RIGA TECHNICAL UNIVERSITY

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AIRFRAME INSPECTION PLANNING

Summary of Doctoral Thesis

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RIGA TECHNICAL UNIVERSITY

Institute of Aeronautics

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DOCTORAL THESIS PROPOSED TO RIGA TECHNICAL UNIVERSITY FOR THE PROMOTION TO THE SCIENTIFIC DEGREE OF DOCTOR OF ENGINEERING SCIENCES

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DECLARATION OF ACADEMIC INTEGRITY

I hereby declare that the Doctoral Thesis submitted for the review to Riga Technical University for the promotion to the scientific degree of Doctor of Engineering Sciences, is my own and does not contain any unacknowledged material from any source. I confirm that the present Doctoral Thesis has not been submitted to any other university for the promotion to any other scientific degree.

Maris Hauka .... (Signature)

Date: ………………………

The Doctoral Thesis has been written in Latvian. The Doctoral Thesis comprises an introduction, 5 chapters, conclusions, bibliography with 47 reference sources and 7 appendices. It has been illustrated by 58 figures and 7 tables. The total volume of the present Doctoral Thesis is 151 pages.

CONTENTS

GENERAL DESCRIPTION OF THE PRESENT RESEARCH

Topicality of the Research

Safety of airplanes is affected by cracks of different shapes and sizes that occur due to metal fatigue. The key elements of plane are also subjected to the fatigue crack development risk. Uncontrolled expansion of a fatigue crack can lead to a total failure of the affected element, thus causing an airplane crash. The problem becomes more acute if the plane is actively exploited for a prolonged period of time. There are two methods to avoid the probability of plane crash. The first method is to limit aircraft time in service based on experimental data of probability of fatigue crack development on key elements. The second method is to create an inspection program with the goal to find fatigue cracks before they expand to a size that may endanger the airplane.

Many scientific publications are dedicated to the choice of inspection intervals. However, most of these methods have serious flaws making them hard to use or even completely useless in creating actual inspection programs for fatigue crack detection in aviation construction elements. No research takes into account the characteristic features of fatigue crack emergence and evolution. The existing methodologies take into account only the distribution law till maintenance and till break-down, which is considered to be a known variable, thus enabling probability calculations. This particular methodology takes into account the characteristic features of fatigue crack emergence and evolution. The parameters of a respective fatigue crack are set based on results of specific test reports, during which it is considered whether to send the construction for recycling should the results be inadequate. This allows using the principle of final decision, which would be based on the MinMax criteria. The name of the developed methodology is MinMax. These methods are mainly based on the precise knowledge of random process parameters of fatigue crack development and sorting of known defects.

In fact, it is necessary to perform the calculation of mathematical statistics, where instead of a parameter the evaluation of occurrence of a parameter is used. However, in reality there are results of a very limited number of fatigue tests of airframe and very random estimates of parameters of fatigue crack growth trajectory.

To plan inspection intervals, not only reliability of airframe but also economical parameters of this program should be taken into account.

The Aim of the Research

The aim of the research is to develop a method to choose the interval of inspection of a fatigue-prone airframe and a computer program for this method.

The Tasks of the Research

To achieve the goal set, it is necessary to put forward the following tasks:

1) to create an inspection program development method based on very limited fatigue crack statistical information about the growth random process and economic requirements;

2) to create software that allows implementing such an inspection program.

Methods of the Research

The Doctoral Thesis has been developed applying the following analytical methods:

- The theory of fatigue crack growth in airframe constructions;
- The theory of probability;
- Mathematical statistics;
- Markov chain theory and semi-Markov processes with reward;
- Visually oriented programming.

In the research, for mathematical data processing and mathematical modelling MathWorks MatLab computer program has been used. To carry out the calculations, AERTI scientific stations have been used.

Thesis Statement

In this research, the airplane structure inspection frequency planning method is offered, which is based on actual assembly fatigue assumption experiments. The results of those experiments are used to assess parameters of the fatigue crack models.

The offered methods are based on choosing frequency of maintenance and ensure high possible income within required safety parameters.

The methodology is based on the semi-Markov chain process theory with gain; MiniMax approach is used.

MatLab computer program is also offered and examples are provided.

Scientific Novelty

A modern maintenance frequency planning program based on renewal (restoration) theory usually as a must takes into account only the service life of the assembly. The offered method also takes into account the fatigue crack growth trajectory.

Existing methods consider the law of service life layout to be a given value, and a probability theory calculation is carried out. The developed methodology evaluates parameters of fatigue crack growth based on examination results.

According to the developed method, if the examination proves that the set requirements of the prototype assembly are not met, the construction is not accepted into exploitation unless flaw is removed. This enables the use of MinMax approach. As a result, the safety of the exploitation of the construction is ensured at any fatigue crack growth parameter and layout peculiarity (since only constructions that meet the set requirements are accepted into exploitation, while inadequate assemblies are not).

The theory of the semi-Markov chain processes with income parameters is used for the calculations. As a result, two major tasks are accomplished:

- Task No. 1: to ensure safety;
- Task No. 2: to manage exploitation expenses.

Properly chosen frequency of the maintenance ensures the maximum amount of income while maintaining the required safety.

Research Results

1. The solution to the problem of developing the inspection program for an aircraft under requirements of reliability of an aircraft and airline has been proposed.
2. Specific features of the corresponding statistical decision function are the following:

- Specific model of fatigue crack growth process has been used, some parameters of which have been estimated taking into account the results of real full-scale fatigue cracks;
- The theory of Markov chains and semi-Markov processes with reward have been used to calculate the reliability of aircraft and both the fatigue failure rate and the reward of airline;
- Offered statistical decision function provides required reliability independent of unknown parameter because the minimax approach has been used based on the results of approval tests, when the redesign of the tested airframe should be made if these results do not meet specific requirements.

Practical Relevance of the Research

The developed models of airplane and airline reliability can be used to develop the inspection program of fatigue cracks in an airframe based on the results of full-scale fatigue tests of airframe, taking into account the following aspects:

- compliance with the requirements imposed on the results of approved fatigue tests and conditions of redesign of an airframe if these requirements are not met,
- the required reliability of airplane and limitations of fatigue failure intensity of airplane,
- effectiveness of inspection technology,
- economic parameters: inspection costs, crack detection costs, fatigue failure costs and cost of a new airplane.

The program package can be used for both research and educational purposes.

The Approval of Research Results

The main results of the research have been presented in the following **international scientific conferences:**

1. The 4th International Conference on Scientific Aspects of Unmanned Aerial Vehicle, “Inspection Program Development for Stationary Operation of Fatigue-prone Aircraft Park”, M.Hauka, J.Paramonovs.
2. The 10th International Vilnius Conference on Probability Theory and Mathematical Statistics, 28/06/2010-02/07/2010, “Reliability of Park of Aircraft at Minimum Cost”, M.Hauka, J.Paramonovs.
3. Reliability and Statistics in Transportation and Communication (RelStat 10) 20/10/2010-23/10/2011, Reliability of Aircraft and Airline; M. Hauka, J. Paramonovs, Transport and telecommunication, Riga.
4. The 9th Tartu Conference on MULTIVARIATE STATISTICS & the 20th International Workshop on Matrices and Statistics. 26.06.2011 - 1.07.2011, Tartu, Estonia; “Minimax Decision for Solution of the Problem of Aircraft and Airline Reliability Processing Results of Acceptance Full-scale Fatigue Test of Airframe”, M.Hauka, J. Paramonovs.
5. International Conference on Stochastic Modelling Techniques and Data Analysis (SMTDA 2012) 5 June-8 July, 2012 Greece, Chania, “MinMax Choice of Inspection Program of Fatigue-Prone Airplane Structure”, M.Hauka, J. Paramonovs.
6. International Conference “Probability Theory and Its Applications”, Russia, Moscow, 26-30 June, 2012, “Reliability of Aircraft and Airline”, J. Paramonovs, M.Hauka.

