K. Morozova*, R. Jäger, A. Zarins, J. Balodis, I. Varna, and G. Silabriedis

Evaluation of quasi-geoid model based on astrogeodetic measurements: case of Latvia

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Abstract: Since the development of GNSS techniques, the determination of a precise quasi-geoid model has become even more actual. In terms of this project the staff of the Institute of Geodesy and Geoinformatics (GGI) has developed a new quasi-geoid model based on DFHRS (Digital Finite-element Height Reference Surface) approach additionally using astrogeodetic measurements – vertical deflections (VD), which can be observed by a Digital zenith camera. This paper evaluates a quasi-geoid model results based on vertical deflections, as a study area using the territory of Latvia: the standard deviation of the solution is equal to 0.006 m with observation residuals after the adjustment of minimum and maximum differences −0.012 and 0.012 accordingly. The standard deviation of quasi-geoid heights and h-H values from LGIA database is equal to 0.012 m with minimum and maximum differences −0.026 and 0.025 accordingly. The post-processed terrestrial VD observations have been compared to VD derivatives from EGM2008 and GGMplus geopotential models. The developed quasi-geoid has been compared to the national quasi-geoid model LV'14 and to the Scandinavian NKG2015.

Keywords: Digital zenith camera, EGM2008, GGMplus, quasi-geoid, vertical deflection

***Corresponding author: K. Morozova,** Institute of Geodesy and Geoinformatics, University of Latvia, Riga, Latvia; and Department of Geomatics, Faculty of Building and Civil Engineering, Riga Technical University, Riga, Latvia; and Institute of Applied Research (IAF), Hochschule Karlsruhe – University of Applied Sciences, Karlsruhe, Germany, e-mail: Katerina.morozova@lu.lv, ORCID: https://orcid.org/0000-0003-2303-6818
R. Jäger, Institute of Applied Research (IAF), Hochschule Karlsruhe – University of Applied Sciences, Karlsruhe, Germany, e-mail: reiner.jaeger@web.de
A. Zarins, J. Balodis, I. Varna, G. Silabriedis, Institute of Geodesy and Geoinformatics, University of Latvia, Riga, Latvia, e-mails: ansis.zarins@lu.lv, Janis.balodis@lu.lv, inese.varna@lu.lv, gunars.silabriedis@lu.lv

1 Introduction

Depending on a national height system – orthometric or normal one, there are several geoid/quasi-geoid models developed around the world with a precision up to 1 cm – Estonia [1], Netherlands [2], Denmark [3], Poland [4] etc. The quasi-geoid is a surface which fits best the mean sea level and coincide with the geoid in the oceans and continues under the Earth's surface. In difference to the geoid, the quasi-geoid is not an equipotential surface (gravity vector is not perpendicular to it). The height anomaly is the distance along the normal plumb line between the Earth's surface and the telluroid or between the ellipsoid and the quasi-geoid. Physical height systems are depicted in Fig. 1. [5]. Vertical deflection is the angular difference between direction of the plumb line (the normal to the geoid) and the vertical direction on the ellipsoid (the normal to the ellipsoid). The vertical deflection consists of a North-South component ξ and an East-West component η. The amplitude of vertical deflection is typically up to 10 arcseconds and can reach 1 arcminute in mountainous regions.

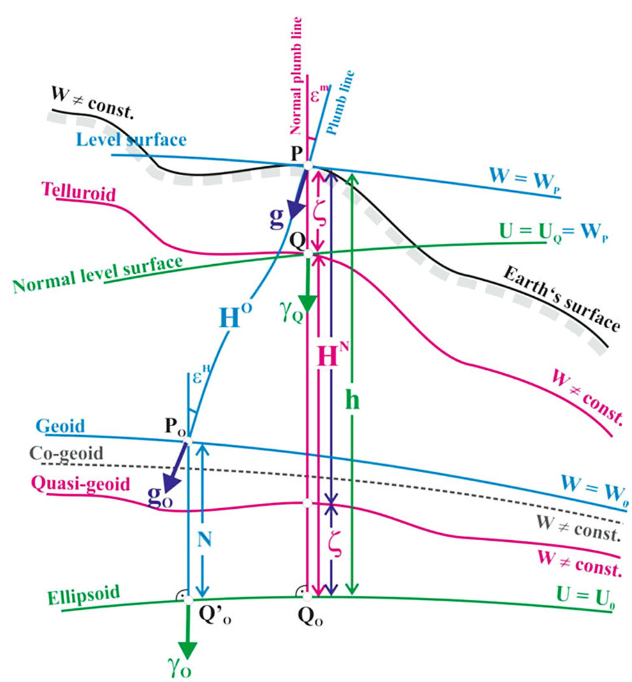

Figure 1: Physical height systems [5].

