METAMODEĻI NESOŠU GOFRĒTA PROFILA PLĀTŅU OPTIMĀLAI PROJEKTĒŠANAI

METAMODELS FOR THE OPTIMUM DESIGN OF CORRUGATE LOAD-BEARING PROFILE PLATES

Kalniņš K., Jēkabsons G., Skuķis E., Sproģis R.

Optimum design, response surface method, corrugate load-bearing profile plates, finite element analysis

Introduction

Corrugate load-bearing profile plates are widely used in load-carrying structures, e.g. civil and industrial buildings, bunkers, offshore structures and bridges, considering different loadings conditions of compression, bending or shear loads. Common engineering design procedures do not embrace load-bearing profile plates in general structural stability estimations. Therefore, an optimum design methodology has been evaluated to incorporate the load-bearing stiffness properties into different engineering analyses. Finite element calculations are extensively used in civil engineering to perform structural strength and stability analyses. However, the detailed representation of corrugate plate structures can cause unacceptable singularities in stress concentrations in models, and unreasonably high computing costs prevent engineers from considering the stiffness advantages of corrugate plates. In order to simplify the finite element model, it is proposed to substitute corrugate load-bearing profile plates with the equivalent orthotropic plate metamodel. Therefore, a significant reduction in the design cycle time can be obtained by constructing metamodels that could be implemented in structural design procedures. Such procedures could reduce the "time to market" time dramatically.

Design problem

The aim of this study is to develop an effective methodology for substituting corrugate load-bearing profile plates with an appropriate orthotopic plate model. Existing procedures [1] for evaluating load-carrying capacity assume that the transverse normal stiffness is infinite, thus keeping the distance between the centroids of the faces constant. Such a theory [2] was developed for orthotropic plates, with x- and y- axes bending the principal axes of orthotropy, and for which the properties are constant thought the panel. The general drawback of this methodology is the incompatibility for using such a procedure in more general structural analyses.

Therefore, in the current paper we evaluate a different optimum design methodology that can be used more effectively to determine structural stability limitations. The methodology is based on the response surface method, [3] with a minimisation of variance between two models. The first model is a full-scale load-bearing plate structure, in comparison with the second model, which is a simple rectangular plate with equivalent orthotropic material properties. Both models are analysed with the finite element code ANSYS, [4] extracting maximum bending deflection and stress values. The minimisation of variance allows one to determine an effective substituting metamodel for the design of corrugate load-bearing profile plates. The choice of corrugate plate geometrical parameters is based on the most frequently used load-bearing profile roof structure geometrical dimensions.

To avoid the numerical noise, the simplification of profile dimensions is presented as constant corrugate angle and equivalent top and bottom intermediate spans. The full-scale three-wave corrugate profile plate geometry presented in Fig. 1.a. is designed with the following parameters: *a* – corrugate profile intermediate span width, *t* – plate thickness, *L* – span length. The total width *B* of the full-scale corrugate panel can be determined:

$$B = 3\left(2a + \frac{4a}{\tan(60^\circ)}\right) \tag{1}$$

considering the panel thickness *t*=1 mm and constant corrugate profile angle 60^0.

For the construction of the equivalent orthotropic plate model Fig. 1.b., the plate thickness is introduced as $t_e = 2a \cdot l_1$, where l_1 is the thickness reduction factor. The total panel length *L* and panel width *B* are used in the same manner as in the full-scale corrugate plate model.

The uniformly distributed bending load case is considered for both models. On the full-scale corrugate load-bearing profile plate, uniformly distributed pressure-loading *Q* is applied on all three top intermediate spans. The load *Q* was recalculated and applied to the upper surfaces in terms of:

$$Q = q \cdot \frac{2a + \dfrac{4a}{\tan(60)}}{a} = q\left(2 + \frac{4}{\tan(60)}\right) \tag{2}$$