7. Intelligent transport systems 2012 (ITS'12), 18 - 20 July 2012, Latvia, Riga. "MinMax Approach for Aircraft Inspection Program Interval Selection in order to Ensure Airline Safety", M. Hauka, J. Paramonovs.
8. The 53rd International Scientific Conference of Riga Technical University, 11-12 October 2012, Latvia, Riga, "Minmax Approach for Aircraft Inspection Program Interval Selection in order to Ensure Airline Safety", M. Hauka, J. Paramonovs.
9. The 7th International Workshop on Simulation, May 21-25, 2013, Rimini, Italy, "Minimax Decision for Reliability of Aircraft Fleet and Airline", J. Paramonovs, M.Hauka, S.Tretjakovs.
10. The 8th IMA International Conference on Modelling in Industrial Maintenance and Reliability (MIMAR),10 - 12 July 2014, Wadham and St Catherine's Colleges, University of Oxford, "Minimax Inspection Program for Reliability of Aircraft Fleet and Airline", M. Hauka, S. Tretjakovs, and J. Paramonovs.

The Research results (papers) that have been **published in scientific and technical proceedings:**

1. Hauka, M., Paramonovs, J. Semi-Markov Model of Aircraft and Airline Reliability with Minimax Decision in Case of Unknown Parameter of Fatigue Crack Growth // International Review of Aerospace Engineering. - Vol.6, No.3. (2010) pp. 329-336.
2. Hauka, M., Paramonovs, J. Reliability of Aircraft an Airline // Transport and Telecommunication. - Vol.11, No.4. (2010) pp. 59-65.
3. Hauka, M., Paramonovs, J. Inspection Program Development for Stationary Operation of Fatigue-Prone Aircraft Park // Scientific Aspects of Unmanned Mobile Vehicle, Kielce University of Technology, 2010. – pp. 559-572.
4. Hauka M., Paramonovs J. Reliability of Park of Aircraft at Minimum Cost // The 10th International Vilnius Conference on Probability Theory and Mathematical Statistics, Lithuania, Vilnus, 28 June-2 July 2010. – pp. 161-161.
5. Hauka, M., Paramonovs, J. Inspection Program Development for Stationary Operation of Fatigue-Prone Aircraft Park // The 4th International Conference on Scientific Aspects of Unmanned Aerial Vehicle (SAUAV-2010), Poland, Kielce, 5-7 May 2010. – pp. 276-284.

6. Hauka, M., Paramonovs, J. Minimax Decision for Solution of the Problem of Aircraft and Airline Reliability Processing Results of Acceptance Full-Scale Fatigue Test of Airframe // The 9th Tartu Conference on Multivariate Statistics & the 20th International Workshop on Matrices and Statistics, Estonia, Tartu, 26 June-1 July, 2011. – pp. 29-29.
7. Hauka, M., Paramonovs, J. Inspection Program Development Using Minimax Method // The 11th International Conference “Reliability and Statistics in Transportation and Communication – 2011”, Riga, Latvia, 19–22 October 2011, pp. 105-114.
8. Hauka, M., Paramonovs, J. MinMax Choice of Inspection Program of Fatigue-Prone Airplane Structure // International Conference on Stochastic Modelling Techniques and Data Analysis (SMTDA 2012), Greece, Chania, 5 June-8 July, 2012. – pp. 47-48.
9. Hauka, M., Paramonovs, J. Minimax Reliability of Aircraft and Airline // International Conference “Probability Theory and Its Applications”, Russia, Moscow, 26-30 June 2012. – pp. 282-283.
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11. Paramonovs, J., Hauka, M., Tretjakovs, S. Planning of Inspection Interval to Provide Reliability of Fatigue-Prone Aircraft Using Result of Acceptance Fatigue Test // Reliability and Statistics in Transportation and Communication: The 13th International Conference (RelStat’13), Latvia, Riga, 16-19 October 2013. Riga: TSI, 2013, pp. 39-47. ISBN 9789984818597.
12. Hauka, M., Paramonovs, J. Airline and Aircraft Reliability // Transport and Aerospace Engineering. No. 1, 2014, pp. 9-14. ISSN 2255-968X. Available: doi:10.7250/tae.2014.002.
13. Hauka, M., Tretjakovs, S., Paramonovs, J. Minimax Inspection Program for Reliability of Aircraft Fleet and Airline // Proceedings of the 8th IMA International Conference on Modelling in Industrial Maintenance and Reliability (MIMAR),10 - 12 July 2014, Wadham and St Catherine’s Colleges, University of Oxford, pp. 125-130.

The Structure of the Doctoral Thesis

The Doctoral Thesis consists of the introduction, 5 chapters, conclusions, bibliography with 47 reference sources and 7 appendices. It has been illustrated by 58 figures and 7 tables. The total volume of the Doctoral Thesis is 151 pages.

- Introduction – the summary of breakdowns caused by metal fatigue and methods to prevent them.
- Chapter 1 – guidelines of evolution of the structure safety philosophies and accidents related to them. History of this evolution can be divided into stages based on air crashes caused by metal fatigue.
- Chapter 2 – it describes a simplified maintenance program, its creation (development); structure and classification to demonstrate the essence and characteristics of aircraft maintenance programs.
- Chapter 3 – it studies different types of restoration (and renewal) functions (methods). Each approach has its advantages and drawbacks. Exploration of these theories allows drawing significant conclusions that will make perfect safety of spacecraft (SC) maintenance programs.
- Chapter 4 – it examines ideology of the choice of inspection (maintenance) programs. This ideology allows for the choice of optimal inspection (maintenance) program that takes into account possible risks and expenses as well as retains high credibility even in case of a small number of conducted full-spectrum fatigue crack growth experiments. Basic mathematical models of explored (and suggested) examples are also included.
- Chapter 5 – it studies the new MinMax methodology with the included example and related calculations. Main difference from other methods is the ability to determine not just breakage probability or brakeage intensity, but also economic effectiveness (cost-efficiency), as well as assess influence of deviations on parameters and allow processing data that are based on a small number of conducted full-scale fatigue crack growth experiments (calculation methodology already described in Chapter 4).
- Conclusions – advantages of the developed methodology are presented and the solution of the confirmed exercise is proposed.
- Appendices – the extended version of explanation of the theory of renewal (restoration) and printout of scripts of the developed computer program.

THE SCOPE OF RESEARCH

Chapter 1. Civil Aviation Design and Safety Philosophy History

Chapter 1 describes history of the problem and gives an overview of the most important studies devoted to its investigation. In accordance with “A Survey of Aircraft Accidents Involving Fatigue Fracture” by G.S. Cambell, R. Lahey in 1981, during period from 1927 to 1981 there were 1,885 identified cases of plane crashes caused by structural fatigue cracks. During this period, the plane crashes took the lives of 2,240 people.

On 23 September 1927, Dornier Merkur suffered a wing failure. In just one year, on average 67 such failures were registered.

Most noted catastrophes are related to the Comet (during 1953 and 1954), Fokker F-27 (1968), F-111 (1969), Hawker Siddeley (1976) and Boeing 707-321C (1977). The most massive structural damage with a successful landing was registered for B-737 (1988) (at its 89,000th flight). Last registered case of wide publicity in press caused by structural fatigue belongs to Airbus A380. In December 2011, a fatigue crack was found in its wing. All these cases stress the need for further research on airplane structural fatigue crack development and forecasts.

There is a set of studies devoted to the development of the airframe inspection programs with the goal to discover a fatigue crack before its size reaches some critical size by the following authors: B. Lundberg, F. H. Hook, D. N. Young, V. Y. Sennik, V. V. Nikonov, N. N. Smirnov, V. S. Streljaev, I. Nesterenko and others. Great contribution to the development of this direction was also made by works supervised by H. B. Kordonsky and J. A. Martynov.

However, the above-mentioned authors have not taken into account in their studies that the probability of failure depends not only on the random nature of fatigue crack growth model parameters and inspection techniques, but also on the final decision-making procedure, and, in particular, they have not considered that in a certain case the airframe project itself may be re-designed.

The early development of mathematical models in the given direction can be found in the works of Y. M. Paramonov and P. M. Sobolev, and then research was continued by N. M. Kimlik, A. Kuznetsov and K. Nechval.

