THE TRANSITION TO CHAOS AND CRITERIA OF CHAOTIC BEHAVIOR IN DYNAMICAL SYSTEMS WITH LINEAR AND NONLINEAR DAMPING

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The achieved result is the elaboration of the basic theory for searching chaotic oscillations and nonlinear effects based on the concepts of periodic skeletons, complete bifurcation groups and method of rare attractors. The chaotic attractor possesses the characteristic amount of periodic skeletons and order of bifurcation groups. The approach allows us to forecast and to find new groups with rare catastrophic or rare useful phenomena with chaotic attractors in the nonlinear driven systems with linear and nonlinear damping. The results of work can be used in the engineering recommendations on how to use complex periodic, chaotic and rare attractors in the engineering systems for the purpose of favorable usage of nonlinear effects in the vibro-technology. It can also be used to forecast catastrophic situations and chaotic oscillations in the mechanics or other nonlinear dynamic systems.

Introduction

The work is devoted to the research of the chaotic determined oscillations in the non-linear dynamical systems describing processes and phenomena in nature, physics, economics and social sphere that are principally different from others. Such chaotic oscillations are generated by strictly causal determined rules and are widespread in the nature and engineering. The understanding of the essence, a substantiation of reasons and study of possibilities of application-oriented usage of chaotic dynamics is a fundamental interdisciplinary problem.

Modern researches of dynamical dissipative systems are inseparably linked with the study of global dynamics of behavior of these systems in the given space of parameters and states [1-7]. At a variation of parameters in non-linear oscillating systems there are qualitative changes in the system behavior which lead to a loss of stability of the periodic solution, to a stage of bifurcations of doubling of the period, to formation of areas with the infinite number of unstable periodic infinitums (UPI), to a birth of chaotic attractors, to existence of steady periodic regimes within the existence of a chaotic attractor and to other non-linear effects.

Complete bifurcation analysis for chaotic dynamics research

The work presents the analysis of an origin of chaotic oscillations in the nonlinear driven systems with linear and nonlinear damping with application of the systematic approach on the basis of a method of complete bifurcation groups[8-14]. The method of complete bifurcation groups and the complex of algorithms and programs allows us to systematically find complete bifurcation groups for non-linear dynamical models and to study interaction of various bifurcation groups. Such approach to the study of chaotic oscillations and their identification on the basis of the analysis of bifurcation groups forming a skeleton of non-linear dynamical system within the researched parameters allows carrying out forecasting of chaotic behavior and control of chaotic oscillations.

The method of complete bifurcation groups assumes the following:

1. Search of all bifurcation groups (nT - designation of the bifurcation group the basis of which is the regime of n^{th} order) with periodic regimes corresponding to them (Pn - designation of a regime of n^{th} order). For search of regimes of the various orders it is recommended to use values of parameters from area with the infinite number of unstable periodic infinitums (UPI).

2. An estimation of stability of all found periodic regimes with calculation of multiplicators at the selected values of space of parameters.

3. Continuation on a parameter (on a solution branch) of stable and unstable periodic regimes of each bifurcation group: the main 1T and subharmonic nT; display of complete bifurcation groups in bifurcation charts.

All propositions of the theory of research of chaotic oscillations on the basis of a method of complete bifurcation groups are published in the papers made under supervision of professor M.V.Zakrzhevsky [12-15].

At the systematic approach the following activities for the certain parameters of researched system after display of complete bifurcation groups are carried out: the additional analysis of various periodic attractors, forecasting of chaotic oscillations and chaotic attractors, projection of areas of attraction of attractors on Poincare's plane in case of multiplicity phenomena, and the study of scenarios of a birth of chaotic attractors. All results have been obtained by direct numerical simulation using software NLO and SPRING [10,15].

It is offered to apply the systematic approach to the research of rare attractors, as well as chaotic oscillations accompanying them. In this case the birth of a rare attractor is connected to appearance of fold bifurcation and a birth of chaotic oscillations as a result of a stage of bifurcations of doubling of the period at a change of bifurcation parameter, and, steady chaotic oscillations exist in a small range of a changing parameter.

