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Comparative Analysis of Fuzzy Set Defuzzification Methods in the Context of Ecological Risk Assessment

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***Abstract* – Fuzzy inference systems are widely used in various areas of human activity. Their most widespread use lies in the field of fuzzy control of technical devices of different kind. Another direction of using fuzzy inference systems is modelling and assessment of different kind of risks under insufficient or missing objective initial data. Fuzzy inference is concluded by the procedure of defuzzification of the resulting fuzzy sets. A large number of techniques for implementing the defuzzification procedure are available nowadays. The paper presents a comparative analysis of some widespread methods of fuzzy set defuzzification, and proposes the most appropriate methods in the context of ecological risk assessment.**

***Keywords* – Defuzzification methods, ecological risk, ecological risk assessment, fuzzification, fuzzy inference, fuzzy inference system.**

I. Fuzzy Inference Techniques: A Brief Introduction

Fuzzy inference approach has first been proposed in [1] in the context of the task related to water heater and boiler automatic control. The main idea of the author was to connect through using production rules fuzzy values of the input variables and fuzzy values of the output variable on whose basis the control impact on a technical device was developed. The necessity to introduce such a fuzzy control was caused by the aim to escape from strict correlations between the inputs and outputs of the control system that are characteristic of classical systems of automatic control as well as to ensure the flexibility regarding the changes of the values of the input variable and the output variable. Systems of fuzzy inference have found wide application in the tasks where necessary information sources are either missing or insufficient, and the only source of information is experts' opinions which are easier to express in the fuzzy form.

General scheme of the fuzzy inference system is shown in Fig. 1 [2]. Linguistic (fuzzy) categories are defined and their membership functions in these categories are constructed on measurement scales. The concepts of linguistic variables and their categories are described in detail in [3]. The system of fuzzy production rules connects the input linguistic categories with the output linguistic categories. Based on inference rules and operators, at the specified specific values of the input variables, the pruned values of membership functions of the output linguistic variables are determined. More information about all procedures of fuzzy inference can be found in [4], [5].

By aggregating fuzzy linguistic categories (fuzzy sets) obtained through using different inference rules, the resulting output fuzzy set is composed. The last procedure of fuzzy inference is defuzzification of that fuzzy set. Defuzzification consists in the determination of a deterministic value on the measurement scale of the output variable that in some sense best represents the resulting fuzzy set. Let us note that to model and assess risks, Mamdani fuzzy inference systems are mostly used that completely correspond to the structure of this kind of tasks.

Let us mention some examples to illustrate the use of fuzzy inference systems in risk assessment. In paper [6], operational risks are assessed and analysed using fuzzy inference systems. This kind of systems makes it possible to take into account complex interactions of the inputs as well as their non-linearities.

In [7], risks of various building vibrations are discussed that are caused by different reasons. The paper also provides an overview of the existing techniques aimed at modelling and assessing such risks; a new methodology based on the fuzzy logic is also proposed.

In [8], it is shown how a fuzzy inference system can be used to model and assess risks of the mined territories. During the Second World War, many territories were mined. A great deal of the mines has not been extracted yet. According to the data by the United Nations Organisation, nowadays there are at least 100 mln mines remained in the fields of former battles. The authors have developed a fuzzy inference system that can be embedded in a smartphone. The system enables the user to assess the risk of mines in a specific territory.

In [9], the use of a fuzzy inference system for modelling and assessing ecological risks is validated. In [2], a thorough and detailed analysis of fuzzy approaches to risk assessment and analysis is provided. Much attention in this study is paid to systems of fuzzy inference.