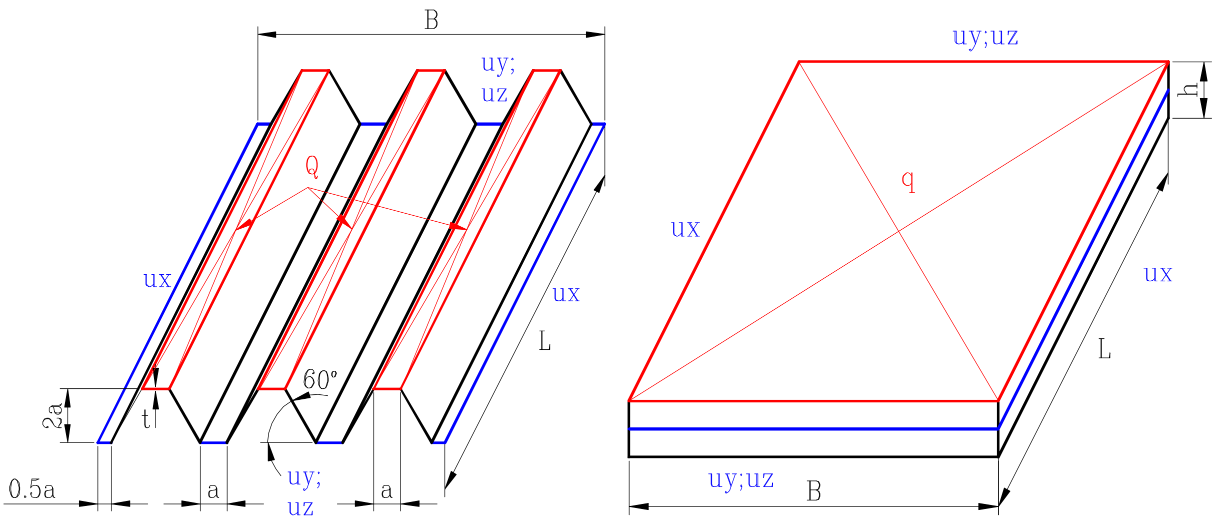


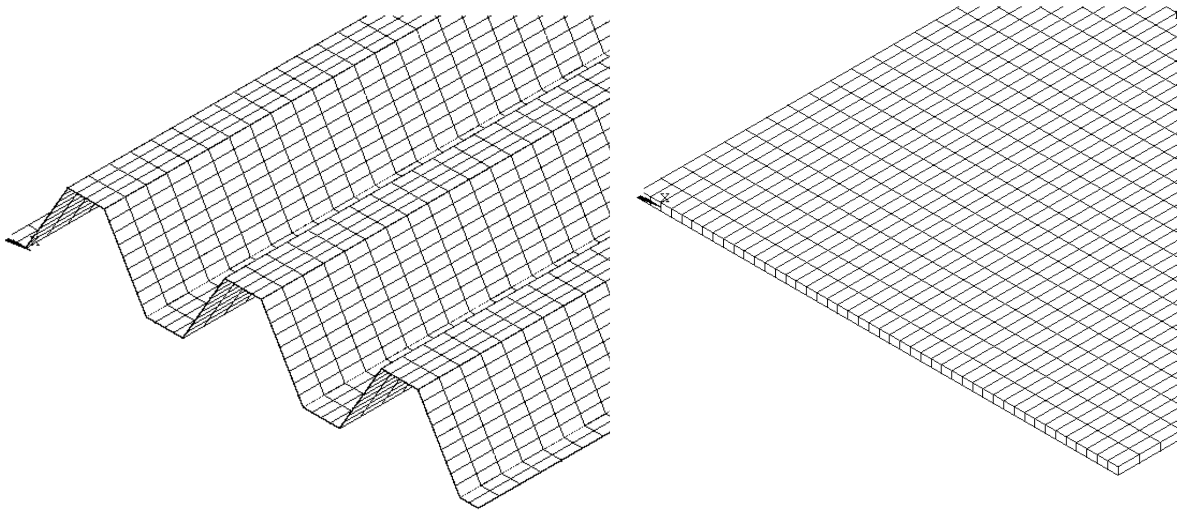
Figure 1. Geometry and boundary conditions: a - full-scale corrugate load-bearing profile plate model; b – orthotropic plate model.

Material properties

All components of corrugate load-bearing profile plates are made from the steel, where the Young modulus is taken as $E = 210\,\text{GPa}$ and the Poisson ration is $u = 0.3$. To estimate the orthotropic plate material properties, the Young modulus and shear modulus is multiplied by correction coefficients: $E_x = 210 \cdot k_1$ GPa; $E_y = 210 \cdot k_1 \cdot k_2$ GPa; $G_{xy} = 210 \cdot k_3$ GPa; $u = 0.3$, where k_1, k_2, k_3 are material elastic property reduction coefficients.