In typical dynamical systems the birth of chaotic oscillations is connected to the main bifurcation group 1T, subharmonic bifurcation groups nT and their coexistence. Chaotic oscillations are possible only in the area with the infinite number of unstable periodic infinitums UPI. Transition process depends on all bifurcation groups located in the field of considered initial conditions. Thus, in the presence of infinite number of unstable periodic infinitums UPI in the system, chaotic transition processes or chaotic attractors (ChA-n) of certain bifurcation groups are observed.

The given research dwells upon formation of typical bifurcation groups and a birth of chaotic oscillations in the dissipative oscillating systems with exterior periodic influence. Figure 1 displays the analysis of regular and chaotic forced oscillations of bifurcation groups 1T, 3T, 5T and 9T on the basis of the systematic approach. Layout of various bifurcation groups in the same range of parameters is typical for non-linear systems.

For display of the irregular stationary regimes (chaotic and quasi-periodic attractors) in bifurcation charts the scanning method is used to determine phase space of the quantitative value of appropriate coordinates of results of integration after termination of the transition process at driving on parameter from one initial condition belonging to the area of an attraction of a stationary regime.

Stationary chaotic oscillations – chaotic attractors – are global or coexistent with other periodic attractors. In case of the symmetric systems, coexisting of two mutually symmetric chaotic attractors is typical. A specific feature of symmetric systems is transition to chaos according to the scenario of Feigenbauma (a stage of bifurcations of doubling of the period) from each of two mutually symmetric regimes which are born as a result of loss of bifurcation symmetry. Thus, in the researched driven symmetric system with cubic restoring force and linear dissipation, there is a birth of two mutually symmetric chaotic attractors (Fig. 2, $h = 5.1$), and then at a change of the bifurcation parameter and a homoclinical contact there is a global bifurcation and a birth of the global chaotic attractor. The system then undergoes further period doubling bifurcations of various bifurcation groups (1T, 3T, 5T, 9T) as the control parameter h (the amplitude of external harmonic forcing) is smoothly varied. For example, at $h=6.0516$, all periodic solutions of various bifurcation groups becomes unstable and given birth to a three chaotic attractors(Fig. 3).