The present study continues the research on the modernization of the model with the aim not only to assess reliability of an aircraft, but also to calculate the fatigue rate and airline reward.

Chapter 2. Maintenance Programs

Chapter 2 explains the process of creation of the SC maintenance program and simplifies structure of a maintenance program to explain the essence and characteristic features of the SC maintenance program. The chapter substantiates the cyclic nature of the time (maintenance frequency) calculations. During the prime of aviation, the maintenance programs were mostly created by the pilots and mechanics themselves based on their own experience and best of their abilities. When aviation evolved into a separate new branch of transportation, different regulations for maintenance and examination started to appear, such as Msg-*I*, Msg-*II* and Msg-*III* guidelines of maintenance program development.

Chapter 3. Renewal Theory

Chapter 3 describes mathematical methods that can be used to create an inspection program.

The theory of renewal is based on the simplest model – after each repair the system returns to its working state; thus, "repair" is a process during which an element regains its full working state.

This idea suits the modal repair method, where a damaged element or a system is fully substituted with a new one. When implemented, the theory shows results almost identical to the real situation. Regardless of the fact that the theory was created for simple models, it can be used for more complex repairs, especially in the theoretical part of technical repairs and revisions.

Periodic renewal is known as renewal depending on age. It is used when a system is renewed after a failure. Also, if a system has worked flawlessly for an interval of time τ, then the substitution of the system is made.

Renewal made after a failure is called an emergency renewal. Both emergency and preventive repairs are complete and thorough – the entire system is substituted with a new one.

Renewal by blocks has a set moment of preventive actions. The method is implemented, when for the repairs a large number of preparations are needed, and the system itself can be taken out of service for the needs of repairs for only a short period of time.

In case of a failure, the system undergoes an emergency renewal. Regardless of the age of the system during time moments τ, 2τ, etc. a renewal is made. The model can be of different types. For example: after a failure the system may be returned to service only at the beginning of the next operation cycle or any other type.

Minimal emergency renewal with periodic full substitution. The methods mentioned above consider that the system is fully renewed in both emergency and preventive repair cases. In reality, however, the repairs are not always implemented. This means that the system will be fully renewed only during next periodic renewal procedure, but in case of non-planned repairs, such as failures, it will undergo only partial renewal. There are cases when non-planned or partial renewal cannot be implemented. This occurs in cases of serious and extreme damage to the system, in these cases full renewal is required.

Renewal of doubled systems. In order to increase the safety of technical systems, the safety of the elements of this system can be increased, but this method has its limits both technically and economically. The use of inspection and preventive repair programs cannot always ensure compliance with the prescribed safety requirements. Thus, sometimes, systems with increased safety have to be implemented as well.

Control with renewal. The method considers that operation of the system does not finish with discovery of the damage. The system regains a full working state after full renewal. The process of control and renewal is never-ending. Both time and expenses are taken into account.

These methods do not take into account specific features of the process of fatigue cracks, which can appear and grow in airframe structures.

Chapter 4. Choosing Inspection Program by Logic and Methods

Chapter 4 describes the ideology of choosing an optimal inspection program, where the possible risks and expenses are taken into account. Description of basic mathematical models and initial information are given below.

Crack and Limiting Parameters

Fatigue crack expansion model is given. To create an inspection program, the results of full-scale fatigue test of airframe have to be used. To perform calculations, a fatigue crack expansion model is required (4.1).

$$a(t) = a(0)\exp(Qt), \qquad (4.1)$$

where

$a_0 = a(0)$ – equivalent size of the crack at the beginning of modelling;

Q – speed of expansion of fatigue crack.

For approximation of real data obtained during a fatigue test of Tu-134 airplane, an exponential function can be used, see Fig. 1.

In this mathematical model, the speed of expansion of fatigue crack (on a logarithmic scale) is parameter Q, the equivalent size of the crack at the beginning of modelling is $a_0 = a(0)$. This formula allows calculating the moment of time T_c, at which the fatigue crack reaches its critical size a_c; time T_d, at which the fatigue crack can be discovered (the size of the fatigue crack is equal to or greater than a detectable size a_d) and the period of time, at which the fatigue crack can be detected (T_d ; T_c), see Fig. 1. A failure of system occurs when the fatigue crack reaches its critical size before the inspection takes place (there is no inspection in time period (T_d ; T_c)).

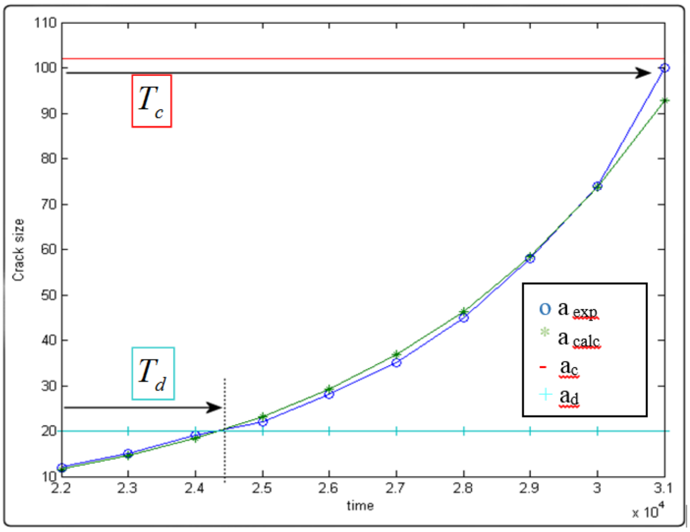


Fig. 1. Processing of fatigue cracks, where a_d – detectable crack size, a_c – critical crack size, a calc – crack calculated size, a exp – crack experimental size

Fatigue cracks for three different a_0, three different Q are shown in Fig. 2.

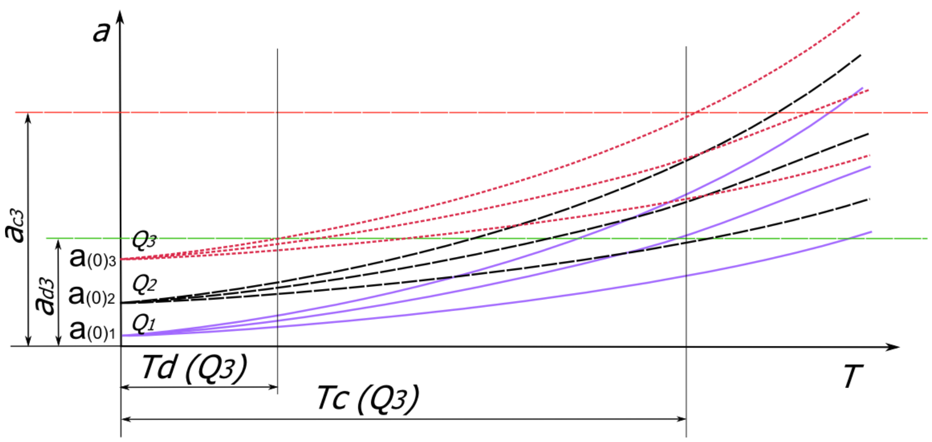


Fig. 2. Fatigue cracks for three different a_0, three different Q

Let us denote $X = \log Q$ and $Y = \log C_c$, where $C_c = \log a_c - \log \alpha$. From the analysis of the fatigue test data, it can be assumed that $\log T_c = \log C_c - \log Q$ is distributed normally. It comes from the additive property of the normal distribution that it can take place if either both $\log C_c$ and $\log Q$ are normally distributed or if one of these components is normally distributed while the other is constant. The current research focuses on the first case: it is supposed that the vector $(X, Y) = (\log(Q), \log(C_c))$ has two-dimensional normal distribution with vector parameter $\theta = (\mu_X, \mu_Y, \sigma_X, \sigma_Y, r)$. It is worth noting that when a_c and a_d are constants, then cdf of C_d is completely defined by the distribution of C_c because $C_d = C_c - \delta$, where $\delta = \log(a_c / a_d)$.

Calculation of the Failure Probability

Calculation of aircraft fatigue failure probability (FFP) is considered. Calculation can be performed using Monte Carlo method, specific formula for FFP and Markov chain theory.