The aim of this study is to achieve up to a 1 cm precise quasi-geoid model, but traditional relative gravimetry requires a lot of measurement points for this task. The alternative method of the DFHRS approach [6, 7] was used, additionally using VDs, which were observed by the GGI developed Digital zenith Camera – VESTA (Vertical by STArs) [8]. In 1998 the first quasi-geoid model LV'98 [9] had been computed using Remove-Restore technique, implemented in GRAVSOFT software [10]. ~12000 relative gravity data points were digitized from 1:200,000 scale Soviet gravity maps for this purpose. The precision of computed quasi-geoid was evaluated by 6–8 cm. In 2014 Latvia transited to a new physical height system – EVRS2007 (European Vertical Reference System) [11], and the realization for Latvia – LAS-2000.5 (Latvian Height System, epoch 2000.5). So, the need for a new quasi-geoid model became obvious. Latvian Geospatial Information Agency (LGIA) has developed LV'14 [12], a quasi-geoid model using 4886 relative gravity points measured with a spring gravimeter Scintrex CG-5 [13], however these measurements can not be considered to be of a good quality, as the observations were not made in a closed loop. Digitized points were not included in the computation. Precision of the LV'14 quasi-geoid model evaluated to be 3–4 cm. So, our approach is based on Digital Zenith Camera (DZC) observations and can be used as an additional method to relative gravity measurements or as a single technique for quasi-geoid determination. The density of VD measurements is about 1 point per 12 × 12 km.

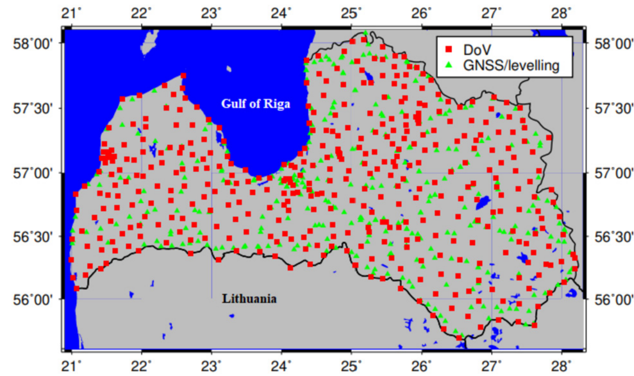

Figure 2: The scheme of measured GNSS/levelling points and vertical deflections.

2 Deflections of the vertical

Deflections of the vertical can be calculated using astronomical coordinates (Φ, Λ) and geodetic (ellipsoidal) coordinates reading [14]:

$$\xi = \Phi - \varphi \tag{1}$$

$$\eta = (\Lambda - \lambda)\, cos\varphi \tag{2}$$

The component ε in the azimuth α can be computed using ξ and η components:

$$\varepsilon = \xi cos\alpha + \eta sin\alpha \tag{3}$$

VD were implemented in the DFHRS approach v.4.3., based on polynomial modelling [15]. The observation equation in the DFHRS adjustment reads:

$$\xi^j = -\frac{\partial \zeta_{FEM}\,(\varphi, \lambda \mid \boldsymbol{p})\,/\partial\varphi}{M\,(\varphi) + h} + \Delta T_\xi\,(\boldsymbol{d})^j \tag{4}$$

$$\eta^j = -\frac{\partial \zeta_{FEM}\,(\varphi, \lambda \mid \boldsymbol{p})\,/\partial\lambda}{(N\,(\varphi) + +h) \cdot cos\varphi} + \Delta T_\eta\,(\boldsymbol{d})^j \tag{5}$$