Finite elements

The finite element code ANSYS [4] with the shell-63 element is used to simulate the problem in question. Both real and equivalent model meshes are presented in Figure 2.



(a) (b)

Figure 2. Finite element mesh: a – Corrugate load-bearing profile plate; b – orthotropic plate

Simply supported boundary conditions are used in the finite element model to describe the real boundary conditions in corrugate load-bearing plate construction.

Design variables

In summary, the design variables considered for the building of metamodels are: *a* – corrugate profile intermediate span width, *L* – span length, *q* – uniformly distributed pressure load, l_1 – thickness reduction coefficient for the orthotropic plate model, k_1, k_2 – correction coefficients for Young's modulus E_x and E_y, k_3 – correction coefficient for the shear modulus G_{xy}.

The domain of interest for the design parameters is determined based on the most frequently used span dimensions and active load combinations, as shown in Table 1.

Table 1. Bounds of the design parameters

Notation		Lower bound	Upper Bound	Units
a	x_1	0.06	0.1	m
q	x_2	1400	2400	Pa
L	x_3	4	6	m
$\boldsymbol{l_1}$	x_4	0.08	0.14	
$\boldsymbol{k_1}$	x_5	15	40	
$\boldsymbol{k_2}$	x_6	0.3	1	
$\boldsymbol{k_3}$	x_7	15	40	

The first three design parameters are used in the full-scale metamodel evaluation, but in the construction of the equivalent orthotropic model, all seven parameters are implemented.

Optimisation problem

The inverse optimisation problem is formulated as a minimum variance between two metamodels. A similar methodology [5] is used in the identification of material properties, where natural experiments are used instead of numerical metamodels.

Objective functions:

$$\hat{Y}_i(x) - \hat{F}_i(x) \to \min \qquad (i = 1,2,3) \tag{3}$$

Design parameters:

$$\begin{aligned} \hat{Y}_i(x) &= \hat{Y}_1(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = \hat{Y}_i(B, q, L, l_1, k_1, k_2, k_3) \\ \hat{F}_i(x) &= \hat{F}_1(x_1, x_2, x_3) = \hat{F}_i(a, q, L) \end{aligned} \tag{4}$$

Constraints:

$$\begin{aligned} &\frac{\hat{F}_i(x) - \hat{Y}_i(x)}{\hat{F}_i(x)} \le 5\% \\ &\hat{Y}_1(x), \hat{F}_1(x) \le [s_{eq}] = 180 \quad \left(\frac{N}{mm^2}\right) \\ &\hat{Y}_2(x), \hat{F}_2(x) \le [w_{gl}] = \frac{L}{250} \quad (mm) \\ &\hat{Y}_1(x), \hat{F}_1(x) \le [s_{sh}] = 80 \quad \left(\frac{N}{mm^2}\right) \end{aligned} \tag{5}$$

Here $\hat{Y}_i(x)$ is assumed to be the metamodel for corrugate load-bearing profile plates and $\hat{F}_i(x)$ is the metamodel for orthotropic plates. Where subscript *i* means: *i*=1 – the maximum equivalent stress, *i*=2 – the maximum deflection, *i*=3 – the maximum shear stress. The material strength design constraints are based on material yield strength, with an estimated safety factor of 0.8. Likewise, the permitted deflection of the roof structure is taken as 1/250 of the roof span, as regulated by the manufacturer [6] and the national building code.

Metamodeling using partial polynomials, function selection

By generalizing the idea of polynomials, the non-linear regression function is the linear combination of any kind of functions. Thereby such a model could be written as follows:

$$\Phi(X) = a_0 F_0(X_1, X_2, \mathbf{K}, X_D) + a_1 F_1(X_1, X_2, \mathbf{K}, X_D) + \mathbf{K} + a_{M-1} F_{M-1}(X_1, X_2, \mathbf{K}, X_D) \tag{6}$$

where $F = \{F_i\}$ is a set of linearly independent functions consisting exclusively of *M* functions.