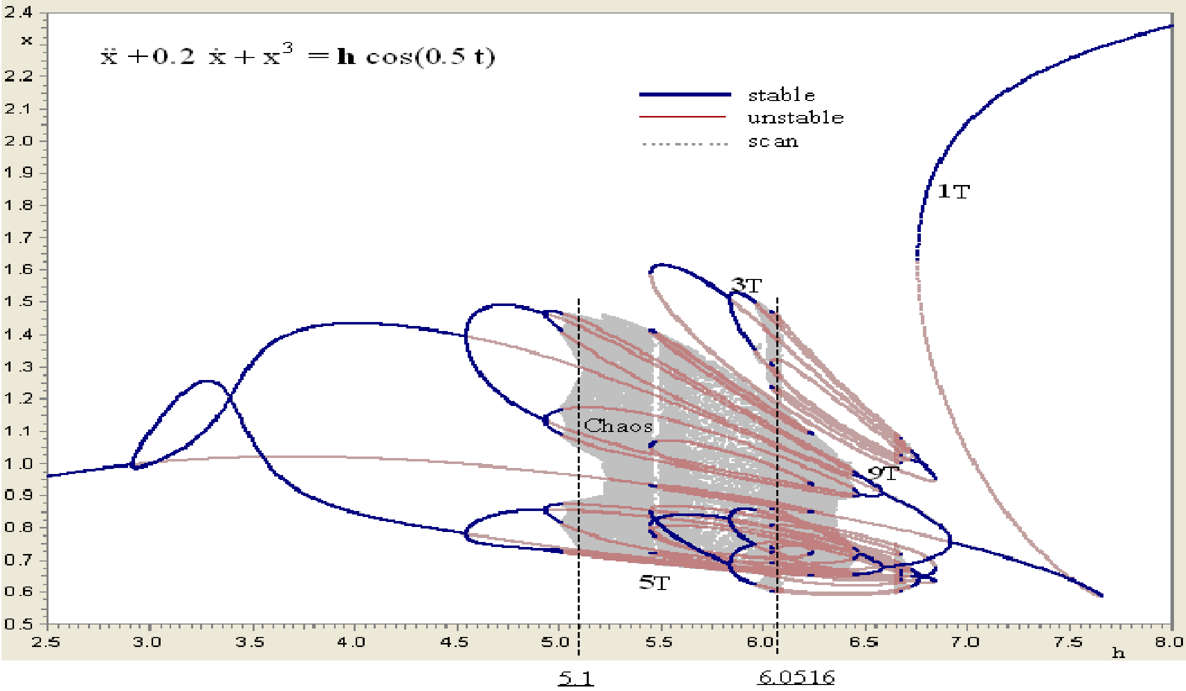

Figure 1. Bifurcation diagram of four bifurcation groups(1T, 3T, 5 and 9T) with rare attract-tors, unstable periodic infinitiums (UPI-1, UPI-3, UPI-5 and UPI-9) and chaotic attractors (see Fig.2-3) of the driven symmetric system with cubic restoring force and linear dissipation (Duffing -Ueda oscillator) under the change of the amplitude of external harmonic forcing.

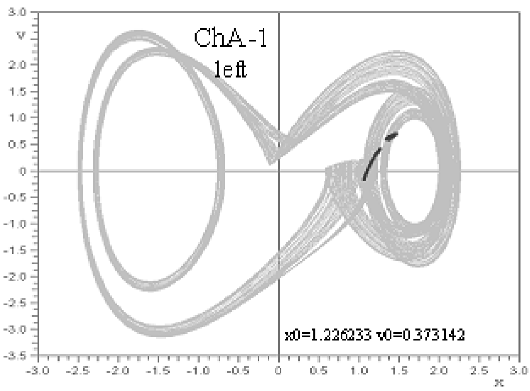

Figure 2. Coexisting of two mutually symmetric chaotic attractors (200 periods) on the Poincaré plane (black) and phase trajectories (gray). in the symmetric system with cubic restoring force, linear dissipation under external harmonic excitation(see Fig.1, h=5.1).

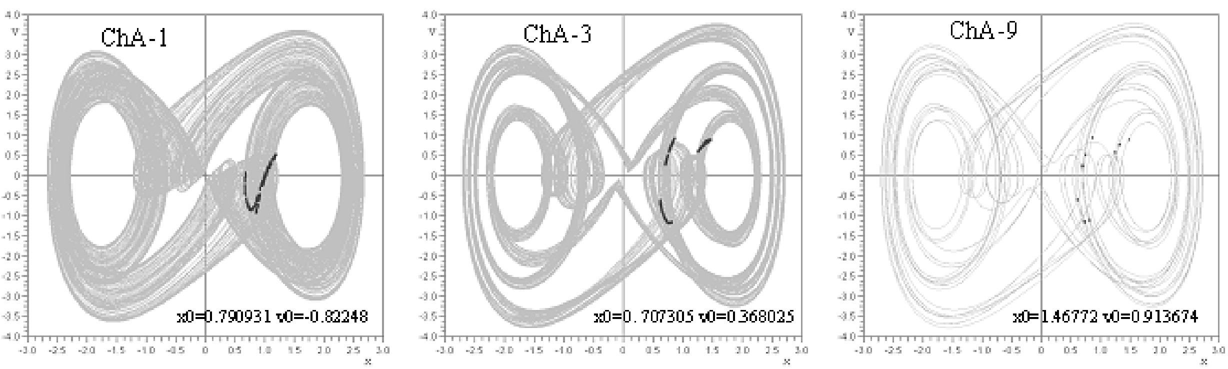

Figure 3. Coexisting of three chaotic attractors ChA-1, ChA-3 and ChA-9 (200 periods) on the Poincaré plane (black) and phase trajectories (gray). in the symmetric system with cubic restoring force, linear dissipation under external harmonic excitation(see Fig.1 h=6.0516).