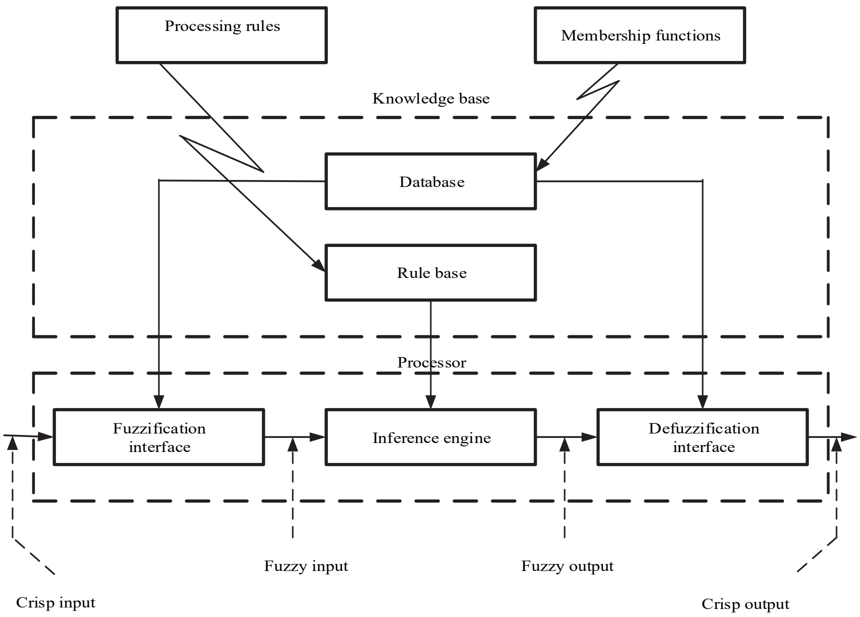

Fig.1. Schematic representation of a fuzzy inference system.

II. Widely Used Methods of Fuzzy Set Defuzzification

Figure 2 below shows graphs of membership functions of three output linguistic variables obtained as a result of using three fuzzy inference rules in a set of the input linguistic variables – risk factors $\tilde{f}_1$, $\tilde{f}_2$ and $\tilde{f}_3$. By applying max principle-based aggregation method to these output data, we obtain a resulting fuzzy set $\tilde{R}$ which is a fuzzy estimate of risk at the specified specific values of risk factors. Due to its fuzziness, this fuzzy set cannot be a basis for making a decision to reduce the risk or declining such a decision. That is why fuzzy set $\tilde{R}$ has to be defuzzificated, i.e., has to be transformed into the form of deterministic numerical estimate, which can serve as a basis for further analysis.

Nowadays, there are a significant number of fuzzy set defuzzification methods that are based on some or other principles of resulting fuzzy set transformation into deterministic values. Most of the methods can be divided into three groups: *maxima methods, distribution methods and area methods.*

A. Maxima Methods [12]

The main principle underlying the methods of this group is as follows: the deterministic value $r \in R$ selected at the corresponding value of the membership function $\mu(\tilde{R})$ is assumed to be the defuzzificated value of fuzzy set $\tilde{R}$. Let us consider the most widespread methods of that group.

First-of-maxima method (FOM)

$$\mathrm{FOM}(\tilde{R}) = \frac{r_{\min}}{\mu_{\tilde{R}}(r)} = \max, \forall r \in R . \tag{1}$$

If the minimal membership value does not have single value r but rather possesses a set (interval) of such values, then the value r corresponding to the middle point of the interval is taken as the value of $\mathrm{FOM}(\tilde{R})$.

Last-of-maxima method (LOM)

The value $r \in R$ having the maximal value of membership function $\mu_{\tilde{R}}(r)$ is assumed to be the defuzzificated value of a fuzzy set

$$\mathrm{LOM}(\tilde{R}) = \frac{r_{\max}}{\mu_{\tilde{R}}(r)} = \max, \forall r \in R . \tag{2}$$

If the set (interval) of values r has the maximum membership value, then the value r corresponding to the middle point of that interval is assumed to be the value $\mathrm{LOM}(\tilde{R})$.

Middle-of-maxima method (MOM)

$$\mathrm{MOM}(\tilde{R}) = \frac{\mathrm{FOM}(\tilde{R}) + \mathrm{LOM}(\tilde{R})}{2} . \tag{3}$$

The main shortcoming of the methods of this group is evident: the defuzzificated value depends only on the values r with extreme values of membership. All the other values of r are not taken into account.