$M(\varphi)$ and $N(\varphi)$ denote the radius of meridian and normal curvatures respectively at the latitude φ. With $\Delta T_\xi(\boldsymbol{d})$ and $\Delta T_\eta(\boldsymbol{d})$ a set of datum parameters is introduced, to model either a datum transition for deflections that refer to another geodetic datum, or local effects that come from systematic error sources, e. g. if VD were derived from GGMs. A parametric model for the local datum-shift, ΔT_ξ and ΔT_η based on 3 shifts $\mathbf{d}^j = (u, v, w)^T$ can be based on the Molodenski transformation [16] as:

$$\Delta T_\xi\,(\boldsymbol{d})^j\,[rad] = \begin{pmatrix} \frac{-sin\varphi\cdot cos\lambda}{M+h} & \frac{-sin\varphi\cdot sin\lambda}{M+h} & \frac{cos\varphi}{M+h} \end{pmatrix} \cdot \begin{pmatrix} u \\ v \\ w \end{pmatrix} \tag{6}$$

And

$$\Delta T_\eta\,(\boldsymbol{d})^j\,[rad] = \begin{pmatrix} \frac{-sin\lambda}{(N+h)\cdot cos\varphi} & \frac{cos\lambda}{(N+h)\cdot cos\varphi} & 0 \end{pmatrix} \cdot \begin{pmatrix} u \\ v \\ w \end{pmatrix} \tag{7}$$

For 3 years GGI has measured 414 vertical deflection observations using digital zenith-camera VESTA of the whole territory of Latvia (Fig. 2). The observations have been done also on the points of the Struve Geodetic arc [17] to determine the values of VD on historical points, which are included in UNESCO World heritage. Though it is a time-consuming process: it is possible to measure about 6–8 points per night, and further up to 20 times less points are needed in comparison to relative gravity measurements [18, 19, 20]. From Kaula's rule [21] the omission error:

$$\sigma_{om} \approx \frac{64}{n_{max}}\,[\mathrm{m}], \tag{8}$$

where n_{max} is a maximum expansion degree of the spherical harmonic function. So, in order to achieve 1 cm uncertainty quasi-geoid, the maximum expansion degree

Figure 3: Digital Zenith Camera VESTA of GGI University of Latvia.

should be ~6400, but our computations with the DFHRS software show, that the quasi-geoid undulations taken from the n = m = 2120 EGM2008, as a global geopotential model, can be introduced in a so-called patching [22], means with datum-parameters (based on the height component of Molodenski transformation) and (30 × 30) km patches can be with an accuracy of (1–3) cm. From [23] the density of gravity data for a 1 cm geoid should be around 15 points/100 km^2 or ~1 point/100 km^2 of VD observations. In terms of our project, based on VD, we have observed 414 VD measurements to improve the Latvian quasi-geoid model using the developed Digital Zenith Camera.

3 The Digital Zenith camera VESTA

The Digital Zenith Camera VESTA (Fig. 3) consists of a small 8-inch catadioptric telescope Meade LX200 (F = 2 m), rotating assembly with levelling system, high resolution two-axis tiltmeter (resolution ~0.02 milliarcseconds), Hemisphere GNSS antenna and receiver A222 (accuracy <0.6 m with SBAS, time accuracy <1 mksec), CCD (Charge-coupled device) camera (8.3 MPx), linear actuators with a resolution of <0.01 µm, on board control computer, wireless data transmission equipment and data processing software.

Accurate levelling, setting of azimuth, control of background oscillations and prescribed schedule of observations are done automatically by control and data processing software developed at GGI. Before starting the terrestrial VD observations on the territory of Latvia, for half a year the instrument was tested in one place, and the accuracy was estimated to be 0.1–0.15 arc seconds. In 2018–2019 3 new cameras were developed with some updates: transition to GAIA data 2nd release star catalog, data processing improvements, and revision of mechanical design. In the result, the accuracy was improved to 0.1 arcsec. The duration of the observation session time usually lasts at least 30 min, providing ~320 frames during 2 full rotations in each direction [8]. Exposure duration of 0.3–0.5 sec has proven to be optimal. Image elongation becomes pronounced for longer exposures; shorter exposures result in a smaller number of stars and in some loss of accuracy – while star position residual dispersion in a frame is a bit smaller for shorter exposures, estimated zenith position dispersion increases, probably due to lesser extent of averaging of air turbulence effects. At the above exposure settings, images of stars up to 13.5–14 magnitude are automatically recognized. That ensures typically 10 to 100 stars per frame; frames with less than 10 stars occasionally can occur only when the imaged area is far from the galactic plane [8].