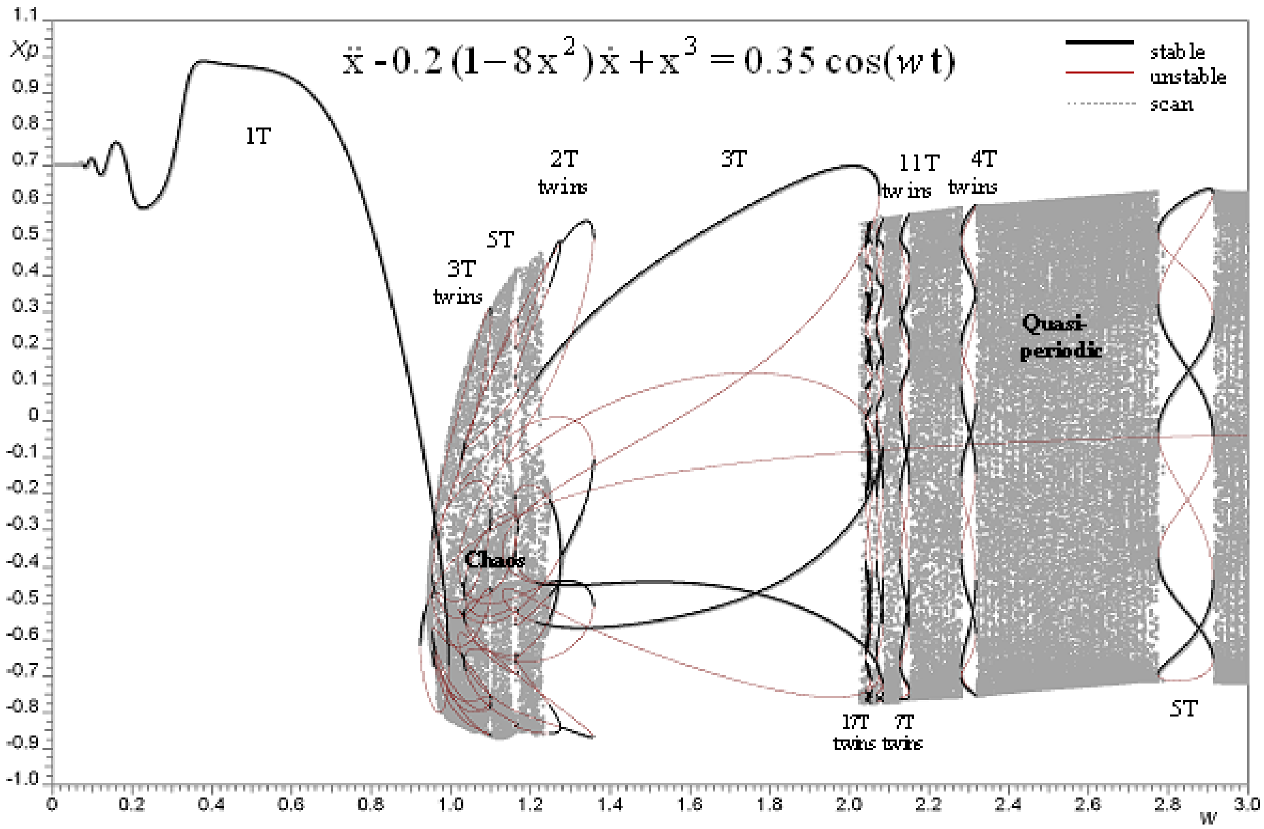

Figure 4. Complete bifurcation diagram of stable, unstable periodic solutions, quasi-periodic motions and chaotic attractors of 16 bifurcation groups (1T, two 2T, three 3T , two 4T, two 5T, two 7T, two 11T, two 17T) of the driven van der Pol oscillator with cubic restoring force (van der Pol-Ueda's equation) under the change of the frequency of external harmonic forcing.