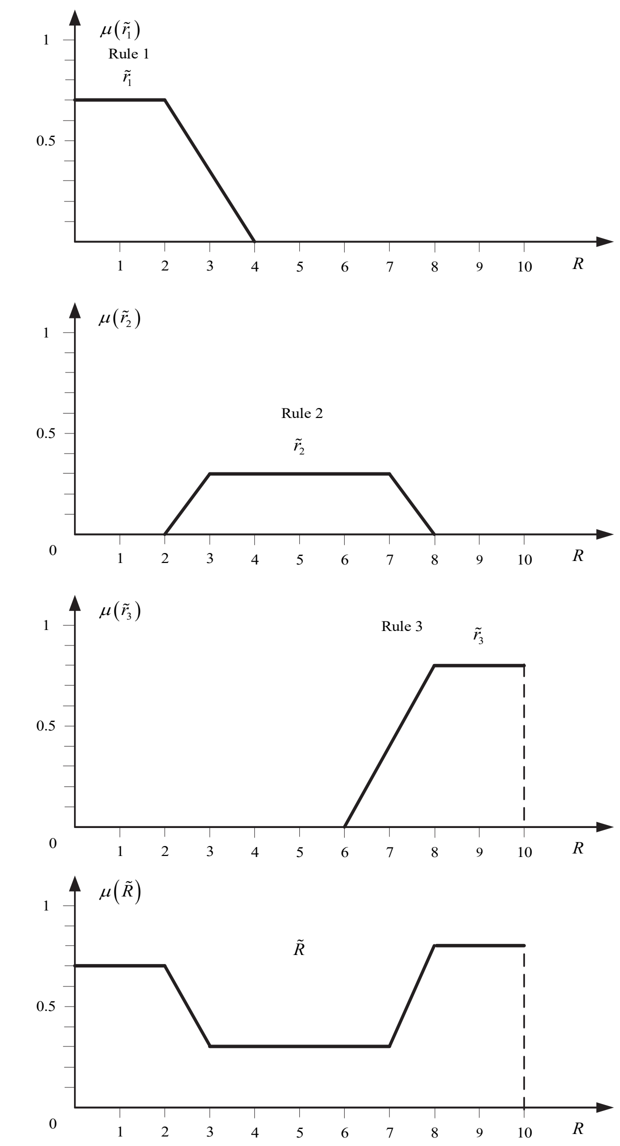

Fig. 2. Graphical representation of the output linguistic variables of categories $\tilde{r}_i$, i = 1, 2, 3 and the resulting fuzzy set $\tilde{R}$.

B. Distribution Methods

Centre of gravity method (COG)

$$\mathrm{COG}\left(\tilde{R}\right)=\frac{\int r\cdot\mu_{\tilde{R}}\left(r\right)\mathrm{d}r}{\int\mu_{\tilde{R}}\left(r\right)}, \tag{4a}$$

where integrals are taken on the ground of fuzzy set $\tilde{R}$ [12].

If the discrete values r are taken only at specific points of the ground of a fuzzy set $\tilde{R}$, then

$$\mathrm{COG}\left(\tilde{R}\right)=\frac{\sum_{i=1}^{n}r_i\cdot\mu_{\tilde{R}}\left(r_i\right)}{\sum_{i=1}^{n}\mu_{\tilde{R}}\left(r_i\right)}, \tag{4b}$$

where n is the number of reference points.

Weighted average method (WAM)

The essence of this method [12, 14] is as follows. First, the defuzzification of each output fuzzy set $\tilde{r}_i$ is performed, and the corresponding deterministic value $\overline{r}_i$ is determined. The values $\overline{r}_i$ are defined through using any suitable defuzzification method. Then, the resulting defuzzification value $\mathrm{WAM}\left(\tilde{R}\right)$ is calculated using expression 5:

$$\mathrm{WAM}\left(\tilde{R}\right)=\frac{\sum_{i=1}^{n_R}\overline{r}_i\cdot\mu_{\tilde{R}}\left(\overline{r}_i\right)}{\sum_{i=1}^{n_R}\mu_{\tilde{R}}\left(\overline{r}_i\right)}. \tag{5}$$

Basic defuzzification method (BADD) [15]

$$\mathrm{BADD}\left(\tilde{R},\gamma\right)=\frac{\sum_{i=1}^{n}r_i\cdot\left(\mu_{\tilde{R}}\left(r_i\right)\right)^{\gamma}}{\sum_{i=1}^{n}\left(\mu_{\tilde{R}}\left(r_i\right)\right)^{\gamma}}, \tag{6}$$

where $\gamma\in\left[0,\infty\right]$ represents the level of confidence in the results of fuzzy inference.