The problem of function selection is to take a set of candidate functions and select a subset that performs the best. This procedure can provide better regression accuracy due to finite sample size effects – irrelevant functions may negatively affect the accuracy of regression [7,8]. In addition, reducing the number of functions may help decrease the cost of acquiring data and might make the regression models easier to understand.

Formally for solving the function selection problem, the subset $F' \subseteq F$ should be found (or generated): $J(F') = \min_{F'' \subseteq F, |F''| = m} J(F'')$, where $J(.)$ stands for the function subset evaluation criterion that should be minimised; m is the number of functions included in subset F'.

A convenient paradigm for viewing the function selection approach is that of heuristic search, with each state in the search space specifying a subset of the possible functions.

Clearly, an exhaustive search of the space is impractical, as there exist 2^M possible subsets of functions. A more realistic approach relies on a greedy method to traverse the space. At each point in the search, one considers local changes to the current set of attributes, selects one, and then iterates. For instance, the hill-climbing approach considers both adding and removing functions at each decision point, which lets one retract an earlier decision without keeping explicit track of the search path. Within these options, one can consider all states generated by the operators and then select the best, or one can simply choose the first state that improves accuracy over the current set.

Two of the most promising sequential search algorithms are those proposed in [9], namely, the Sequential Forward Floating Selection (SFFS) algorithm and the Sequential Backward Floating Selection (SBFS) algorithm. They improve the standard SFS and SBS techniques by dynamically changing the number of features included (SFFS) or removed (SBFS) at each step and by allowing the reconsideration of the features included or removed at the previous steps.

Another effective algorithm is Random-Mutation Hill Climbing (RMHC) [10]. RMHC is a stochastic meta-algorithm. It simply runs an outer loop over hill-climbing which stochastically iterates in any direction as long as it is possible to increase the value of the criterion function. Each step of the outer loop chooses a random initial condition x_0 to start hill-climbing. The best x_m is kept: if a new run of hill-climbing produces a better x_m than the stored state, it replaces the stored state.

In the current paper, metamodels as polynomial regression functions are generated by using the data of numerical simulation. FEM simulations were performed in 150 sample points for data set with seven variables.

Evaluation of metamodels

The constructed metamodels can be evaluated by leave-one-out cross-validation [11]. The leave-one-out cross-validation error is calculated as

$$s_{cr} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\hat{F}_{-i} - F_i\right)^2} \quad (7)$$

where F_i is the response value at the i-th sample point x^i of experimental design and $\hat{F}_{-i}$ denotes the prediction of the response on x^i using the metamodel created on N-1 sample points with point i moved out (i=1,2,...,N).

Similarly, the leave-one-out cross validation percentage error is calculated as

$$s_{cr\%} = \frac{s_{cr}}{STD}100\% \quad (8)$$

where STD stands for standard deviation from the mean value of experimental responses. This measure shows how much smaller the standard deviation of approximation is in comparison with the approximation with a constant value.

For the final evaluation of our metamodels we will use the test set mean absolute error T_{abs} and percentage error $T_{\%}$ [8,12]. The test set mean absolute error can be written in the following expression:

$$T_{abs} = \frac{1}{|W|}\sum_{i=1}^{|W|}\left|\Phi(X_{iW}) - Y_{iW}\right| \quad (9)$$

where W is the test set; $\Phi(X_{iW})$ is the model's estimated value for the test set's i-th data sample; Y_{iW} is test set's i-th data sample affirmative indication value.

Likewise, the test set percentage error can be written in following expression:

$$T_{\%} = \frac{T}{TSTD}\cdot 100\% \quad (10)$$

where $T = \sqrt{\frac{1}{|W|}\sum_{i=1}^{|W|}\left(\Phi(X_{iW}) - Y_{iW}\right)^2}$; $TSTD = \sqrt{\frac{1}{|W|}\sum_{i=1}^{|W|}\left(\overline{Y}_W - Y_{iW}\right)^2}$; $\overline{Y}_W$ stands for all test set affirmative indication mean values.

Parametric studies

The above-mentioned design problem data are analysed using polynomial regression with full 2nd and 3rd order polynomial functions (RP2 and PR3). In addition, partial polynomial functions that do not exceed the third order are investigated. To generate the partial polynomial functions, two heuristic state space search algorithms were chosen – RMHC and SFFS.

