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CREEP APPROXIMATION METHOD FOR BENT WOOD MATERIAL ELEMENTS

LIEKTU KOKSNES MATERIĀLU ELEMENTU ŠĻŪDES DEFORMĀCIJU APROKSIMĀCIJAS METODE

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Introduction

Both in the production engineering of construction elements from wood, and in the load-bearing capacity calculations, where real operating conditions are taken into account, there are necessary to consider the material creep. Often for describing of the wood deformational properties the rheological equation for typical viscous-elastic body is used [1], [2]

$$nE\dot{\varepsilon} + H\varepsilon = n\dot{\sigma} + \sigma \, , \tag{1}$$

where, n – relaxation time;

E – momentary elastic modulus;
H – long-term elastic modulus;
ε – deformation;
σ – stress.

However this differential equation in many cases incompletely describes experimental results, therefore B.Ogarkov [3] suggests to improve the rheological equation of typical viscous-elastic body with the creep curve shape coefficient α (hereafter abbreviated called for "complemented rheological equation"):

$$nE\dot{\varepsilon} + \alpha H\varepsilon\left[\left(\frac{t}{n}\right)^{1-\alpha}\right]^{-1} = n\dot{\sigma} + \alpha\sigma\left[\left(\frac{t}{n}\right)^{1-\alpha}\right]^{-1} \tag{2}$$

Unambiguous method for the determination of coefficients n, H, α in equation (2), is not developed, therefore the method for determination of these coefficients was developed and advantages what opens this equation for describing of various wood material creep curves were checked.

Determination of coefficients for complemented rheological equation

Relevancies for determination of coefficients are acquired and checked for creep in case of constant stress, but as for foundation of produced method is taken the rheological equation for typical viscous-elastic body which can be used in solving of different rheological, then probably the determined coefficients are applicable also in other rheological tasks, for example, in case of stress relaxation.

For description of creep curves the dependence, which is obtained by integration of differential equation (2) in case of constant stress, is used.

$$\varepsilon = \frac{\sigma}{E} + \sigma(\frac{1}{H} - \frac{1}{E})\left\{1 - e^{-\frac{H}{E}\left(\frac{t}{n}\right)^{\alpha}}\right\} \tag{3}$$

Creep curve shape coefficient α is obtained from the equation (3) by equalizing the long-term elastic modulus H to the experimentally obtained elastic modulus at the end of experiment E_B, and transforming the equation in a form

$$\frac{\varepsilon_B - \varepsilon}{\varepsilon_B - \varepsilon_0} = e^{-\frac{E_B}{E}\left(\frac{t}{n}\right)^{\alpha}} \text{, where} \tag{4}$$

$$E_B = \frac{\sigma}{\varepsilon_{\mathbf{B}}}, \tag{5}$$

and ε_B - final experimentally obtained longitudinal deformation;

ε_0 - initial deformation of sample.

The deformations in time moment's t_1 and t_2 determined using equation (4) could be united in equation system

$$\begin{cases} \dfrac{\varepsilon_B - \varepsilon_1}{\varepsilon_B - \varepsilon_0} = e^{-\frac{E_B \cdot t_1^{\alpha}}{E \cdot n^{\alpha}}} \\ \dfrac{\varepsilon_B - \varepsilon_2}{\varepsilon_B - \varepsilon_0} = e^{-\frac{E_B \cdot t_2^{\alpha}}{E \cdot n^{\alpha}}} \end{cases} \text{, where} \tag{6}$$

$$t_2 = t_B / 2, \tag{7}$$

$$t_1 = t_2 / 2, \tag{8}$$

$$\varepsilon_2 = \varepsilon\big|_{t_2}, \tag{9}$$

$$\varepsilon_1 = \varepsilon\big|_{t_1}, \tag{10}$$

and t_B – experiment duration.