C. Area Methods

Let us introduce one of the most known methods of that group, ***bisector or centre of area method (COA)***. Using this method, the defuzzificated value $\mathrm{COA}\left(\tilde{R}\right)$ divides the initial fuzzy set $\tilde{R}$ into two subsets with the same weighted squares. The value $\mathrm{COA}\left(\tilde{R}\right)$ is calculated using the condition below:

$$\int_{r_{\inf}}^{\mathrm{COA}\left(\tilde{R}\right)} r*\mu_{\tilde{R}}\left(r\right)\mathrm{d}r=\int_{\mathrm{COA}\left(\tilde{R}\right)}^{r_{\sup}} r\cdot\mu_{\tilde{R}}\left(r\right)\mathrm{d}r\,. \tag{7a}$$

The discrete version of that condition looks as follows:

$$\sum_{r_{\inf}}^{\mathrm{COA}\left(\tilde{R}\right)} r_i\cdot\mu_{\tilde{R}}\left(r_i\right)=\sum_{\mathrm{COA}\left(\tilde{R}\right)}^{r_{\sup}} r_j\cdot\mu_{\tilde{R}}\left(r_j\right), \tag{7b}$$

where i and are the selected reference points of the values r to the left and to the right of the value $\mathrm{COA}\left(\tilde{R}\right)$.

To perform an analysis of the presented methods, let us calculate the defuzzificated values for fuzzy set $\tilde{R}$ shown in Fig. 2. Calculation results are given in Table I.

TABLE I

RESULTS OF CALCULATION OF THE DEFUZZIFICATED VALUES FOR FUZZY SET $\tilde{R}$ SHOWN IN FIG. 2

Method	Expression to calculate	Defuzzificated value
FOM	(1)	8.00
LOM	(2)	10.00
MOM	(3)	7.00
COG	(4b)	5.89
WAM	(5)	5.22
BADD ($\gamma = 2$)	(6)	5.47
COA	(7b)	≈ 8.50

III. ANALYSIS OF THE PRESENTED DEFUZZIFICATION METHODS

Defuzzification is a mathematical process used to convert a fuzzy set to a singleton. This step is quite important because fuzzy sets generated by means of fuzzy inference have to somehow be combined to obtain a single number at the system output.

To obtain validated and reliable results, a defuzzification operator has to possess some basic properties. These basic features have to represent both theoretical background and orientation regarding their application.

In [10], the following list of such properties is provided (cited according to [11]):

1. Consistency: When a defuzzification maps convex crisp sets to their centroid, it is called consistent.
2. Section invariance: When a magnification of a regarded section does not affect the results, the defuzzification is called section invariant.
3. Monotonicity: If the defuzzification results remain unchanged or move toward a single element when its membership grade increases or if by decreasing the membership grade of a single element the defuzzification result moves to the opposite direction or remains constant, it is called monotonous defuzzification.
4. Linearity: A linear defuzzification result is maintained after affine transformation such as rotation, reflection, translation and scaling.
5. Offset and scale invariance: If membership value offset or scaling does not affect the defuzzification results, it is called offset invariant defuzzification, respectively.
6. Compatibility: The defuzzification method chosen must be compatible with the inference, composition, and other operations used in the fuzzy system.
7. Arithmetic compatibility: A defuzzification is arithmetically compatible, if it defuzzifies "about a" to "a" which equals to the mean value with a membership grade of 1.
8. Exclusion: In exclusive defuzzification methods negative information is recognised with a nonzero membership value.

In [13], several constraints are proposed that characterise rational defuzzification strategies. In [11], a wide list of defuzzification methods is provided along with checking them for correspondence to these constraints. Papers [14], [15], [16] are devoted to various aspects of the analysis of fuzzy set defuzzification methods.

