}$ value of the number of roughness peaks on level [γ, γ+dγ] can be determined by the expression [6]:

$$E\left\{n_p(\gamma)\right\} = \frac{E^2\{m_1\}}{3} c \cdot \left\{ \begin{array}{l} \frac{\lambda}{4}\sqrt{3\left(8-3\lambda^2\right)} \cdot \gamma e^{-\frac{4}{8-\lambda^2}\gamma^2} + \\ + \frac{3\sqrt{2\pi}}{4}\lambda^2\left(\gamma^2-1\right)e^{-\frac{\gamma^2}{2}}\phi\left(\sqrt{\frac{3\lambda^2}{8-3\lambda^2}}\gamma\right) + \\ +4\sqrt{\frac{2\pi}{3\left(4-\lambda^2\right)}}e^{-\frac{2}{4-\lambda^2}\gamma^2} \cdot \\ \cdot\phi\left(\sqrt{\frac{4\lambda^2}{\left(4-\lambda^2\right)\left(8-3\lambda^2\right)}}\right) \end{array} \right\}, \quad (3)$$

where:

indexes 1 and 2 refer to the direction of parameter setting, respectively 1 – perpendicular to the treatment direction, 2 – perpendicular to direction 1.

$E\{m_1\}$ – ME value of surface maximums per unit of length;

λ – non-dimensional parameter $\lambda = \frac{E\{n_1(0)\}}{E\{m_1\}}$;

c – $E\{n(0)\}$ coefficient of anisotropy $c = \frac{E\{n_1(0)\}}{E\{n_2(0)\}}$;

$\phi(\ldots)$ – Laplacian function: $\phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{t^2}{2}dt}$, its numerical values can be found [7]

But the ME value of the number of $E\left\{M_p(\gamma)\right\}$ surface peaks above level γ:

$$E\left\{M_p(\gamma)\right\} = c \cdot \frac{2\pi}{3\sqrt{3}} E^2\{m_1\} \cdot \ell_1, \quad (4)$$

where ℓ_1 – function of roughness parameter λ and level γ determined according to the expression:

$$\ell_1 = \frac{3\lambda}{8\pi}\sqrt{8-3\lambda^2} \cdot e^{-\frac{4\gamma^2}{8-3\lambda^2}} + \frac{3\sqrt{3}\lambda^2}{4\sqrt{2\pi}} \cdot \gamma \cdot e^{-\frac{\gamma^2}{2}}\phi\left(\sqrt{\frac{3\lambda^2}{8-3\lambda^2}}\gamma\right) + \left[1-\phi\left(\sqrt{\frac{4}{4-\lambda^2}}\gamma\right)\right] + 2\sqrt{\frac{2}{\pi\left(4-\lambda^2\right)}} \cdot \int_{\gamma}^{\infty}\phi\left(\sqrt{\frac{4\lambda^2}{\left(4-\lambda^2\right)\left(8-3\lambda^2\right)}}\gamma\right) \cdot e^{\frac{2\gamma^2}{4-\lambda^2}} \cdot d\gamma \quad (5)$$

As mentioned before, in contact tasks one should determine the ME value of the height of those surface peaks that are above the set deformation level u, or standardised deformation level γ. Thus, by transforming formula (1) the ME value of surface peak height above level γ can be determined as follows:

$$E_{1\gamma}\left\{\xi_p\right\} = \int_{\gamma}^{\infty}\xi_p \cdot f_{1\gamma}\left(\xi_p\right) \cdot d\xi_p, \quad (6)$$

Inserting expressions (2), (4) and (5) in the equation (6), after integration [8], we get:

$$E_{1_\gamma} = C_1 \cdot \frac{\sqrt{3}}{2\pi}\left\{ \begin{array}{l} \frac{3\lambda^2\sqrt{2\pi}}{4} \cdot \left(\gamma^2+1\right) \cdot e^{-\frac{\gamma^2}{2}} \cdot \phi\left(\beta_6\gamma\right) + \frac{4\sqrt{\pi}\lambda}{\sqrt{3}} \cdot \left[1-\phi\left(\beta_7\gamma\right)\right] + \\ + \frac{\sqrt{6}\lambda}{\beta_7} \cdot \gamma e^{-\frac{\beta_7^2\gamma^2}{2}} + 2\sqrt{\frac{\pi}{6}\left(4-\lambda^2\right)} \cdot e^{-\frac{2\gamma^2}{4-\lambda^2}} \cdot \phi\left(\beta_8\gamma\right) \end{array} \right\} \quad (7)$$

where:

C_1 – standardised multiplier;

$$\beta_6 = \sqrt{\frac{3 \cdot \lambda^2}{8-3\lambda^2}}; \quad \beta_7 = \sqrt{\frac{8}{8-3\lambda^2}}; \quad \beta_8 = \sqrt{\frac{4\lambda^2}{\left(4-\lambda^2\right)\left(8-3\lambda^2\right)}}.$$

Coefficient C_1 is determined from the relation:

$$C_1\int_{\gamma}^{\infty} f_1\left(\xi_p\right)d\xi_p = 1 \quad (8)$$

The value of coefficient C_1 changes depending on deformation level γ and roughness parameter λ, and its values are given in Table 1.