In case of Monte Carlo method, the modelling of random variables X and Y is made using the following formulae (4.2).

$$Y = \eta_1 \sigma_Y + \mu_Y, \quad X = \eta_1 \sigma_X r + \eta_2 \sigma_X \sqrt{1 - r^2} + \mu_X \tag{4.2}$$

where

η_1 and η_2 have a standard normal distribution.

If the number of cracks observed is large enough, it is possible to assume the part of missed cracks among all the cracks in the series as the estimate of probability of failure for a particular inspection program (4.3).

$$\hat{P}_f \underset{N_{total} \to \infty}{\to} P_f = \lim_{N_{total} \to \infty} \left(\frac{N_{missed}}{N_{total}} \right), \tag{4.3}$$

where

$N_{total} = N_{missed} + N_{discovered}$;

N_{total} – total number of cracks;

N_{missed} – number of missed cracks.

The particular inspection program is defined by the sequence of inspection time moments, $t = (t_1, t_2, ..., t_n)$, where t_i is the time moment of i-th inspection, $i=1,2,...,n$, n is the inspection number, $t_{n+1} = t_{SL}$. To calculate the failure probability for this inspection program, it is necessary to sum up all failure probabilities in all intervals (4.4) where failure probabilities in one interval are (4.5).

$$p_f = \sum_{i=1}^{n+1} q_i \tag{4.4}$$

$$q_i = P(t_{i-1} < T_d \le T_c \le t_i) = P\left(t_{i-1} < \frac{C_d}{Q} \le \frac{C_c}{Q} \le t_i \right), \tag{4.5}$$

where

q_i – i- point failure probability;

t_{i-1} – i-1 check time moment;

t_i – i check time moment;

T_c – time to reach a critical size of the crack;

T_d – time to reach detectible size of the crack;

C_d – crack detectible size;

C_c – critical crack size;

Q – crack growth rate.

If, in the simplest case, we assume that C_c and C_d are some constants, $C_c \ge C_d$, then use equation (4.6), where for the normal distribution of random variable $X = \log Q$ failure probability is (4.7).

$$q_i = \begin{cases} 0, \ \text{if } t_i \le t_{i-1}\dfrac{C_c}{C_d}, \\ q_i^+, \ \text{if } t_i > t_{i-1}\dfrac{C_c}{C_d}, \end{cases} \tag{4.6}$$

$$q_i^+ = P\left(\frac{C_c}{t_i} \le Q < \frac{C_d}{t_{i-1}}\right) = \Phi\left(\frac{\ln(C_d/t_{i-1}) - \theta_0}{\theta_1} \right) - \Phi\left(\frac{\ln(C_c/t_i) - \theta_0}{\theta_1} \right). \tag{4.7}$$

For a two-parameter model, assuming that normal distribution of $X = \log Q$ and $Y = \log C_c$, we should take into account that (4.8) where δ is some constant. Then failure probability is (4.9)

$$C_c - C_d = \log(a_c) - \log(a_d) = \log(a_c / a_d) = \delta \,, \tag{4.8}$$

where

δ – some constant

$$q_i^+ = P\left(\log C_c - \log t_i \le \log Q < \log(C_c - \delta) - \log t_{i-1}\right) = \int_{\ln\delta}^{+\infty} \left(g_i^+(y)\right) d\Phi\left(\frac{y-\mu_Y}{\sigma_Y}\right), \tag{4.9}$$

where

$\Phi(.)$ – normal distribution function.

$$g_i^+(y) = \max\left(0, \Phi\left(\frac{\left(\log\left(e^y - \delta\right) - \log t_{i-1}\right) - \mu_{X/y}}{\sigma_{X/y}}\right) - \Phi\left(\frac{\left(y - \log t_i\right) - \mu_{X/y}}{\sigma_{X/y}}\right)\right), \tag{4.10}$$

$$\mu_{X/y} = \mu_X + r\frac{\sigma_X}{\sigma_Y}\left(y - \mu_Y\right),\ \sigma_{X/y} = \sigma_X\sqrt{1-r^2}\ , \tag{4.11}$$

where

δ – some constant;

μ_x, μ_y – the mean value of the parameter (the parameter Q and Cc, respectively);

σ_x, σ_y – standard deviation (parameter Q and Cc, respectively);

t_{i-1} – i-1 point time moment;

t_i – i-Point time moment.

Here we suppose that parameters σ_X, σ_Y and r depend on technology, which does not change (for a new aircraft), and these parameters can be estimated using information of previous designs. We suppose that they are fixed and known values. Then an unknown parameter, θ, has only two components: $\theta = (\mu_X, \mu_Y)$. The process of operation of AC can be viewed as absorbing MC with $(n+4)$ states. The states $E_1, E_2, ..., E_{n+1}$ correspond to AC operation in time intervals $[t_0, t_1), [t_1, t_2), ..., [t_n, t_{SL})$. States E_{n+2}, E_{n+3}, and E_{n+4} are absorbing states: AC is discarded from service when the SL is reached, or a fatigue failure (FF) or fatigue crack detection (CD) takes place. Let us assume that in the transition probability matrix, P_{AC}, for corresponding process of AC operation the probability of crack detection during the inspection number i is denoted as v_i; the probability of failure in service time interval $t \in (t_{i-1}, t_i]$ is q_i and the probability of successful transition to the next state is $u_i = 1 - v_i - q_i$. In our model we also assume that an aircraft is discarded from service at t_{SL} even if there are no cracks discovered by inspection at the time moment t_{SL}. This inspection at the end of (n+1)-th interval (in state En+1) does not change the reliability, but it is made in order to know the state of aircraft (whether there is a fatigue crack or there is no fatigue crack). It can be shown in (4.12- 4.15).

$$u_i = P(T_d > t_i \mid T_d > t_{i-1}) = P(Q < C_d / t_i) / P(Q < C_d / t_{i-1}) = a_i / a_{i-1} \tag{4.12}$$

$$q_i = P(t_{i-1} < T_d < T_c < t_i \mid Td > t_{i-1}) = \begin{cases} 0, & if \quad t_{i-1}C_c / C_d > t_i, \\ b_i / a_{i-1}, & if \quad t_{i-1}C_c / C_d \le t_i, \end{cases} \tag{4.13}$$

$i = 1,\ldots,n+1,$

$$a_i = \Phi(\ln(C_d / t_i) - \theta_0) / \theta_1 \quad , \tag{4.14}$$

$$b_i = \Phi(\ln(C_c / t_i) - \theta_0) / \theta_1 \quad , \tag{4.15}$$

where

u_i – probability that the process will go to the next interval;

q_i – probability of fuller;

v_i – probability that there will be a discovered crack;

b_i, a_i - $\Phi(.)$ – standard normal distribution function.

The transition probability matrix of this process, P_{AC}, can be composed as it is presented in Fig. 3. Example of transition diagram for the case of two inspections is shown in Fig. 4.

	E_1	E_2	E_3	...	E_{n-1}	E_n	E_{n+1}	E_{n+2} (SL)	E_{n+3} (FF)	E_{n+4} (CD)
E_1	0	u_1	0	...	0	0	0	0	q_1	v_1
E_2	0	0	u_2	...	0	0	0	0	q_2	v_2
E_3	0	0	0	...	0	0	0	0	q_3	v_3
...	...	...	...	...	...	...	...	...	...	...
E_{n-1}	0	0	0	...	0	u_{n-1}	0	0	q_{n-1}	v_{n-1}
E_n	0	0	0	...	0	0	u_n	0	q_n	v_n
E_{n+1}	0	0	0	...	0	0	0	u_{n+1}	q_{n+1}	v_{n+1}
E_{n+2} (SL)	0	0	0	...	0	0	0	1	0	0
E_{n+3} (FF)	0	0	0	...	0	0	0	0	1	0
E_{n+4} (CD)	0	0	0	...	0	0	0	0	0	1

Fig. 3. Matrix P_{AC}

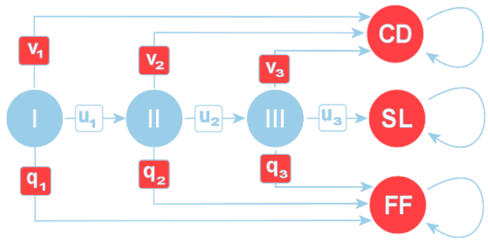


Fig. 4. Transition diagram

The use of this matrix makes it possible to calculate the absorption probability in different absorbing states. The structure of the matrix under consideration can be described in the following way where I is a matrix of identity corresponding to absorbing states, 0 is a matrix of zeros presented in Fig. 5.