As the time t_2 is two times larger than time t_1, the equation system (6) is transformed in the form

$$\begin{cases} \dfrac{\varepsilon_B - \varepsilon_1}{\varepsilon_B - \varepsilon_0} = e^{-\frac{E_B \cdot t_1^{\alpha}}{E \cdot n^{\alpha}}} \\ \dfrac{\varepsilon_B - \varepsilon_2}{\varepsilon_B - \varepsilon_0} = \left(e^{-\frac{E_B \cdot t_1^{\alpha}}{E \cdot n^{\alpha}}}\right)^{2^{\alpha}} \end{cases} . \tag{11}$$

The relevance for determination of creep curve shape coefficient α, characterized with four experimentally obtained values of longitudinal deformations in different time moments, is stated from equation system (11):

$$\alpha = \log_2 \left(\frac{\ln\left(\frac{\varepsilon_B - \varepsilon_2}{\varepsilon_B - \varepsilon_0} \right)}{\ln\left(\frac{\varepsilon_B - \varepsilon_1}{\varepsilon_B - \varepsilon_0} \right)} \right) . \tag{12}$$

Relaxation time *n* is determined from the equation (3) by equalizing the power function of exponent to one, and replacing time *t* with time t_k

$$n = \frac{t_k}{\left(\frac{E}{H} \right)^{\frac{1}{\alpha}}} \text{ , where} \tag{13}$$

t_k – time moment, when the experimental longitudinal deformation has reached the value ε_k, according to [4].

$$\varepsilon_k = \varepsilon_0 + (\varepsilon_B - \varepsilon_0) \cdot (1 - e^{-1}) \tag{14}$$

Long-term elastic modulus *H* is determined in several stages. Initially its value is assumed equal with experimentally obtained elastic modulus in the end of experiment E_B. Afterwards a value is gradually changed with the purpose to bring the calculated deformation in the end of creep curve nearer to the experimentally obtained until they match. Relaxation time n_j at each selected value of elastic module E_{Bj} is determined from dependence

$$n_j = \frac{t_k}{\left(\frac{E}{E_{Bj}} \right)^{\frac{1}{\alpha}}} . \tag{11}$$

The last determined value of elastic module E_{Bj}, using which the calculated end point of creep curve matches with the experimentally obtained, is the long-term elastic modulus *H* of material.

Experimental results

The advantages of developed method are analysed for experimentally obtained creep curves of five different wood material elements in bending - Douglas-fir beam [5], three-layer medium density particleboard plate, 3-ply plywood plate [6], composite hardboard-webbed I-beam with Douglass-fir flanges [7] (hereafter abbreviated called for "I-beam-1") and composite plywood webbed I-beam with Douglass-fir flanges [7] (hereafter abbreviated called for "I-beam-2"). Most of creep experiments took place in uncontrolled interior environments: tests with Douglas-fir beams were conducted in an unheated, enclosed building with natural ventilation [5], tests with particleboard plate and composite hardboard-webbed I-beams were conducted in building heated in winter, but there were no humidity control [7], only in an experiment with plywood plate, the temperature in the time of loading was constantly 21.1ºC [6].

Creep curve calculated using "complemented rheological equation", with the values of rheological coefficients H, n, α (see 10th row in table 1) determined using developed method (hereafter abbreviated called for "gradual approximation method") are graphically compared with creep curves obtained using:

1. equation for typical viscous-elastic body [1];
2. "complemented rheological equation", with creep curve shape coefficient α calculated using formula (12) from lit.[4], relaxation time n calculated using formula (7) from lit.[4] and long term elastic modulus H is equalized to E_B (hereafter abbreviated called for "complemented rheological equation with coefficients α, n, $H=E_B$");
3. "complemented rheological equation", with creep curve shape coefficient α calculated using formula (12) from lit.[4], relaxation time n calculated using formula (7) from lit.[4] and long term elastic modulus H is gradually approximated to its true value so that the end point of calculated creep curve matches with the experimentally obtained (hereafter abbreviated called for "complemented rheological equation with coefficients α, $n_j=f(H)$, $H=E_{Bj}$"). The elastic modulus can be approximated in the case of every rheological equation, therefore to objectively compare existing method for determination of coefficients α and n with the developed "gradual approximation method", the approximation of long-term elastic modulus for "complemented rheological equation" was performed;
4. "complemented rheological equation", with creep curve shape coefficients α and n calculated using formulas (4) and (11) of developed method, and long term elastic modulus H equalized to E_B (hereafter abbreviated called for "complemented rheological equation with coefficients α, $n_j=f(\alpha, H)$, $H=E_B$"). This curve is constructed in a compare intention with the "complemented rheological equation", which is taken for foundation of produced method, and where gradual approximation of long-term elastic modulus is not applied too. In the calculations of load bearing constructions determinant is the resulting deformations, not the deformations in the process of creep, therefore in the calculations of building constructions approximation is obligatory.