To find a possible anomalous refraction effect and evaluate random dispersion and thermal drift, observation data was processed using a moving average time interval windows. Figure 4 (A, B) shows variations of VD components estimated using moving average time windows of 4 sizes: 12, 24, 30 and 60 positions, corresponding to time intervals of 18, 36, 45 and 90 minutes.

The main consideration is on anomalous refraction which is the main error source of VD observations causing irregular angular displacements of observed stars and therefore evoking time dependent variations of estimated VD values. Variations of VD caused by anomalous refraction have amplitudes of up to 0.1–0.2″ with periods of 0.5–4 hours. Figure 4 (A, B) shows that the effect of anomalous refraction can be averaged out by larger moving time windows but that also requires an extension of observation time which is not the optimal solution for regular observations. Therefore, for a regular 30-minute-long observation session, VD estimation accuracy now is limited to ~0.1″ with anomalous refraction in zenith being the main limiting factor. Of course, longer measurement sessions can

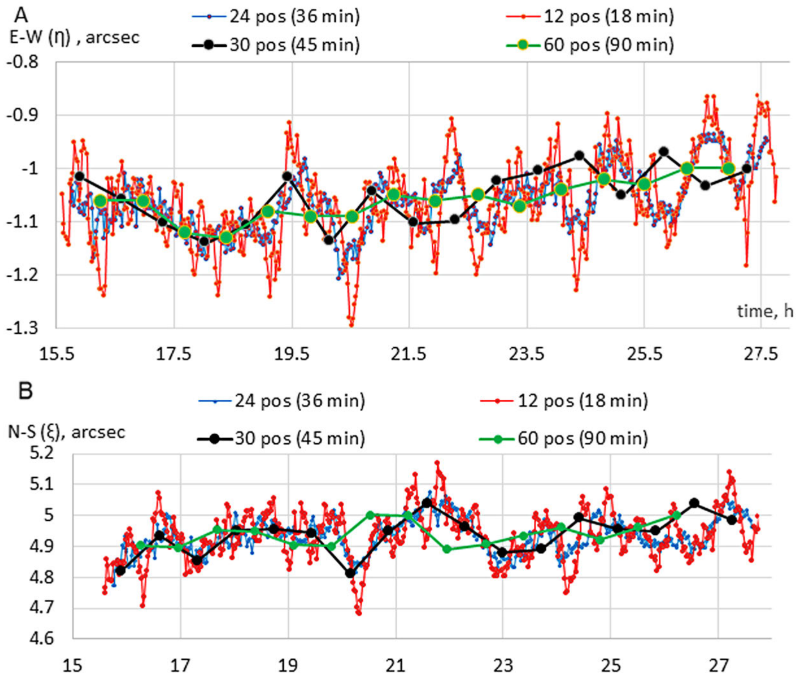

Figure 4: (A, B) Variations of ξ and η component estimated using moving average time windows of different width (12 h uninterrupted measurements).

Table 1: The statistical results of 3 solutions for quasi-geoid model evaluation using 3 data sets [in units of m].

Data set	SD	Min	Max	Mean
GNSS/levelling points + VD derivatives from EGM2008 + observed VD by DZC	0.006	−0.012	0.012	0.000
EGM2008 (without VD) + GNSS/levelling points + VD by DZC	0.017	−0.068	0.074	0.001
EGM2008 (without VD) + GNSS/levelling points	0.038	−0.106	0.246	0.006

represent the behavior of anomalous refraction more fully, possibly giving several times more accurate vertical deflection estimates.