For generated full-scale second-order polynomial metamodels, cross-validation errors do not exceed a 2% margin. Therefore, our main attention is paid to reducing the modelling error for equivalent orthotropic plate metamodels. The results of sampling data approximation through the use of the in-house code EDAOPT [13] with 2nd and 3rd order polynomials and the use of the in-house code FUNSEL [12,14] with partial 3rd order polynomials are summarized in Table 2.

Table 2. vonMisses metamodel prediction errors

	PR2	**PR3**	**SFFS**	**RMHC**	**RMHC_50**	**RMHC_40**
Regression coefficients	36	120	29	79	42	40
$\boldsymbol{CV_{\%}}$ **(10 folds)**	0.154	0.539	0.0953	0.0303	0.0623	0.0705
$\boldsymbol{T_{\%}}$**_full-scale**	0.136	0.187	0.205	0.199	**0.116**	**0.115**
$\boldsymbol{T_{\%}}$**_orthotropic**	0.228	0.251	0.234	0.160	**0.193**	**0.149**

From the approximation results summarized in Table 2 one can see that the best model can be obtained by using the state space search RMHC algorithm. Unfortunately, the SFFS algorithm found only local minimum values. Additionally, RMHC algorithm searches are restricted to a maximum of 50 and 40 regression coefficients (RMHC_50 and RMHC_40) to meet the criteria of minimization of partial regression coefficients. The most suitable model was found with 40 regression coefficients, in comparison with second-order polynomial approximation, which has 36 regression coefficients. The leave-one-out cross validation percentage error has thus decreased about two times.

Parametric studies were carried out additionally for the designer's convenience to investigate the influence of different design parameters on behavioural functions. This can be done by displaying contour plots or 3D graphs of approximating functions. The dependencies of equivalent stresses in different load-bearing plate design parameters are given in Figures 3 and 4. The ultimate design stress level is within the middle of the design space. Therefore, the design parameter boundaries are finely foreseen. More extensive parametric studies are given in references [8,15].

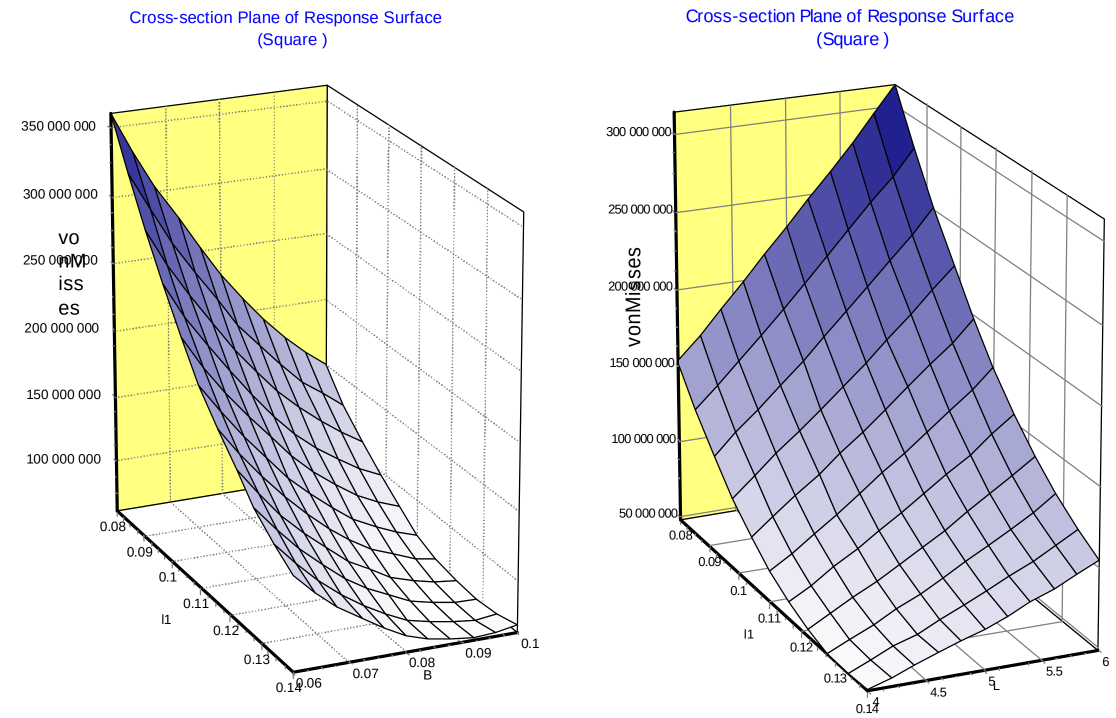


Figure 3. The dependencies of equivalent stresses over l1&B and l1&L design parameters