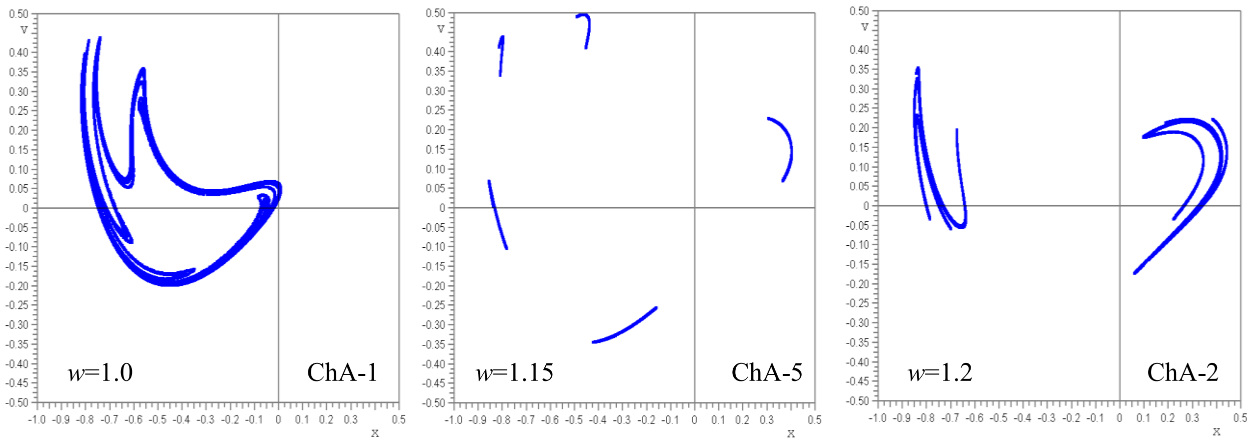

Figure 5. Chaotic attractors ChA-1, ChA-5, ChA-2 of various bifurcation groups of the driven van der Pol oscillator with cubic restoring force (van der Pol-Ueda's equation) under the change of the frequency of external harmonic forcing (see Fig.4 , w=1.0, w=1.15, w=1.2).

The systematic approach allows to do global analysis using the method of bifurcation groups for the driven van der Pol oscillator with cubic restoring force (van der Pol-Ueda's equation) under the change of the frequency of external harmonic forcing (Fig.4). In the presence of infinite number of unstable periodic infinitums (UPI) of various bifurcation groups, as shown In Fig. 5, exist different chaotic attractors.

In typical dynamical systems the birth of chaotic oscillations at change of dissipation is shown. It is shown that change of linear (see Fig.6) and nonlinear (see Fig. 7) dissipation lead to occurrence of nonlinear effects in systems with nonlinear positional forces. Some nonlinear phenomena are arising at increase of dissipation level. Example is the birth of additional stationary regimes after increase of dissipation, some of which have the greater amplitude, than amplitude of the main regime. The listed variety of nonlinear effects allows underlining in many cases paradoxical influence of change of dissipation. As a result of researches of a great number of nonlinear systems at change of linear and nonlinear dissipation has been established that the method complete bifurcation groups allows to regularly find nonlinear effects and to investigate conditions of their occurrence. The nonlinear effects arising at change of dissipation, in many cases can be useful to realization of operating conditions of nonlinear dynamical systems of the different nature.