Ecological risks constitute a specific group of risks. These risks can be divided into two large groups:

risks related to the negative impact of human economic activity on the environment and risks related to the negative impact of the environment on economic and social activity of humans.

Ecological risks can also be classified by the character of manifestation of factors related to them. For example, flood is a one-time event. The consequences of the floods are not accumulated provided that the flood occurs quite seldom. On the other hand, there are a lot of risk factors that act during a long time and the consequences are accumulated. These can be long-acting sources of the environmental pollution: pollutant emissions into atmosphere produced by transport vehicles and industrial enterprises, draining the unrefined flows etc.

Fuzzy inference systems best suit modelling and assessment of this kind of risk. They can be used to assess risks of one-time events like flood, volcanic eruption, tsunami and others provided that the correlations between the occurrence of unfavourable events and the factors that cause them are reliably identified. However, the main application area of these systems is modelling and assessment of risks associated with long-acting unfavourable factors. Here, an analogy can be traced with fuzzy control of technical devices. The relevant fuzzy inference systems operate continuously. The system reacts to the changes in operating parameters of the device; if the parameter values exceed the specified boundary conditions, a new signal is formed which implements the necessary controlling action.

When developing and adjusting this kind of systems, many requirements have to be satisfied. The main requirements are as follows: correct construction of membership functions of the input and output linguistic categories, accurate formulation of fuzzy inference rules, specification of thresholds for operating parameters of the controlled device and formation of controlling actions for relevant response to the changes in the input parameters (fuzzy control accuracy).

The same requirements can completely be transferred to the fuzzy inference systems intended for assessing ecological risks. The difference between the systems is that in case of assessing ecological risks the systems do not produce any controlling actions. The main intended use of such systems is to assess the dynamics of risks and provide data for making relevant controlling decisions.

Important issue related to assessing ecological risks is to take into account specific values of all relevant factors; therefore, the changes in the resulting assessment of risks have to be associated with the changes in any risk factor or in any set of the factors.

What fuzzification methods are most suitable for fuzzy assessment of ecological risks? It is evident that no methods of the maxima group suit that purpose. The reason is that when these methods are used, a single deterministic output value is determined based on the extreme values of membership in the resulting fuzzy set. This can lead to a situation when a fuzzy inference system does not respond to a significant change in risk factors in the case when an extreme value of membership function of the resulting fuzzy set does not change at the changes in certain risk factors.

Based on the above-mentioned considerations, a definite conclusion can be drawn: to ensure successful functioning of a fuzzy inference system intended for modelling and assessment of ecological risks, the defuzzification of the resulting fuzzy set has to be based on the values of the membership function at all points of the ground of that set. The centre of gravity method (COG) is the most preferred method in the sense of satisfying that requirement. The advantage of the method is that in case if functions of membership in the output linguistic categories are symmetrical, the extent of overlapping of some membership functions does not affect the result of defuzzification. However, if the output membership functions are significantly asymmetric, such an overlapping can affect defuzification result. Then, the weighted average method (WAM) can be used as an alternative.

Bisector or centre of area method (COA), though producing the results close to those produced by COG and WAM methods, requires complicated computational procedures. Therefore, the method should be considered a less preferred one than COG and WAM methods.

Another advantage of COG and WAM methods is that defuzzification principles underlying these methods are simple and intuitively supportable, which raises the degree of confidence in the results provided by a fuzzy inference system.

IV. Conclusion

The paper has concisely described the basic principles of fuzzy inference system operation and validated possible application of this kind of systems for assessing dynamics of ecological risk variation associated with long-acting unfavourable factors. To assess ecological risks, Mamdani fuzzy inference systems are solely appropriate, which in a fuzzy manner translate real values of factors into specific values of the output linguistic variables.

Considerable importance for the output assessment of risk lies in the method of resulting fuzzy set defuzzification. The paper has demonstrated that methods of the maxima methods group do not suit this procedure in cases of ecological risk assessment. In case of symmetric membership functions of the output linguistic variables, the most appropriate method is the centre of gravity (COG). Alternatively, if relevant membership functions are not symmetric, the weighted average method (WAM) is an appropriate alternative.