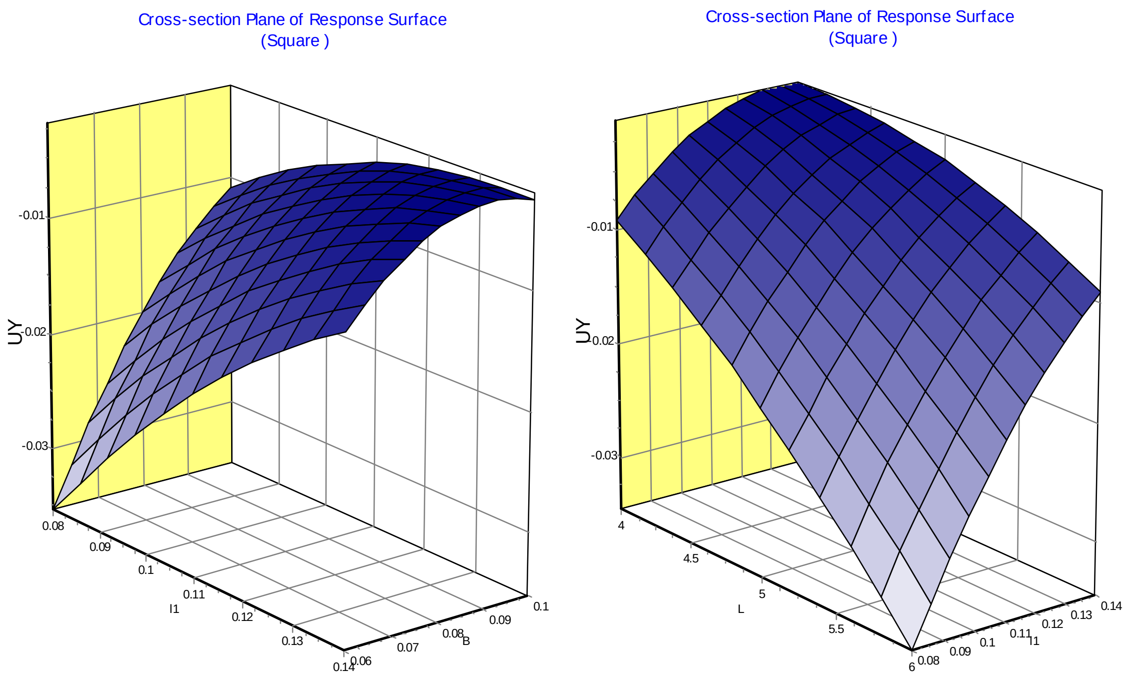


Figure 4. The dependencies of plate deflection over l1&B and l1&L design parameters

The minimization problem between two metamodels is solved by the random search method. Some particular results where the span length and loading factor is fixed are presented in Table 3. The study of metamodel prediction errors shows high compatibility between the numerical results obtained by the finite element code ANSYS and response surface metamodels.