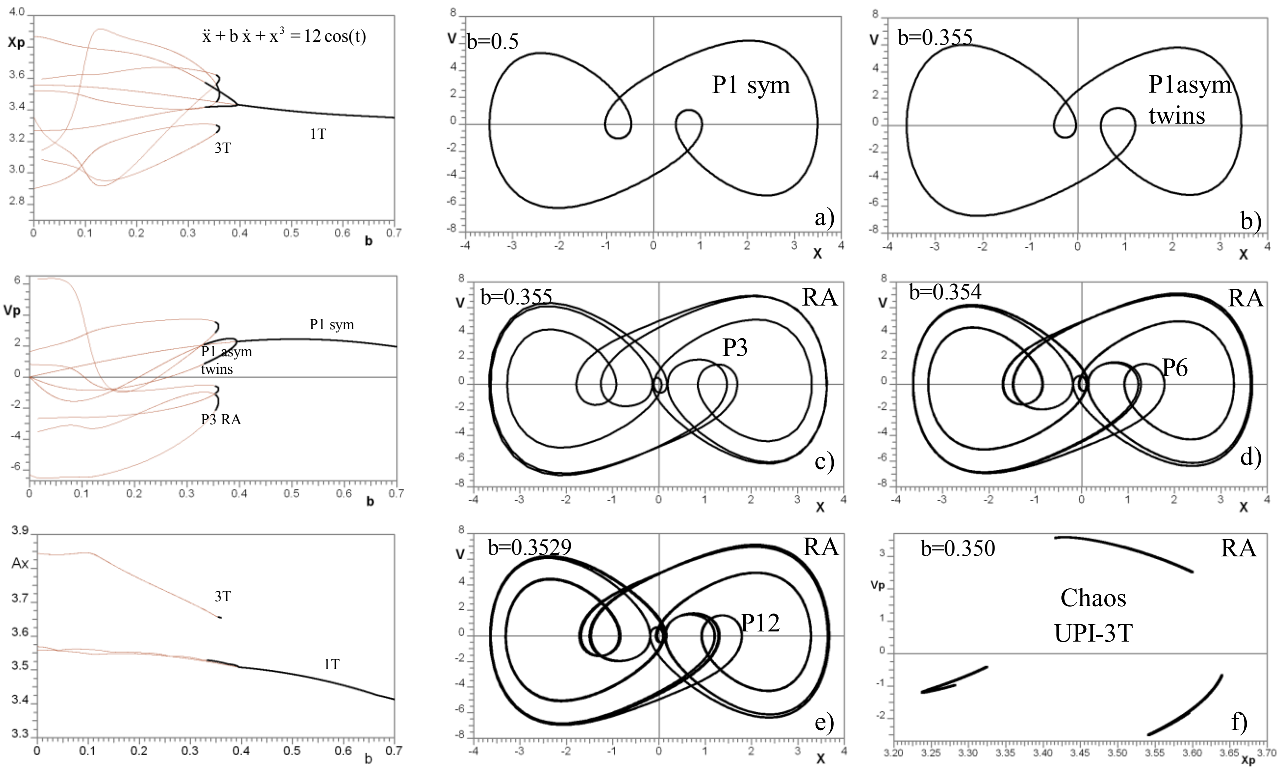

Figure 6. Bifurcation diagrams (displacement (Xp), velocity (Vp) coordinates and amplitudes (Ax) of regimes) of stable and unstable periodic solutions of bifurcations groups 1T and 3T in Duffing system ($\ddot{x} + b\dot{x} + x^3 = h_1 \cos(\omega t)$) under the change of linear dissipation coefficient. New nonlinear phenomena have been discovered: rare chaotic attractor P3 (f). Phase portraits of attractors of 1T bifurcations group (a-b) and phase portraits of rare attractors (RA) of 3T bifurcation group (c-e) under external harmonic forcing with linear damping under the change of linear dissipation coefficient. A rare chaotic attractor (f) of 3T bifurcation group on the Poincaré plane has just burst out, the dissipation coefficient being b=0.35. The Duffing system parameters: b = var, ω=1.0, h1= 12.0.