1.table. The parameters of experimentally checked samples used in the calculations of creep curves

Nr. p.k.	*Element, its material* *Notation of parameter*	*Wood beam (see fig.1)*	*Particleboard plate (see fig.2)*	*Plywood plate (see fig.3)*	*"I-beam-1" (see fig.4)*	*"I-beam-2" (see fig.5)*
1.	σ, [kN/cm^2]	*0.797*	*0.682*	*0.327*	*0.345*	*0.329*
2.	M, [kN·cm]	*0.399*	*1.867*	*0.911*	*299.365*	*299.365*
3.	t_B, [years]	*12.27*	*21.55*	*0.23*	*3.49*	*3.49*
4.	u_0, [mm]	*8.74*	*2.25*	*1.73*	*2.16*	*3.98*
5.	u_B, [mm]	*25.77*	*9.16*	*2.59*	*4.04*	*6.87*
6.	*Coefficients of creep curve for the 1 case* [*1]	*(327.18; 0.768; -)*	*(132.43; 1.455; -)*	*(363.10; 0.013; -)*	*(771.46; 0.282; -)*	*(428.77, 0.324; -)*
7.	*Coefficients of creep curve for the 2 case* [*2]	*(327.18; 0.768; 0.340)*	*(132.42; 1.455; 0.466)*	*(363.10; 0.013; 0.580)*	*(771.46; 0.282; 0.373)*	*(428.77, 0.324; 0.534)*
8.	*Coefficients of creep curve for the 3 case* [*3]	*(180.00; 0.423; 0.340)*	*(30.00; 1.455; 0.466)*	*(358.00; 0.013; 0.580)*	*(650.00; 0.237; 0.373)*	*(405.00, 0.306; 0.534)*
9.	*Coefficients of creep curve for the 4 case* [*4]	*(327.18; 0.233; 0.476)*	*(132.42; 0.219; 0.425)*	*(363.10; 0.013; 0.939)*	*(771.46; 0.170; 0.552)*	*(428.77, 0.238; 0.644)*
10.	*Coefficients of creep curve for the 5 case* [*5]	*(303.00; 0.198; 0.476)*	*(116.00; 0.160; 0.425)*	*(363.10; 0.013; 0.939)*	*(750.00; 0.161; 0.552)*	*(425.00, 0.235; 0.644)*

[*] *In the rows 6.÷10. are declared values of rheological coefficients (H; n; α), when describing creep curves with:*

[1] *equation for typical viscous-elastic body;*
[2] *"complemented rheological equation with coefficients* α *, n,* $H=E_B$*";*
[3] *"complemented rheological equation with coefficients* α *,* $n_j=f(H)$*,* $H=E_{Bj}$*";*
[4] *"complemented rheological equation with coefficients* α *,* $n_j=f(\alpha, H)$*,* $H=E_B$*";*
[5] *"gradual approximation method".*

Rheological coefficients H, n, α are calculated for every material from experimentally obtained creep curves (see fig.1÷5). In the calculations of creep curves were assumed that bending stress distribution in the cross-sections of wood elements is constant all the time of element loading and stresses in the compressed and tensioned parts of cross-section are equal. The elastic modules of materials in the appropriate time moments were calculated from the deflections declared in publications [5]÷[7] (deflections of specimens at the experiment beginning u_0 and in the end u_B see in table 1). The maximal longitudinal deformations of specimen were calculated using Hooks law with long term elastic modulus H. The maximal bending stresses σ (see table 1) in outer fibers were calculated as a division of maximal bending moment M and section modulus of cross-section.