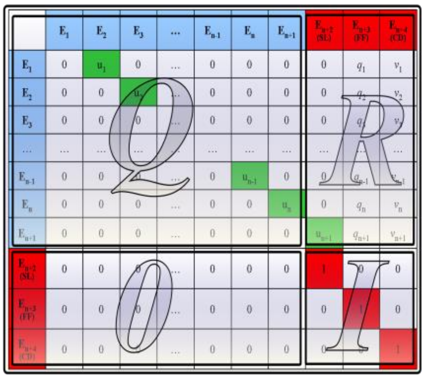


Fig. 5. Sub-matrices of transition probability matrix

Then the matrix of absorption probabilities in different absorbing states for different initial transient states is defined by the following formula (4.16).

$$B = (I - Q)^{-1} R \tag{4.16}$$

The first row of matrix B defines the absorption probabilities in states SL, FF, CD; in particular, item $B(1,2)$ defines the failure probability for a new aircraft, which begins operation within the first interval.

Calculation of Fatigue Failure Rate

Calculation of fatigue failure rate (FFR) and income of airline (AL) are considered.

Recall that in matrix P_{AC} there are three units in the last three lines in a diagonal matrix because states E_{n+2}, E_{n+3}, and E_{n+4} are the absorbing states: AC is discarded from service when the SL is reached, or a fatigue failure (FF) or fatigue crack detection (CD) takes place.

In the corresponding matrix for operation process of AL, P_{AL}, the states E_{n+2}, E_{n+3} and E_{n+4} are not absorbing but correspond to return of MC to state E_1 (AL operation returns to the first interval). The other lines of P_{AC} and P_{AL} are the same. The example of P_{AL} of a transition diagram for the case of two inspections is shown in Fig. 6 and Fig. 7.

	E_1	E_2	E_3	...	E_{n-1}	E_n	E_{n+1}	E_{n+2} (SL)	E_{n+3} (FF)	E_{n+4} (CD)
E_1	0	u_1	0	...	0	0	0	0	q_1	v_1
E_2	0	0	u_2	...	0	0	0	0	q_2	v_2
E_3	0	0	0	...	0	0	0	0	q_3	v_3
...	...	...	...	...	...	...	...	...	...	...
E_{n-1}	0	0	0	...	0	u_{n-1}	0	0	q_{n-1}	v_{n-1}
E_n	0	0	0	...	0	0	u_n	0	q_n	v_n
E_{n+1}	0	0	0	...	0	0	0	u_{n+1}	q_{n-1}	v_{n+1}
E_{n+2} (SL)	1	0	0	...	0	0	0	0	0	0
E_{n+3} (FF)	1	0	0	...	0	0	0	0	0	0
E_{n+4} (CD)	1	0	0	...	0	0	0	0	0	0

Fig. 6. Matrix P_{AL}

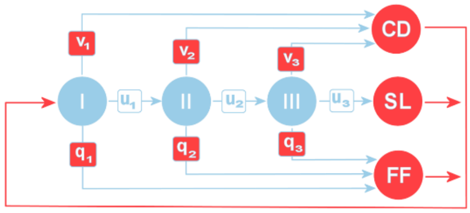


Fig. 7. Transition diagram

Using the theory of semi-Markov processes with reward (SMPW), the matrix of transition probability, P_{AL}, and matrix of reward R are presented in Fig.8.

	E_1	E_2	E_3	...	E_{n-1}	E_n	E_{n+1}	E_{n+2} (SL)	E_{n+3} (FF)	E_{n+4} (CD)
E_1	0	a_1	0	...	0	0	0	0	b_1	c_1
E_2	0	0	a_2	...	0	0	0	0	b_2	c_2
E_3	0	0	0	...	0	0	0	0	b_3	c_3
...	...	...	...	...	...	...	...	...	...	...
E_{n-1}	0	0	0	...	0	a_{n-1}	0	0	b_{n-1}	c_{n-1}
E_n	0	0	0	...	0	0	a_n	0	b_n	c_n
E_{n+1}	0	0	0	...	0	0	0	a_{n+1}	b_{n+1}	c_{n+1}
E_{n+2} (SL)	d_{n+2}	0	0	...	0	0	0	0	0	0
E_{n+3} (FF)	d_{n+3}	0	0	...	0	0	0	0	0	0
E_{n+4} (CD)	d_{n+4}	0	0	...	0	0	0	0	0	0

Fig. 8. Matrix of reward R

It is possible to obtain the airline gain (4.17), where π is the vector of stationary probabilities, which is defined by the equation system (4.19).

$$g(n) = \sum_{i=1}^{n+4} \pi_i g_i(n), \tag{4.17}$$

where

$\pi = (\pi_1, ..., \pi_{n+4})$ – vector of stationary probabilities

$$\pi P = \pi, \quad \sum_{i=1}^{n+4} \pi_i = 1$$

$$\pi P = \pi, \quad \sum_{i=1}^{n+4} \pi_i = 1 \tag{4.18}$$

$$g_i(n) = \begin{cases} a_i \cdot u_i + b_i \cdot q_i + c_i \cdot v_i, & i = 1, ... n+1, \\ d_i, & i = n+2, ..., n+4, \end{cases} \tag{4.19}$$

where

a_i – reward related to successful transition from one operation interval to the subsequent one;

b_i – reward related to fuller;

c_i – reward related to crack detection;

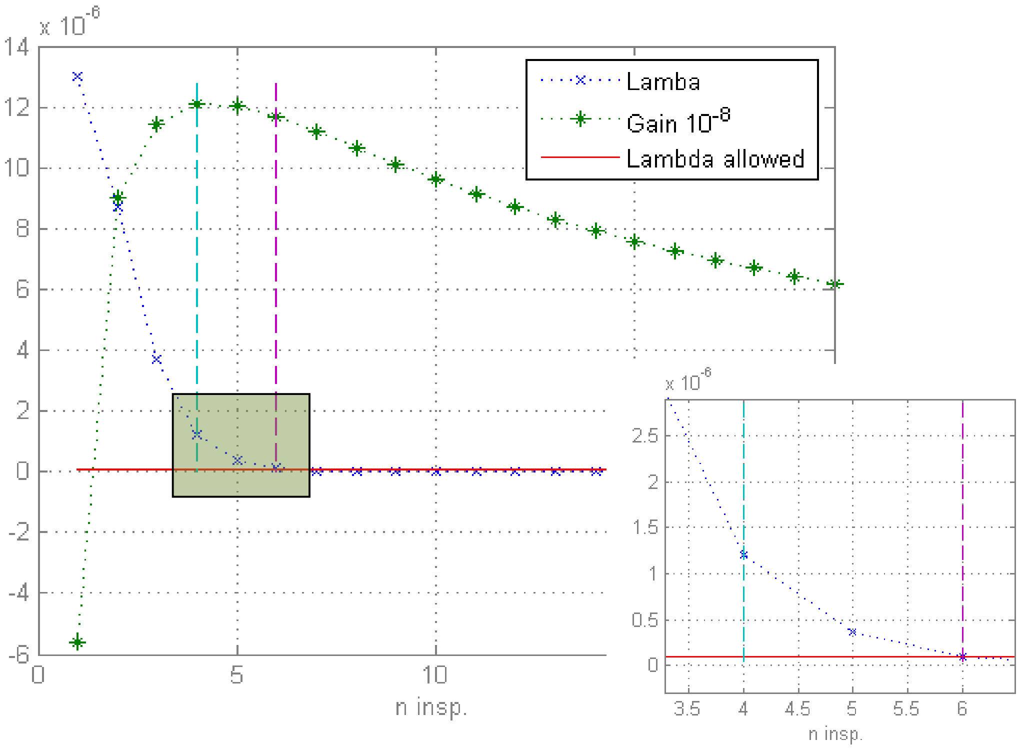
d_i – reward associated with the purchase of new aircraft.