References

[1] E. H. Mamdani, "Application of fuzzy algorithms for control of simple dynamic plant," *Proceedings of the Institution of Electrical Engineers*, vol. 121, no. 12, p. 1585, 1974. https://doi.org/10.1049/piee.1974.0328

[2] A. F Shapiro and M. -S. Koissi. *Risk Assessment Application of Fuzzy Logic*. Society of Actuaries Canadian Institute of Actuaries, 2015.

[3] L. A. Zadeh, "Fuzzy logic = computing with words," *IEEE Transactions on Fuzzy Systems*, vol. 4, no. 2, pp. 103–111, May 1996. https://doi.org/10.1109/91.493904

[4] T. J. Ross, *Fuzzy Logic with Engineering Applications*, 2nd ed. John Wiley and Sons Ltd, 2004.

[5] R. R. Yager and L. A. Zadeh Ed., *An Introduction to Fuzzy Logic Applications in Intelligent Systems*. Springer Science; Business Media, LLC, 1992.

[6] C. León, "Operational Risk Management Using a Fuzzy Logic Inference System," *SSRN Electronic Journal*, 2009. https://doi.org/10.2139/ssrn.1473614

[7] Ridong Du, Yongbo Yuan, and Miao Chen, "Construction Vibration Risk Assessment Based on Fuzzy Logic," *International Journal on Advances in Information Sciences and Service Sciences*, vol. 5, no. 4, pp. 664–675, Feb. 2013. https://doi.org/10.4156/aiss.vol5.issue4.81

[8] S. S. M. Khalifa, K. Saadan, and N. Md Norwawi, "Risk Assessment of Mined Areas using Fuzzy Inference," *International Journal of Artificial Intelligence & Applications*, vol. 6, no. 2, pp. 37–51, Mar. 2015. https://doi.org/10.5121/ijaia.2015.6203

[9] A. Radionovs and O. Uzhga-Rebrov, "Environmental Risk Assessment by Fuzzy Multiple Criteria Decision Making Approach", in *International Scientific School "Modelling and Analysis of Safety and Risk in Complex Systems"*, Saint-Petersburg, Russia, pp. 119–124. Oct. 25–28, 2016.

[10] T. A. Runkler, "Extended defuzzification methods and their properties," *Proceedings of IEEE 5th International Fuzzy Systems*, pp. 694–700, 1996. https://doi.org/10.1109/fuzzy.1996.551822

[11] A. Amini and N. Nikraz, "Proposing Two Defuzzification Methods based on Output Fuzzy Set Weights," *International Journal of Intelligent Systems and Applications*, vol. 8, no. 2, pp. 1–12, Feb. 2016. https://doi.org/10.5815/ijisa.2016.02.01

[12] W. V. Leekwijck and E. E. Kerre, "Defuzzification: criteria and classification," *Fuzzy Sets and Systems*, vol. 108, no. 2, pp. 159–178, Dec. 1999. https://doi.org/10.1016/s0165-0114(97)00337-0

[13] T. A. Runkler and M. Glesner, "A set of axioms for defuzzification strategies towards a theory of rational defuzzification operators," *Second IEEE International Conference on Fuzzy Systems*, pp. 1161–1166, 1993. https://doi.org/10.1109/fuzzy.1993.327350

[14] S. Naaz, A. Alam and R. Biswas, "Effect of different defuzzification methods in a fuzzy based load balancing application," *International Journal of Computer Science Issues*, vol. 8, issue 5, pp. 261–267, 2011.

[15] K. K. Uraon and S. Kumar, "Analysis of Defuzzification Method for Rainfall Event," *International Journal of Computer Science and Mobile Computing*, vol. 1, issue 1, pp. 341–354, 2016.

[16] D.Wilczyńska–Sztyma and K.Wielki, "Direction of Research into Methods of Defuzzification for Ordered Fuzzy Numbers", presented at XII International PhD Workshop, 23–26 October, 2010, pp. 105–110.

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