4 Quasi-geoid evaluation and results

4-hour long GNSS observations were made at 325 first and second order levelling points and post-processed in Bernese GNSS Software v.5.2. by GGI [24] as reference points using 9 EPN (European Permanent Network) stations [25]. Station positions were corrected for the effect of solid Earth tides and the ocean tidal loading (FES2004 ocean tide model was used). Obtained positions are without corrections of the atmospheric tidal loading and non-tidal ocean loading (NTOL), which is raised by the Baltic Sea level changes [26]. All observations were made in IGS14 system and transformed to 2017.0 epoch. Levelling data were provided by Latvian Geospatial Information Agency (LGIA). The modelling method of quasi-geoid determination can be found at [27]. The three solutions of the quasi-geoid model were prepared using different data sets: global geopotential model EGM2008 [28, 29] and GNSS/leveling points; EGM2008, GNSS/levelling points and VD observed by DZC; and EGM2008 using additionally VD derivatives from the model, GNSS/levelling points and VD observed by DZC. From the Table 1 the best solution is obtained using both VD derivatives from GGMs and terrestrial VD observations. The standard deviation of this solution is equal to 0.006 m with residuals of minimum and maximum differences −0.012 and 0.012 accordingly. The worst solution is obtained when VD were not included in the modelling: neither VD derivatives from GGMs nor terrestrial VD observed by DZC: the standard deviation for this solution is equal to 0.038 m with residuals of minimum and maximum differences −0.106 and 0.246 accord-

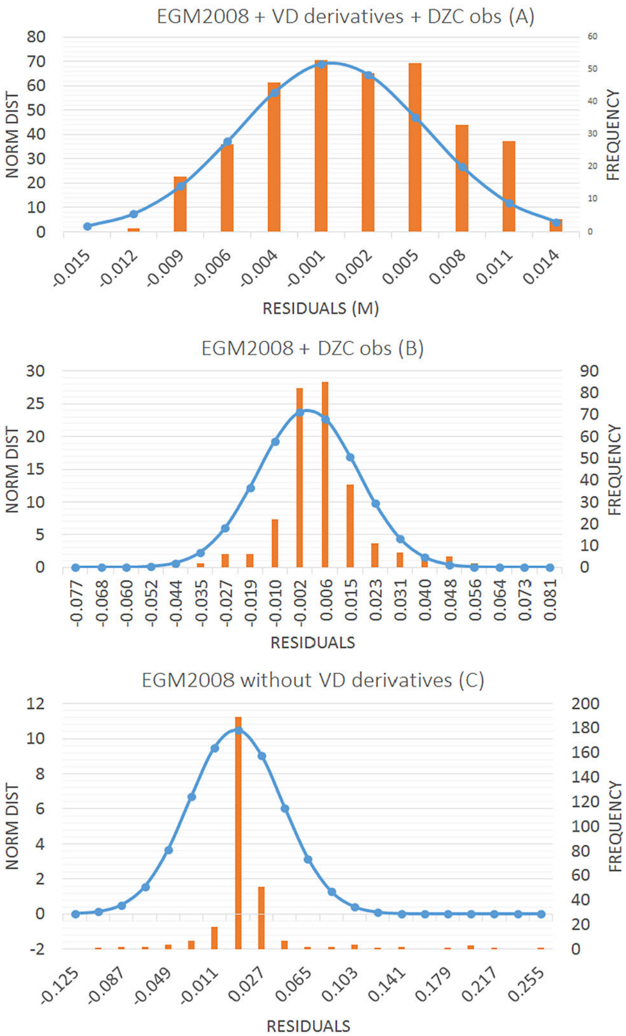

Figure 5: (A, B, C) The normal distribution and histogram of observation residuals after the adjustment.

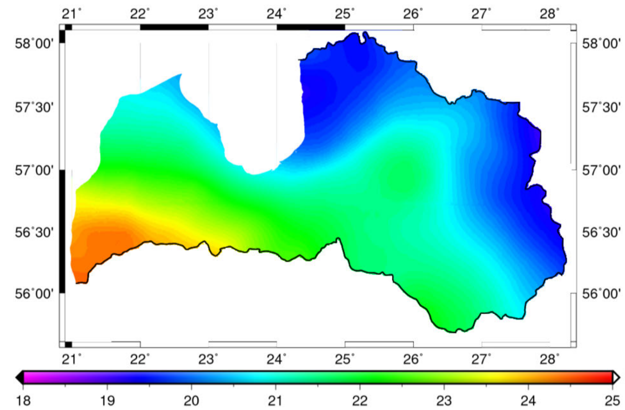

Figure 6: LU_GGI20 quasi-geoid model.