AL operation rewards are defined in (4.25), where a_i is the reward related to successful transition from one operation interval to the subsequent one and the cost of one inspection; b_i, c_i and d_i are related to the transition to states E_{n+3} (FF), E_{n+4} (CD) and E_1.

Let us note that if $a = b = c = 1$, $d = 0$ and time transition to state E_1 is equal to zero, then $\pi_{tj} = \pi_j g_j(n) / g(n)$ defines the time, which is spent by SMP in state E_j, $j = 1, ..., n+1$; $L_j = g(n) / \pi_j$ defines the mean return time for state E_j. Specifically, L_{n+3} is the mean time between FF; so $\lambda_F = 1 / L_{n+3}$ is the FFR.

It is worth mentioning that the same value can be calculated in a different way. This value is equal to the ratio of aircraft failure probability, p_F, to the mean life of a new aircraft, $L_1 = g(n) / \pi_1$ (the mean time of renewal of AC (renewal operation of AL in the first interval)).

There are two very similar versions of reliability requirement: requirement (A) corresponds to limitation of FFR of AL, requirement (B) corresponds to limitation of FFP of AC. Solution to one of the problem version gives an unambiguous solution to the other one. First, it is necessary to consider requirement (A). If θ is known, we calculate the gain as the function of n, $g(n, \theta)$, and choose the number n_g corresponding to the maximum gain: $n_g(\theta) = \arg\max_n g(n, \theta)$. Then we calculate FFR as the function of n, $\lambda_F(n,\theta)$, and choose n_λ in such a way that for any $n \geq n_\lambda$ the function $\lambda_F(n,\theta)$ will be equal to or less than some value λ_{FD} (the designed FFR) : $n_\lambda(\lambda_{FD},\theta) = \min\{n : \lambda_F(n,\theta) \leq \lambda_{FD},\ \text{for all } n \geq n_\lambda(\lambda_{FD},\theta)\}$. Finally we choose $n = n_{g\lambda}(\lambda_{FD},\theta) = \max(n_g, n_\lambda)$, Fig. 9.



A	B
Fig. 9. Choice of $n = n_{g\lambda}(\lambda_{FD},\theta) = \max(n_g, n_\lambda)$ (a), in detail (b).	

Choosing the Number of Inspections in Case of Unknown Crack Growth Parameters

Statistical aspect of the problem under analysis is considered. *In fact, we do not know θ and we can only estimate this parameter, $\hat{\theta}$, using approval test results.* Then, first of all, we should define the parameter space Θ_0 in such a way that if $\hat{\theta} \notin \Theta_0$ then redesign of AC should be made. If instead of $n_{g\lambda}(\lambda_{FD},\theta)$ we use $n_{g\lambda}(\lambda_{FD},\hat{\theta})$ then real intensity FFR will be a function of random variable, $\lambda_F(n_{g\lambda}(\lambda_{FD},\hat{\theta}),\theta)$. Let us define (4.26) and (4.27).

$$\lambda_F(\hat{\theta},\lambda_{FD},\Theta_0) = \lambda_F(n_{g\lambda}(\lambda_{FD},\hat{\theta}),\theta) \text{ if } \hat{\theta} \in \Theta_0 \tag{4.26}$$

$$\lambda_F(\hat{\theta},\lambda_{FD},\Theta_0)=0 \quad \text{if} \quad \hat{\theta}\notin\Theta_0 . \tag{4.27}$$

The expected value of FFR $w(\theta,\lambda_{FD},\Theta_0)=E\{\lambda_F(\hat{\theta},\lambda_{FD},\Theta_0)\}$ as the function of θ has maximum because for "bad $\hat{\theta}$ " it is necessary to redesign an airframe but in case of "very good $\hat{\theta}$ " there is not any need for inspection, Fig.10.

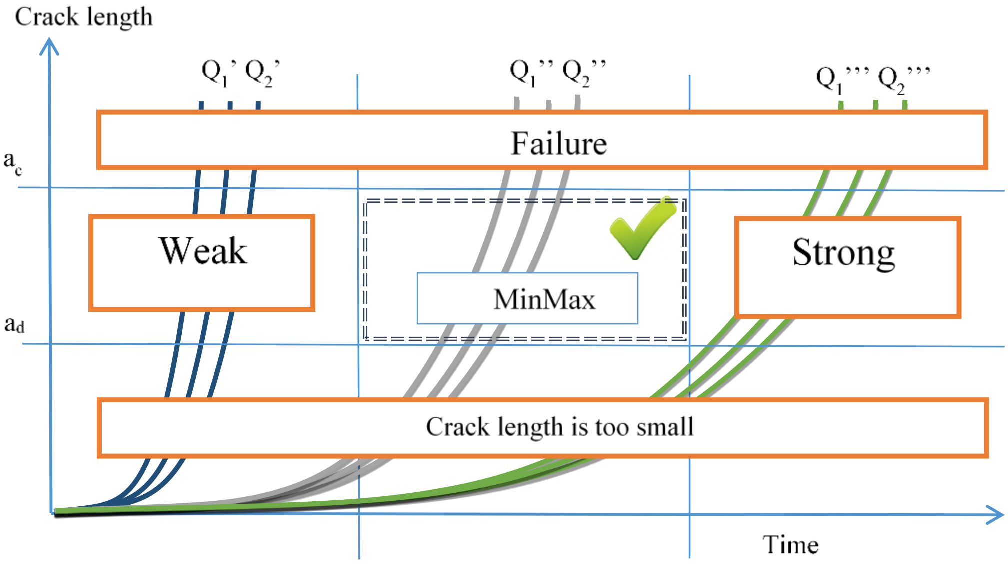


Fig. 10. Fatigue failure probability as the function of mean fatigue life; three sets of fatigue cracks: redesign is needed, reliability is provided by inspection, reliability is provided without inspection

Let us assume that λ^* is required FFR, which is defined by specific aviation regulations. Let $\lambda^*_{FD}(\Theta_0)$ denote the solution to an equation $w(\theta,\lambda_{FD},\Theta_0)=\lambda^*$ if the solution to this equation exists for specific Θ_0. If after an approval test we see that $\hat{\theta}\in\Theta_0$ then the required inspection number is $n=n_{g\lambda}(\lambda^*_{FD},\hat{\theta})$; otherwise, the redesign should be made.

Now the requirement (B) will be considered. In this case, the choice of vector $\vec{t}$ is the choice of p-set function, which in a given case is defined in the following way.

Let Z and X be random vectors (r.v.) and we suppose that the class is known $\{P_\theta,\ \theta\in\Theta\}$ to which the probability distribution of the random vector *W=(Z,X)* is assumed to belong. It is presumably known that the parameter θ, which labels the distribution, lies in a certain set Θ, the parameter space. Let $S_Z(x)=\bigcup_{i=1}^{r} S_{Z,i}(x)$ denote some set of disjoint sets of *z* values as the function of x. If (4.28) then statistical decision function $S_Z(x)$ is p-set function for r.v. Z based on a vector x .

$$\sup_{\theta}\sum_{i=1}^{r} P(Z \in S_{Z,i}(X)) = p \tag{4.28}$$

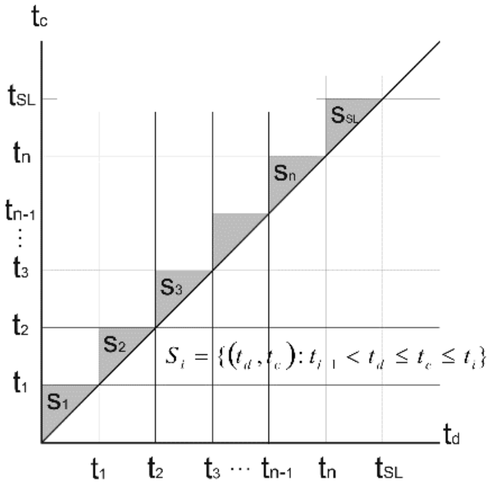


Fig. 11. Example of a set of sets $S_{Z,i}$, $i = 1,...,n+1$. Here $S_{Z,i}$ is denoted as S_i .