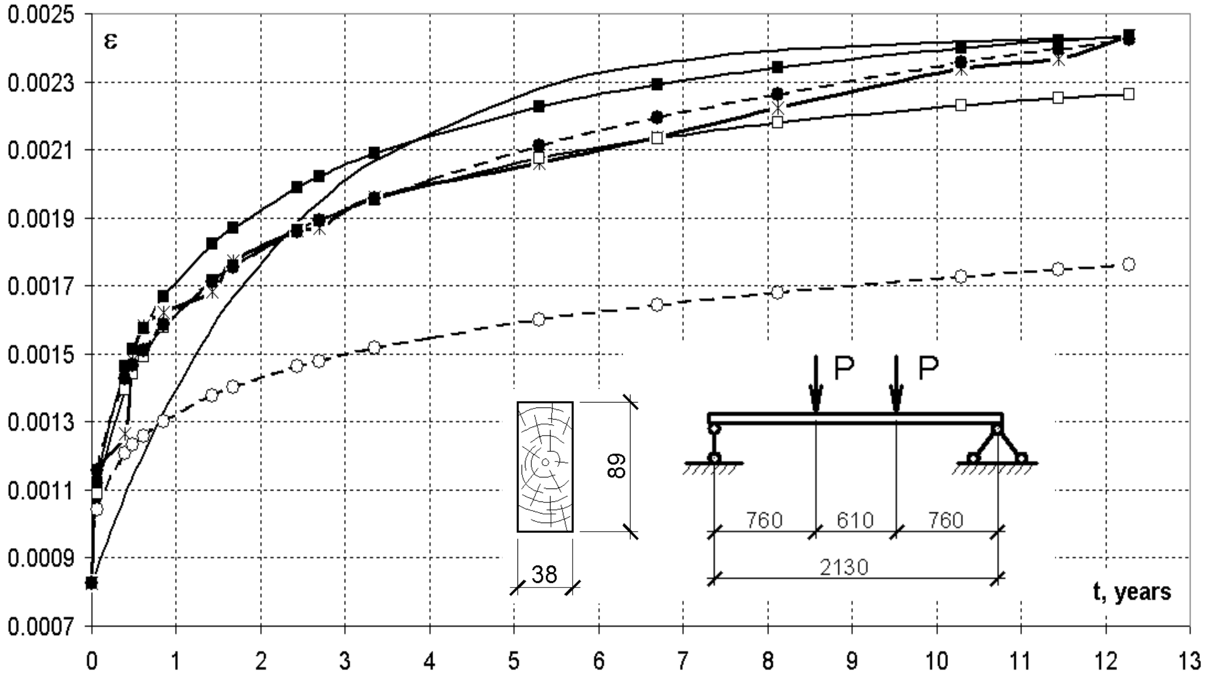

Fig.1. Loading scheme, cross-section and creep curves of Douglas-fir beam, if load P=0.526 kN.
Patterns applied in graphic:

—✳— *Experimental results [5];*
——— *Calculation using rheological equation for typical viscous-elastic body;*
– ○ – *used "complemented rheological equation with coefficients* α *, n,* $H=E_B$*";*
– ● – *used "complemented rheological equation with coefficients* α *,* $n_j=f(H)$*,* $H=E_{Bj}$*";*
—◇— *used "complemented rheological equation with coefficients* α *,* $n_j=f(\alpha, H)$*,* $H=E_B$*";*
– ◆ – *used "gradual approximation method".*

Creep curves calculated using the rheological equation of typical viscous-elastic body, "complemented rheological equation" and "gradual approximation method" were compared with the experimentally obtained creep curves (see table 2). In all the cases maximal difference from the experimentally obtained deformations, are the smallest in case of developed “gradual approximation method”.