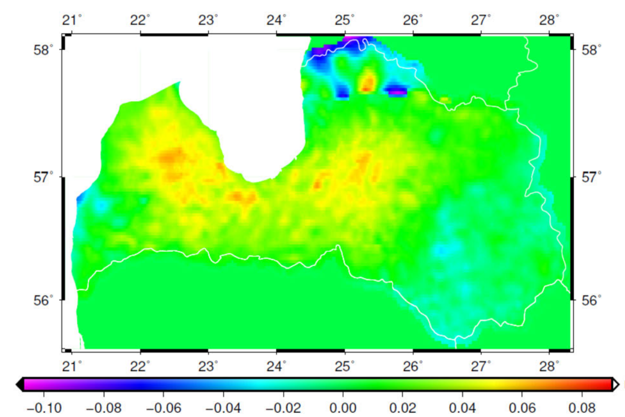

Figure 7: The comparison of LU_GGI20 quasi-geoid and LV'14.

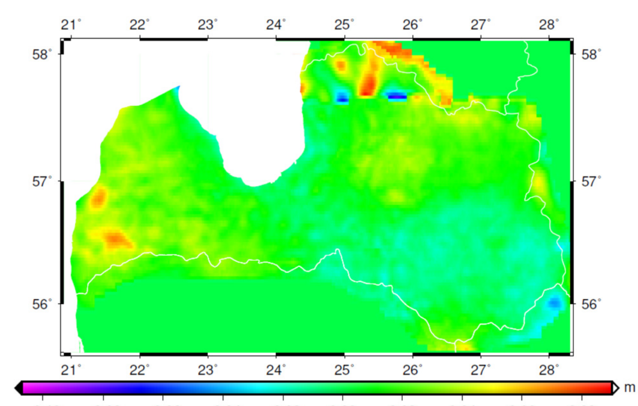

Figure 8: The comparison of LU_GGI quasi-geoid and NKG2015.

ingly. From the second solution which considers terrestrial VD observations it can be concluded that VD observations significantly improve the results: the standard deviation decreased to 0.017 m. The residuals vary from −0.068 m to 0.074 m.

The normal distribution and histogram of observation residuals after the adjustment of 3 solutions are depicted on Fig. 5 (A, B, C).

The GGI developed quasi-geoid model is depicted at Fig. 6. The quasi-geoid heights vary from 18.94 m in the North and North-East at Estonian and Russian borders and 24.44 m in the South-West near the Baltic Sea and the Lithuanian border. The computed quasi-geoid model (A solution) has been compared with the national Latvian model LV'14 and quasi-geoid model NKG2015 [30] computed by the Nordic Geodetic Commission.

The differences of the LU_GGI20 quasi-geoid and the LV'14 vary from −0.098 m in the North of the country near the Estonian border, and up to 0.073 m in the hilly area of Gaizinkalns and the center of Kurzeme region. The comparison of LU_GGI20 and LV'14 is depicted in Fig. 7. The

Table 2: The comparison of LU_GGI20 quasi-geoid model with LV'14, NKG2015 models.

	Min	Max	Avg	STDEV
LV'14	−0.098	0.073	0.009	0.020
NKG2015	−0.065	0.086	0.008	0.017

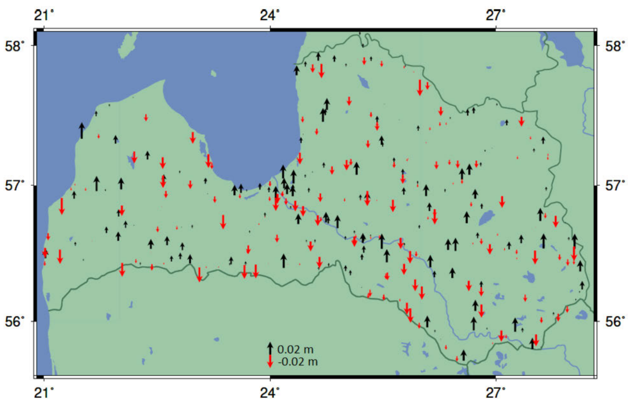

Figure 9: The difference between LU_GGI20 quasi-geoid heights and h-H values from LGIA database.