TABLE 1

VALUES OF COEFFICIENT C_1

γ	λ				
	0	**0.63**	**1.00**	**1.41**	**1.63**
0	2.000	1.304	1.115	1.013	1.00
0.5	3.145	1.721	1.325	1.065	1.004
1.0	6.329	2.667	1.825	1.295	1.085
1.5	14.921	5.000	3.049	1.919	1.466
2.0	43.472	41.631	6.410	3.690	2.667
2.5	161.322	34.483	11.952	8.434	6.579
3.0	769.233	125.022	58.821	26.322	21.741

The determination of ME value of surface peak height by the formula offered by P.R. Nayak is complicated for making engineering calculations, it is essential to find a simpler distribution law, which could substitute the precise formula.

Let's consider two best-known probability distribution laws: normal (Gaussian) distribution law and Rayleigh law.

III. Mathematical expectation value of peak height according to truncated normal distribution law

The ME value of surface peak height according to the truncated normal distribution law is determined as follows:

$$E_2\left\{\xi_{p\gamma}\right\} = \int_{\gamma}^{\infty} \xi_p \cdot f_{2\gamma}\left(\xi_p\right) \cdot d\xi_p = \frac{2}{\sqrt{2\pi} \cdot erfc\left(\frac{\gamma}{\sqrt{2}}\right)} \cdot \int_{\gamma}^{\infty} \xi_p \cdot e^{-\frac{1}{2}\xi_p^{\ 2}} d\xi_p \qquad (9)$$

We mark: $B_2 = \int_{\gamma}^{\infty} \xi_p \cdot e^{-\frac{1}{2}\xi_p^{\ 2}} d\xi_p$

According to [8] integration is made:

$$B_2 = \int_{\gamma}^{\infty} \xi_p \cdot e^{-\frac{1}{2}\xi_p^{\ 2}} \cdot d\xi_p = \frac{e^{-\frac{1}{2}\gamma^2}}{2 \cdot \frac{1}{2}} = e^{-\frac{1}{2}\gamma^2} \qquad (10)$$

By inserting the obtained result in (20), we get that for truncated normal distribution law; the ME value of surface peak height is:

$$E_2\left\{\xi_{p\gamma}\right\} = \frac{2 \cdot e^{-\frac{1}{2}\gamma^2}}{\sqrt{2\pi} \cdot erfc\left(\frac{\gamma}{2}\right)} . \qquad (11)$$

IV. Mathematical expectation value of peak height according to Rayleigh law

Similar to the scheme in case of truncated distribution law, we determine the ME value of surface peak height also for Rayleigh's law:

$$E_3\left\{\xi_{p\gamma}\right\} = \int_{\gamma}^{\infty} \xi_p \cdot f_3\left(\xi_p\right) d\xi_p = e^{\frac{1}{2}\gamma^2} \cdot \int_{\gamma}^{\infty} \xi_p^{\ 2} \cdot e^{-\frac{1}{2}\xi_p^{\ 2}} \cdot d\xi_p . \qquad (12)$$

Making integration in parts according to [9] we get:

$$E_3\left\{\xi_{p\gamma}\right\} = e^{\frac{1}{2}\gamma^2} \cdot \left[\sqrt{\frac{\pi}{2}} \cdot erfc\left(\frac{\gamma}{\sqrt{2}}\right) + \gamma \cdot e^{-\frac{1}{2}\gamma^2}\right] = e^{\frac{1}{2}\gamma^2} \cdot \sqrt{\frac{\pi}{2}} \cdot erfc\left(\frac{\gamma}{\sqrt{2}}\right) + \gamma . \qquad (13)$$

Diagrams of the ME value of surface peak height are given in Figure 2. Figure 2 shows that starting from value $\gamma > 1$ the ME values of surface peak height are drawing together, and according to Rayleigh's law the ME value is closer to the precise value. Thus, Rayleigh's law in the solution of engineering tasks can be used for range $\gamma > 1$.