In our case, Z is a vector (T_d, T_c) related to an aircraft in operation, X is the estimate of parameter θ, i.e., $x = \hat{\theta}$. This estimate can be obtained using results of the fatigue test of an airframe of aircraft of the same type in a laboratory (i.e., observations of fatigue crack: pairs {(time, fatigue crack size) i, $i = 1,...,k$ }$= \{(tt_i, a_i), i = 1,...,k\}$, where k is the number of fatigue crack observations.

$$S_{Z,i} = \{(t_d, t_c): t_{i-1} < t_d, t_c \le t_i\},\ i = 1,...,n+1,\ t_{n+1} = t_{SL} \tag{4.29}$$

Each set $S_{Z,i}$ is a set of points in the plane, {(t_d, t_c)}, defined in (4.29), Fig.11. Vector of time points of inspection $\vec{t}$ is the function of observation of estimate of parameter θ , i.e., the function of $\hat{\theta}$.

Let $E(T_A)$ be the mean time to absorption (corresponding to the matrix P_{AC}) of an aircraft, which begins the service in the first interval, and the failure probability p_f corresponds to the function $\vec{t}(x)$.

$$\lambda = p_f / E(T_A) \tag{4.30}$$

Then the FFR and FFP have unambiguous connection (4.30).

Chapter 5. Numerical Example

Numerical example and comparison of cases with one- and two-dimensional unknown parameters are given.

Ten fatigue cracks discovered during full-scale fatigue tests of an aircraft are observed in Fig. 12.

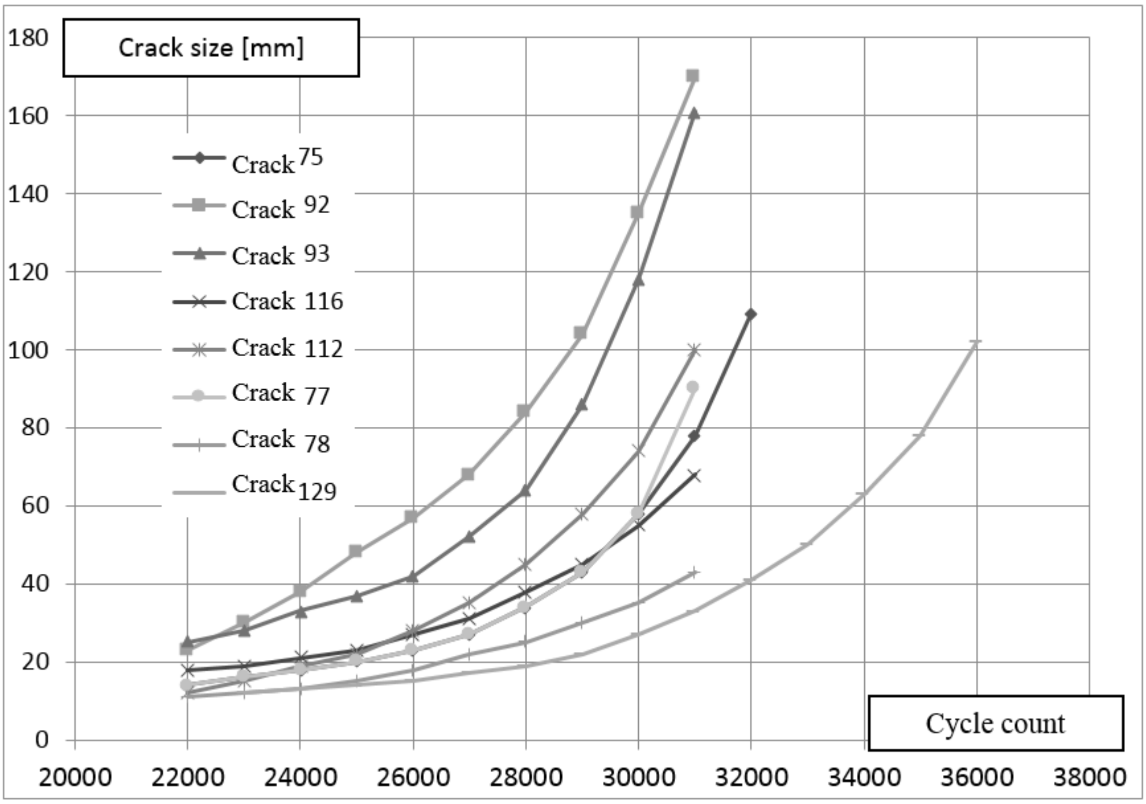


Fig. 12. Observation of 10 fatigue cracks discovered during full-scale fatigue tests of an aircraft

Table 1

Airframe Fatigue Crack Parameters

Nr	Crack #	Ln (a0)	Q	X=Ln(Q)	Y=lnCc
1	75	-1.2513	1.86E-04	-8.58976	1.905519
2	92	-1.8768	1.95E-04	-8.54251	1.994482
3	93	-1.2445	1.61E-04	-8.73411	1.904507
4	116	-1.697	2.20E-04	-8.42188	1.96971
5	112	-1.5102	2.07E-04	-8.48279	1.943306
7	77	-2.5329	2.28E-04	-8.38616	2.080003
8	78	-0.6479	1.54E-04	-8.77856	1.81148
10	129	-1.4226	1.57E-04	-8.75926	1.93068
	Average	-1.5229	0.000189	-8.5868804	1.942461
	StdDev	0.5480844	2.9E-05	0.1551287	0.077889
	CORREL r			0.796	

Suppose we have the following estimate of parameter $\theta = (\mu_X, \mu_Y, \sigma_X, \sigma_Y, r)$: $\hat{\theta}$= (-8.587, 1.942, 0.346, 0.0779, 0.796) (here instead of $\sigma = 0.155128668$ for more clear demonstration we use $\sigma = 0.346$). It is supposed that all inspection intervals are equal, a_c = 237.8 mm and a_d = 20 mm. We use the following definitions of component of AL income: for all $i = 1,...,n+1$ $a_i = a(n) = a_0(n) + d_{insp}t_{SL}$, where $a_0(n) = a_{01}t_{SL} / (n+1)$ is the reward related to successful transition from one operation interval to the subsequent one, a_{01} defines the reward of operation in one time unit (one hour or one flight); $d_{insp}t_{SL}$ is the cost of one inspection (a negative value) which is supposed to be proportional to t_{SL}; $b_i = b_{01}t_{SL}$ is related to FF (a negative value), $c_i = c_{01}a_0(n)$ is the reward related to transitions from any state $E_1,...,E_{n+1}$ to the state E_{n+4} (it is supposed to be proportional to a_0 because it is part of a_0); $d_i = d_{01}t_{SL}$ is a negative reward, the absolute value of which is the cost of new aircraft acquisition, when events SL, FF or CD and transition to E_1 take place. In a numerical example, the following values have been used: b_{01}=-0.3; d_{insp}= - 0.05; a_{01}=1; c_{01}=0.1; d_{01}=- 0.3. Calculation of $w(\theta, \lambda_{FD}, \Theta_0) = E\{\lambda_F(\hat{\theta}, \lambda_{FD}, \Theta_0)\}$ was made for $(7.2029 \le \mu_X \le 9.9709)$, $(1.3972 \le \mu_Y \le 2.4877)$ at $t_{SL} = 40000$. The set Θ_0 corresponds to the decision to make redesign if estimate of $E(T_c) < 0.3t_{SL}$. Calculation is made for different λ_{FD} assuming that the vector (σ_X, σ_Y, r) is known and it is equal to test estimate (0.155128668, 0.0778895, and 0.796437). It is supposed that this vector is the same for different vectors (μ_X, μ_Y).