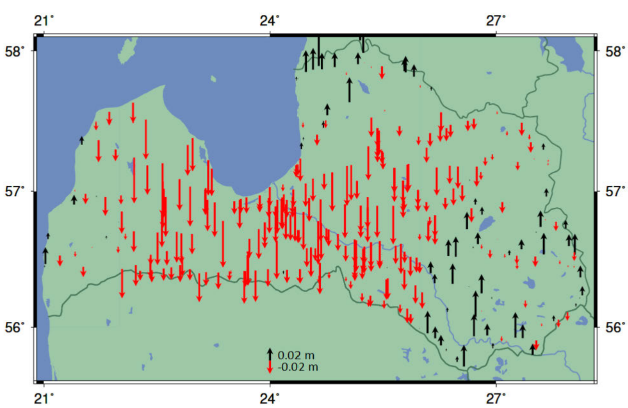

Figure 10: The difference between LV'14 quasi-geoid heights and h-H values from LGIA database.

differences between LU_GGI20 and NKG2015 are depicted in Fig. 8: the differences vary from −0.065 m in the North of the country and −0.054 m in Cape Kolka, where the Baltic Sea meets the waters of the Gulf of Riga and up to 0.069 m in two peaks in the South-West of Latvia and 0.086 m in one peak in the North of the country. The average differences and standard deviations are depicted in Table 2.

The comparison of the quasi-geoid heights and geodetic (h) minus normal heights (H) from LGIA database has also been performed for LU_GGI20 (Fig. 9), LV'14 (Fig. 10) and NKG2015 (Fig. 11).

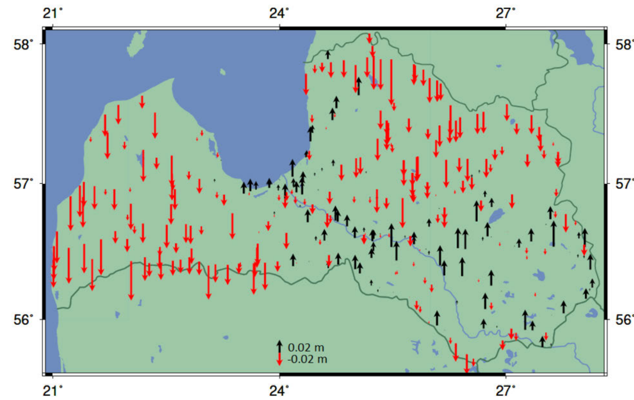

Figure 11: The difference between NKG2015 quasi-geoid heights and h-H values from LGIA database.

Table 3: The comparison of quasi-geoid heights and h-H values from LGIA database (m).

	Min	Max	Avg	STDEV
LU_GGI20	−0.026	0.025	0.000	0.012
LV'14	−0.081	0.082	−0.017	0.026
NKG2015	−0.070	0.040	−0.010	0.021

The summary of these differences are depicted in Table 3. The standard deviation for LU_GGI20 quasi-geoid model and h-H values from LGIA database is equal to 0.012 m and vary from −0.026 to 0.025 m. The standard deviation of the same evaluation for LV'14 quasi-geoid model is equal to 0.026 m and differences vary from −0.081 to 0.082 m. The results for NKG2015 quasi-geoid model are a little bit better in comparison to LV'14: the standard deviation is equal to 0.021 m and differences vary from −0.070 m and 0.040 m.

Terrestrial VDs observed by DZC were compared with VD derivatives from global geopotential models (see Table 4), e. g. GGMplus [31] and EGM2008, and computed the quasi-geoid model LU_GGI20. The results show a better correspondence with the GGMplus model by evaluating the standard deviation: 0.314 and 0.307 arcsec for ξ and η components respectively in comparison to 0.346 and 0.358 arcsec for ξ and η components for the EGM2008 model. The correspondence of terrestrial VD to derivatives computed from the LU_GGI20 quasi-geoid model is significantly better: the standard deviation is 0.055 and 0.046 arcsec for ξ and η respectively. More statistics can be found in Table 4. The comparison of terrestrial VD observations to EGM2008 and GGMplus are depicted in Fig. 12 and Fig. 13 respectively.

Table 4: The comparison of terrestrial VD observations observed by DZC to GGMs, and LU_GGI20 (arcsec).