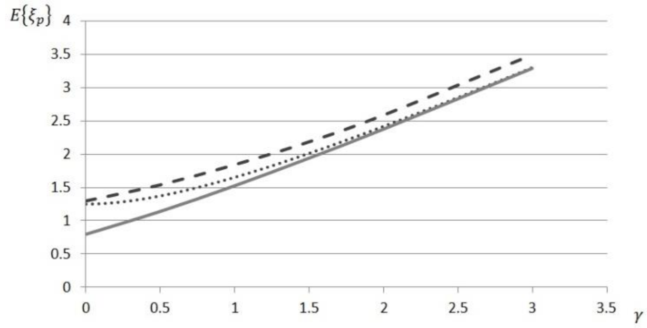

Fig. 2. Mathematical expectation value of surface height
Formula by P.R. Nayak
Truncated normal distribution law
Rayleigh's distribution law

Numerical values of all the above considered distribution laws for the determination of the ME value of peak height and the precise formula at different deformation levels γ are given in Table 2.

TABLE 2

Comparison of mathematical expectation values for different distribution laws

γ	Precise	Truncated distribution law		Rayleigh's law	
	$E_{1\gamma}\{\xi_p\}$	$E_{2\gamma}\{\xi_p\}$	Deviation (%)	$E_{3\gamma}\{\xi_p\}$	Deviation (%)
0	1,3032	0,7979	39%	1,2533	4%
0,5	1,5445	1,1411	26%	1,3764	11%
1,0	1,8254	1,5251	16%	1,6557	9%
1,5	2,1893	1,9387	11%	2,0158	8%
2,0	2,5897	2,3732	8%	2,4214	6%
2,5	2,6424	2,8227	7%	2,8543	8%
3,0	3,4922	3,2831	6%	3,3046	5%

V. Conclusions

Based on the data presented in this work it can be concluded that the precise formula both for the determination of the density of probability distribution of surface roughness height and determination of the ME value of peak height, being complicated for the solution of engineering tasks, can be substituted by a simpler one. The Rayleigh's law is the most suitable for the substitution of the precise formula.

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Jānis Rudzītis, Anita Avišāne, Guntis Spriņģs. Berzes virsmas negludumu augstuma sadalījuma statistika.
Lai noteiktu berzes virsmu deformāciju, viens no būtiskākajiem parametriem ir virsmas negludumu augstums. Kontaktteorijā virsma tiek modelēta kā normāls gadījuma lauks. Šādam normālajam gadījuma laukam izciļņu augstuma varbūtību sadalījuma blīvuma likumu ir ieguvis P.R. Naijaks, kas savukārt ļauj noteikt virsmas izciļņu augstuma matemātiskās sagaidāmās vērtības, taču šī izteiksme ir praktiski nepiemērojama inženieruzdevumu risināšanai, tāpēc šajā darbā ir noskaidrots, ka esošo formulu ir iespējams aizstāt ar vienkāršāku sadalījuma likumu. Šajā rakstā apskatītas un salīdzinātas trīs dažādas formulas virsmas izciļņu augstuma matemātiskās sagaidāmās vērtības noteikšanai tiem nelīdzenumiem, kas atrodas virs nosacīta deformācijas līmeņa γ. Ir apskatīta P.R. Naijaka formula un divi pazīstamākie varbūtību sadalījuma likumi: normālais sadalījuma (Gausa) likums un Releja likums. Salīdzinot šīs trīs formulas ir atrasts vienkāršāks, bet pietiekami precīzs risinājums, ar ko aizstāt sarežģīto formulu. Darbā ir grafiski attēlotās, iegūtās virsmas izciļņu augstuma matemātiskās sagaidāmās vērtības visām trim formulām pie dažādām γ vērtībām.

Янис Рудзитис, Анита Авишане, Гунтис Спрингис. Статистика распределения высоты шероховатости поверхности трения.
Для определения деформации поверхности трения, одним из наиболее важных параметров является высота шероховатости поверхности. В контактной теории поверхность моделируется как нормальное случайное поле. Для такого нормального случайного поля закон плотности распределения вероятности высоты выступов был найден П.Р. Найаком, который, в свою очередь, позволяет определить математическое ожидание (МО) высоты выступов поверхности, но это выражение не подходит для практических инженерных задач. Настоящая работа показала, что существующую формулу можно заменить более простым законом распределения. Данное исследование представляет и сравнивает три различные формулы определение МО высоты выступов поверхности, находящихся выше условного уровня деформации γ. Рассмотрена формула П.Р. Найака и два наиболее известные закона распределения вероятностей: нормальный закон распределения (Гаусса) и закон Релея. Сравнивая эти три формулы, найдено более простое, но достаточно точное решение для инженерных расчетов. В работе наглядно показано графическое изображение значений МО высоты выступов поверхности при различных значениях γ для всех трех формул.