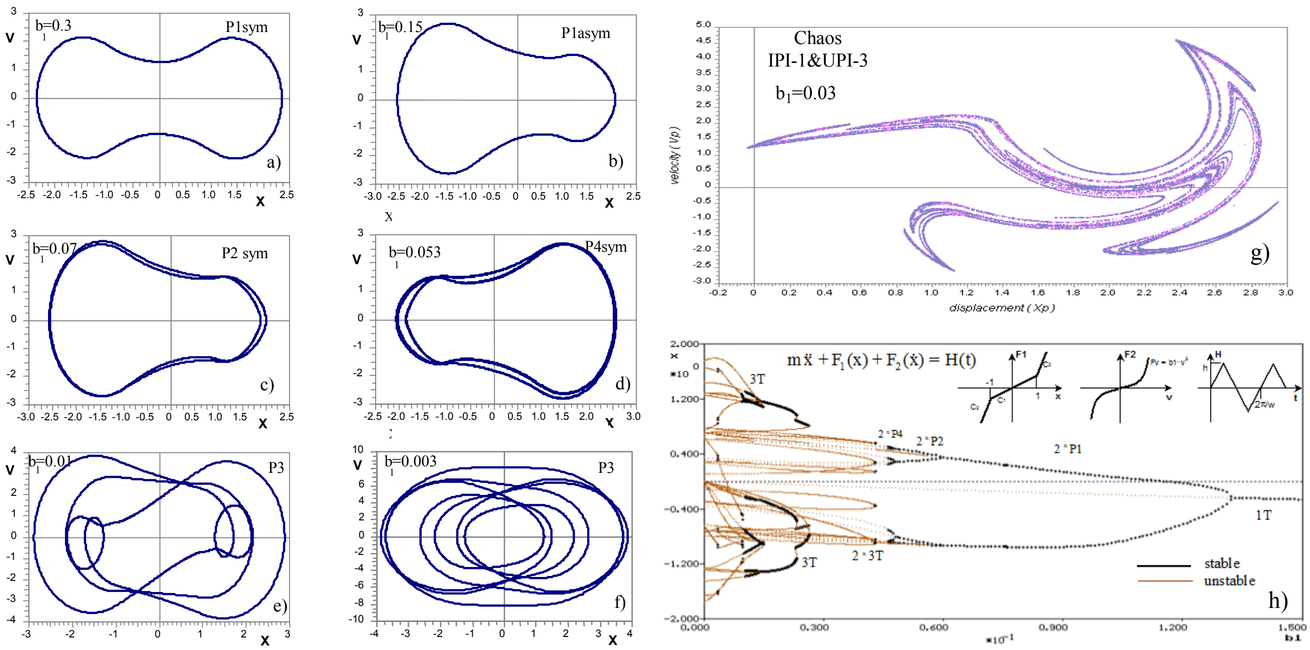

Fig. 7. Trilinear symmetric system bifurcation analysis results under the change of nonlinear dissipation coefficient. Phase portraits of attractors of 1T bifurcations group (a-d) and phase portraits of rare attractors (RA) of 3T bifurcation group (e-f) in under external saw-tooth forcing with cubic turbulent damping. Chaotic attractor coexisting of three bifurcation groups (1T group and two 3T groups) in trilinear symmetric system under external saw-tooth forcing with cubic turbulent damping, the dissipation coefficient being b1=0.03 (g). Bifurcation diagram (h) with five complete bifurcation groups: 1T and four several bifurcations groups 3T in trilinear symmetric system under external saw-tooth forcing with cubic turbulent damping under the change of nonlinear dissipation coefficient. Nonlinear phenomena have been discovered: five rare attractors with chaotic attractors of bifurcations groups 3T (h). System parameters: m=1, $c_1 = 1$, $c_2 = 9$, $\Delta = 1$, $b_1 = var$, ω=1, $h_1 = 7$.

Since deterministic chaos is associated with random behaviour arising from sensitive dependence on initial conditions, it is quite natural to expert quantative criteria to distinguish between chaotic and regular motions should be based on the distinctive characteristics. Indeed there are many such characteristics available in the literature for this purpose and some of the most prominent of them are: Lyapunov exponent, power spectrum, autocorrelation function and dimension. The chaotic attractor possesses: at least one positive Lyapunov exponent, power spectrum with continuous (broad-band), decaying to zero autocorrelation function, noninteger or fractal dimension. These measures are capable of distinguishing different degrees of complexity of attractors and motions. In dissipative systems, in addition to the above characteristic properties the chaotic attractor possesses amount of periodic skeletons and order of bifurcation groups.

Conclusion

Application of the systematic approach while the study of forced oscillations on the basis of a method of complete bifurcation groups allows us to clarify the origin of the researched chaotic oscillations and to classify the transition to chaos and chaotic attractors according to the amount of periodic skeletons and order of bifurcation groups. Results of researches are recommended to be considered for a prediction and prevention of chaotic oscillations and catastrophic situations in mechanical, electrical and other nonlinear dynamical systems.

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