Table 3. Compression between metamodel and numerical ANSYS results

Input data		**a** [mm]	**68**	**62**	**62**	**61**
		q [kPa]	**1.6**	**1.6**	**1.6**	**1.6**
		L [m]	**6**	**5.5**	**5**	**4.5**
		l_1	**0.12**	**0.125**	**0.125**	**0.125**
		k_1	**20.5**	**19**	**18.5**	**18.5**
		k_2	**1**	**1**	**1**	**1**
		k_3	**24**	**21.5**	**21.5**	**21**
Metamodel	**Full-sc.pl.**	s_{eq} [MPa]	178	156	128	103
		w_{gl} [mm]	16.9	14.4	10.1	6.86
		s_{sh} [MPa]	43.9	37.3	31.4	26.1
	Ort. pl.	s_{eq} [MPa]	176	157	127	104
		w_{gl} [mm]	16.9	14.5	10.2	6.82
		s_{sh} [MPa]	43.5	37.4	31.8	25.9
ANSYS	**Full-sc.pl.**	s_{eq} [MPa]	175	158	130	106
		w_{gl} [mm]	16.6	14.1	9.67	6.58
		s_{sh} [MPa]	42.8	38.5	32.5	27.3
	Ort. pl.	s_{eq} [MPa]	166	154	128	107
		w_{gl} [mm]	16.7	14.9	10.4	7.19
		s_{sh} [MPa]	39.8	35.8	31.7	26.9
%	**Full-sc.pl.**	s_{eq} [%]	**1.69**	**1.28**	**1.56**	**2.91**
		w_{gl} [%]	**1.78**	**2.08**	**4.26**	**4.08**
		s_{sh} [%]	**2.51**	**3.22**	**3.50**	**4.60**
	Ort. pl.	s_{eq} [%]	**5.68**	**1.91**	**0.79**	**2.88**
		w_{gl} [%]	**1.18**	**2.76**	**1.96**	**5.43**
		s_{sh} [%]	**8.51**	**4.28**	**0.31**	**3.86**

Conclusions

A load-bearing corrugate panel optimal design problem has been formulated and a methodology based on experimental design and response surface techniques has been developed for the minimisation of variance between two different metamodels. The advantages of partial regression using state space search algorithms have been proven by using different error evaluation criteria. In addition, an acquired, simple but precise metamodel can be extensively implemented by designers even using a “pocket calculator,” instead of a full-scale finite element analysis.

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Kalniņš K., Jēkabsons G., Skuķis E., Sproģis R. Metamodels for optimum design of load-bearing profile plates
Thin walled load-bearing corrugate profile plates are among the most commonly utilized lightweight roof structures in civil engineering. Existing analysis procedures do not involve these load–bearing profiles in the estimation of structural stiffness. Currently finite element analyses have become a common practice in structural analysis, where large-scale structural modelling cannot be effectively segmented into adequate finite elements of corrugate plate profiles. Therefore, an equivalent orthotropic material plate model can be used to substitute the real corrugate profile model. The objective of the present study is to derive the relationship between the geometrical characteristics of corrugate profile and equivalent orthotropic material properties.

Калниньш К., Екабсон Г., Скукис Э., Спрогис Р. Метомодели для оптимального проектирования пластин несущго гофрированого профиля
Тонкостеные несущие профиля широко применяются в строительстве при монтаже конструкций для крышь. Применяемая на практике процедура расчета несущих профилей не учитывает жесткостные характеристики полной конструкции крыши. Анализ масштабных конструкций широко распространен с помощью МКЭ. Однако использование полной модели несущих профилей не целесообразна. Разумно производить замену несущего профиля на эквивалентную ортотропную модель. Целью работы является нахождения связи между геометрическими характеристиками несущего профиля с эквивалентной ортотропной моделью.

Kalniņš K., Jēkabsons G., Skuķis E., Sproģis R. Metamodeļi nesošo gofrēta profila plātņu optimālai projektēšanai
Nesošu gofrētu plānsienu profilu plātnes tiek plaši izmantotas būvniecība, it sevišķi, vieglo pārsegumu konstrukciju projektēšanā. Pastāvošā aprēķina metodika neievērtē šīs nesošās profila plātnes kopējās konstrukcijas stinguma aplēsēs. Šobrīd aprēķini ar galīgiem elementiem ir kļuvuši par plaši izplatītu konstrukciju aprēķina praksi. Tomēr šādus plaša mēroga modeļus nav racionāli sadalīt smalkā galīgo elementu tīklā, kas precīzi spētu aprakstīt gofrētās plātnes ģeometriju. Lai atrisinātu šo problēmu tiek piedāvāts aizvietot gofrēta profila plātni ar ekvivalentu ortotropa materiāla plātni. Darba mērķis ir atrast savstarpējas savietojamības iespējas gofrēta profila plātnes un ortotropa materiāla plātnes modeļiem.