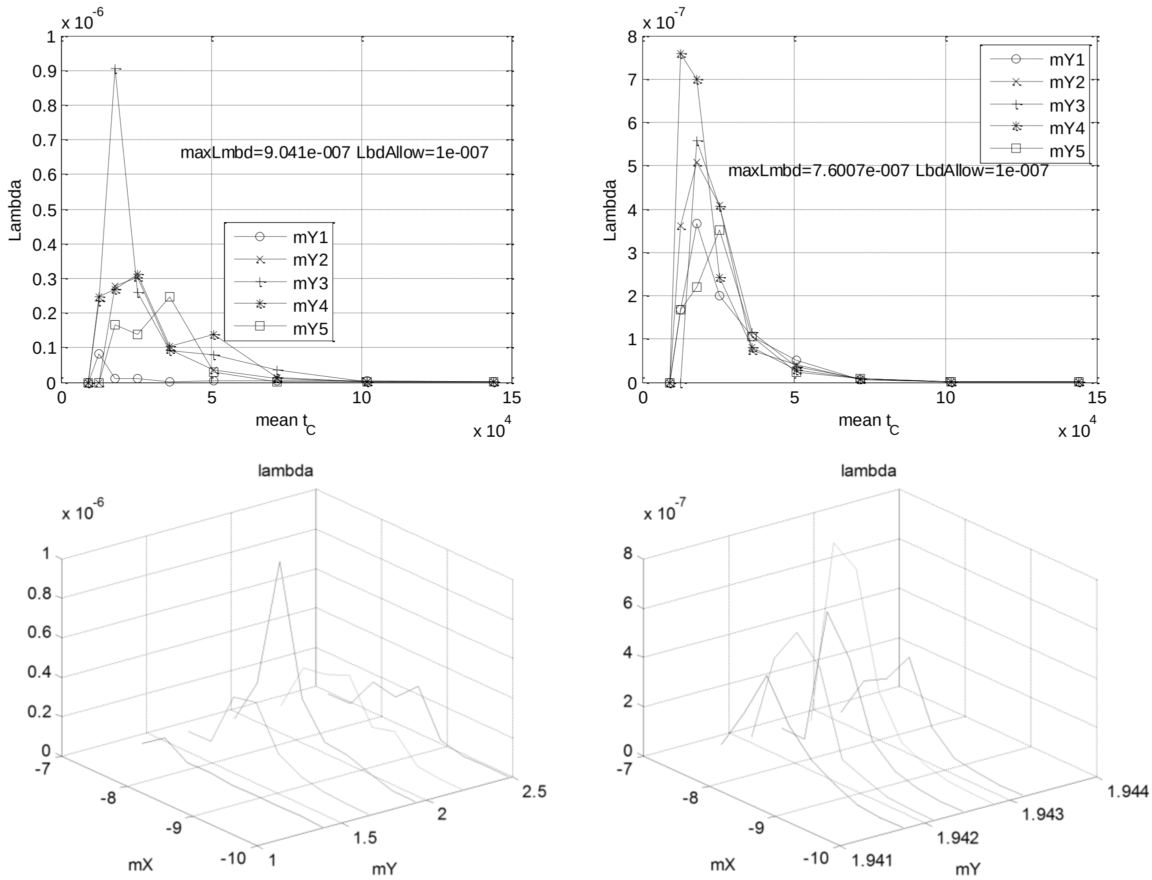


Fig. 13. Values of $w(\theta, \lambda_{FD}, \Theta_0)$ as the function of $E(T_c)$ for different μ_Y for (σ_X, σ_Y, r) = (0.155128668, 0.0778895, 0.796437) (on the left side) and 0.760*10-6 for (σ_X, σ_Y, r) = (0.346, 0.0001, 0) (on the right side)

In Figure 13 , it is possible to observe that $w(\theta, \lambda_{FD}, \Theta_0)$ is the function of equivalent mean value of T_C, calculated as $\exp(\mu_Y - \mu_X)$ for five values of μ_Y $(1.2415 \le \mu_Y \le 2.6435)$ in the vicinity of a maximum value of $w(\theta, \lambda_{FD}, \Theta_0)$ that is equal to 0.9041*10-6 for (σ_X, σ_Y, r) = (0.155128668, 0.0778895, 0.796437) and 0.760*10-6 for (σ_X, σ_Y, r) = (0.346, 0.0001, 0) (see Table 2).

Table 2

Results

Lambda allowed	σ_Y = ***0.0778895*** r_{XY} ***=0.796***	σ_X ***Sy= 0.0001*** r_{XY} ***=0***
0.1*10-6	0.9041*10-6	0.760*10-6

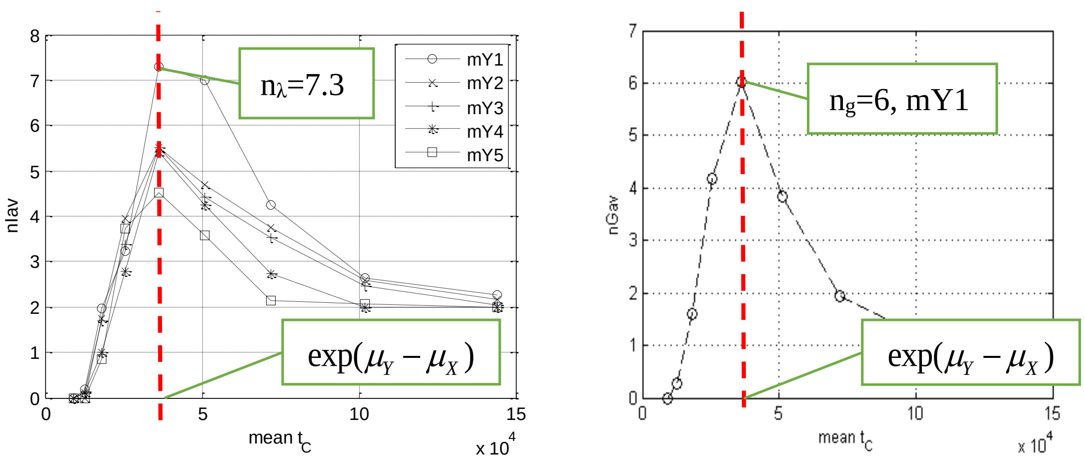


Fig. 14. Mean value of inspection number as the function

of equivalent mean value of $E\{T_C\}=$ $\exp(\mu_Y - \mu_X)$

In this case, the influence of scatter Y (and of EIFS, a_0) does not seem significant. However, this conclusion depends on the other components of the problem. Suppose that (σ_X, σ_Y, r) = (0.155128668, 0.0778895, 0.796437) and the value 0.9041*10-6 coincides with the required FFR of AL. Let us also assume that in the real test we have got $\hat{\mu}_X$ = -8.5885, $\hat{\mu}_Y$ =1.942460769, then the equivalent mean value of T_C $\exp(\mu_Y - \mu_X)$ = 37.4574e+003.

If these estimates are considered to be the real value of a parameter, the mean value of a number of inspections will be equal to 8 (see Fig.14, left), but economically recommended number is 6 (see Figure 14, right).

CONCLUSIONS

1. The method of development of the inspection program for fatigue-prone airframe details has been offered, taking into account the following aspects:

- the required reliability of airplane and limitations of fatigue failure intensity of airline,
- compliance with the requirements imposed on the results of approved fatigue tests and conditions of redesign of an airframe if these requirements are not met,
- effectiveness of inspection technology,
- cost of inspection, crack detection costs, fatigue failure consequence costs, cost of a new airplane and other economic parameters.

2. Specific features of the corresponding statistical decision function are the following:

- Specific model of fatigue crack growth process has been used, some parameters of which have been estimated taking into account the results of real full-scale fatigue cracks;
- In the existing inspection interval calculation methodologies, crack parameter distribution function is already known, thus enabling probability calculations
- Offered statistical decision function provides required reliability independent of unknown parameter because the MiniMax approach has been used based on the results of approval tests, when the redesign of the tested airframe should be made if these results do not meet specific requirements.
- The theory of Markov chains and semi-Markov processes with reward have been used to calculate the reliability of aircraft and both the fatigue failure rate and the reward of airline.

MinMax methodology is based on the use of the principles of minmax. It allows bypassing the relevant probability use. Parameter relevant probability selection based on the safety equation is a difficult task.