	Min		Max		Avg		STDEV	
	ξ	η	ξ	η	ξ	η	ξ	η
LU_GGI20	−0.348	−0.190	0.216	0.162	0.007	−0.002	0.055	0.046
GGMplus	−1.300	−1.370	1.105	1.194	0.008	−0.025	0.314	0.307
EGM2008	−1.351	−1.031	1.747	2.509	0.013	−0.024	0.346	0.358

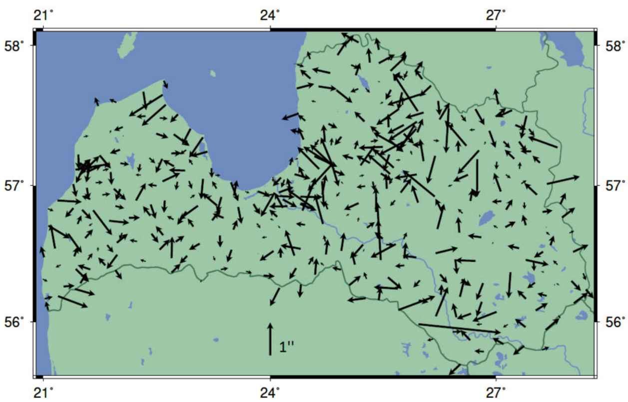

Figure 12: The comparison of terrestrial VD and EGM2008.

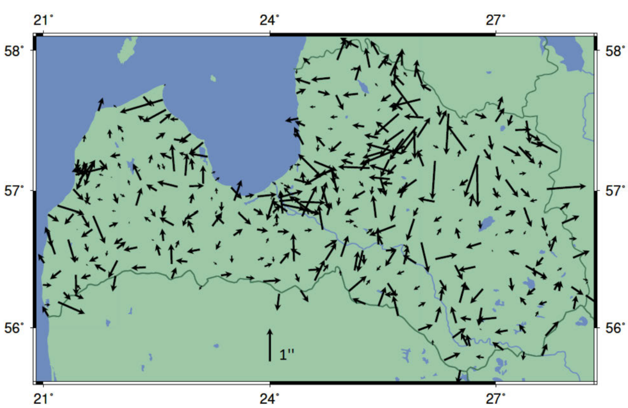

Figure 13: The comparison of terrestrial VD and GGMplus.

5 Conclusions

For the first time a high number of observations of terrestrial vertical deflections (VD) have been done on the territory of Latvia. The application of VD observations for the quasi-geoid model determination has shown the significant improvement of the computed quasi-geoid model. The standard deviation of the observation residuals after the adjustment, considering both VD derivatives from GGMs and terrestrial VD observed by DZC is equal to 0.006 m. Terrestrial VD observations fit the developed quasi-geoid model well, and the standard deviation for ξ and η components are equal to 0.055 and 0.046 arcsec respectively. Comparing the terrestrial VD observations to GGMs: no significant difference in standard deviation between GGMplus and EGM2008 was found, though the maximum difference for η component was 2 times less for GGMplus model. The final LU_GGI20 quasi-geoid model corresponds better to the NKG2015 model: the average difference is equal to 0.008 m in comparison to the LV'14 model, where this difference is equal to 0.009 m. The quasi-geoid heights have also been compared to ellipsoidal minus levelling heights: the standard deviation is equal to 0.012 m with minimum and maximum differences −0.026 m and 0.025 m respectively. The further development of the approach in terms of PhD thesis of the first author [32] concerns the implementation of VD observations in terms of SCH modelling [33, 34] together with gravity measurements. As astrogeodetic deflections of the vertical are an additional observation group, being independent from gravimetric geoid models and gravity observations, the combination of both gravity data and terrestrial VD observations, together with the stochastic prior information of the SCH coefficients from a global gravity field model, would give a complete integrated approach for quasi-geoid determination. In this context, the PhD of the first-mentioned author also deals with the 1st order design problem of the optimal positions of VD and gravity observations for regional gravity field determination.

The next research step concerning the DZC, is to test the DZC in various environments for the investigation of anomalous refractions in zenith. It will include long-term observations for seeking characteristics of anomalous refraction in several test sites during various weather conditions. Simultaneous observations with two adjacent DZCs will be a method to distinguish instrument-attributed variations from changes in the measured quantity itself and find the spatial properties of anomalous refraction effects.

Further in-depth study will be involved aiming at a comprehensive characterization of the DZC VESTA; it will include testing of various instrumental settings, such as analyses of the CCD binning parameter, star magnitude and star color impacts on accuracy.

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