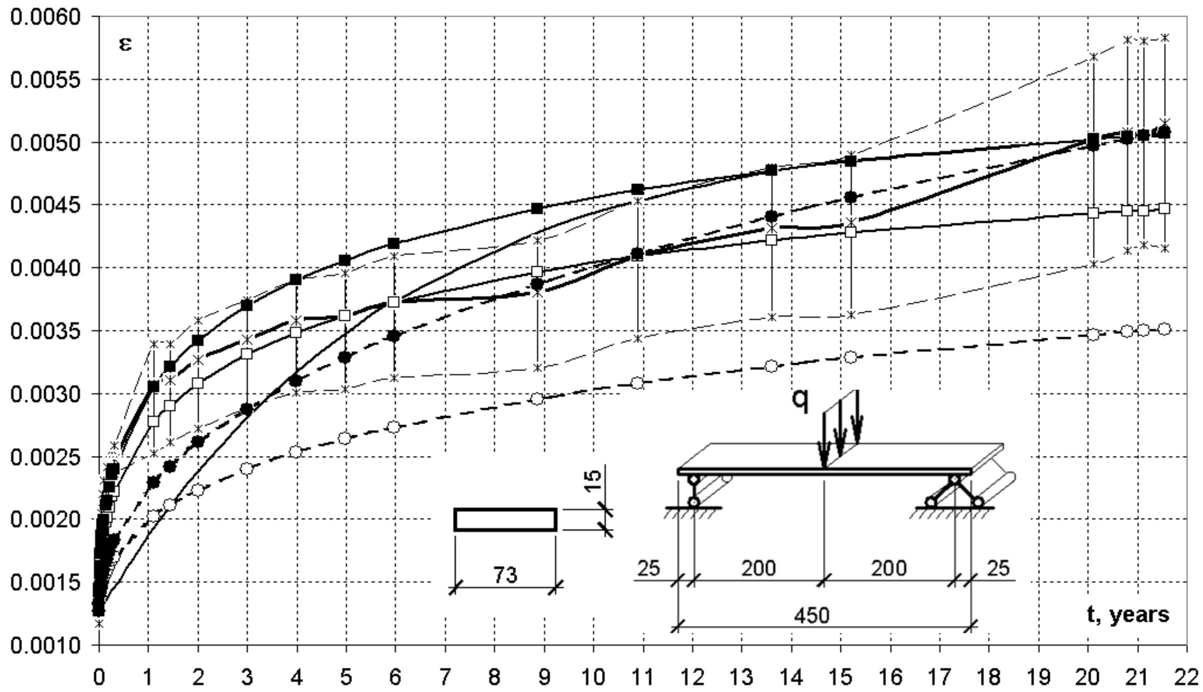

Fig.2. Loading scheme, cross-section and creep curves of particleboard plate, *if load q=2.562 kN·m*

Patterns applied in graphic:

—⁎— *Dispersion borders of the experimental results;*

decoding of patterns —✳— , ——— , – ○ – , —□— , —■— , – ● – *see in fig.1.*

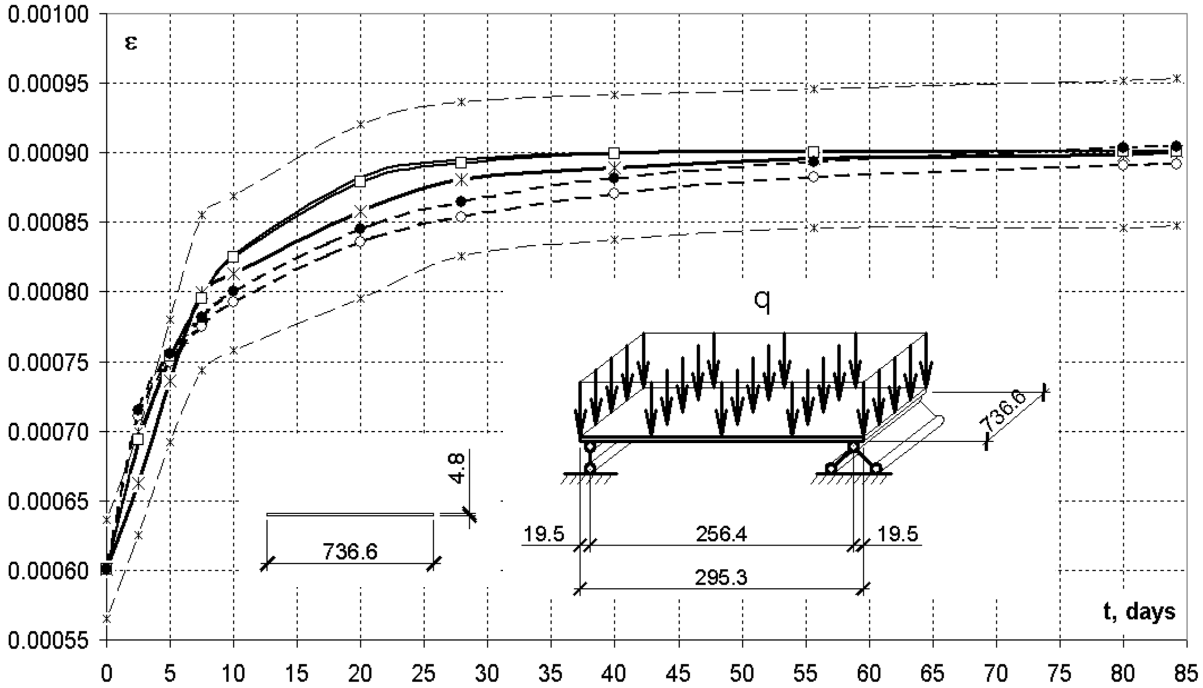

Fig.3. Loading scheme, cross-section and creep curves of plywood plate, if load $q=4.323kN/cm^2$.

Patters applied in graphic:

—✳— *Experimental results [6];*

—⁎— *Dispersion borders of the experimental results [6];*

decoding of patterns ——— , – ○ – , —□— , —■— , – ● – *see in fig.1.*

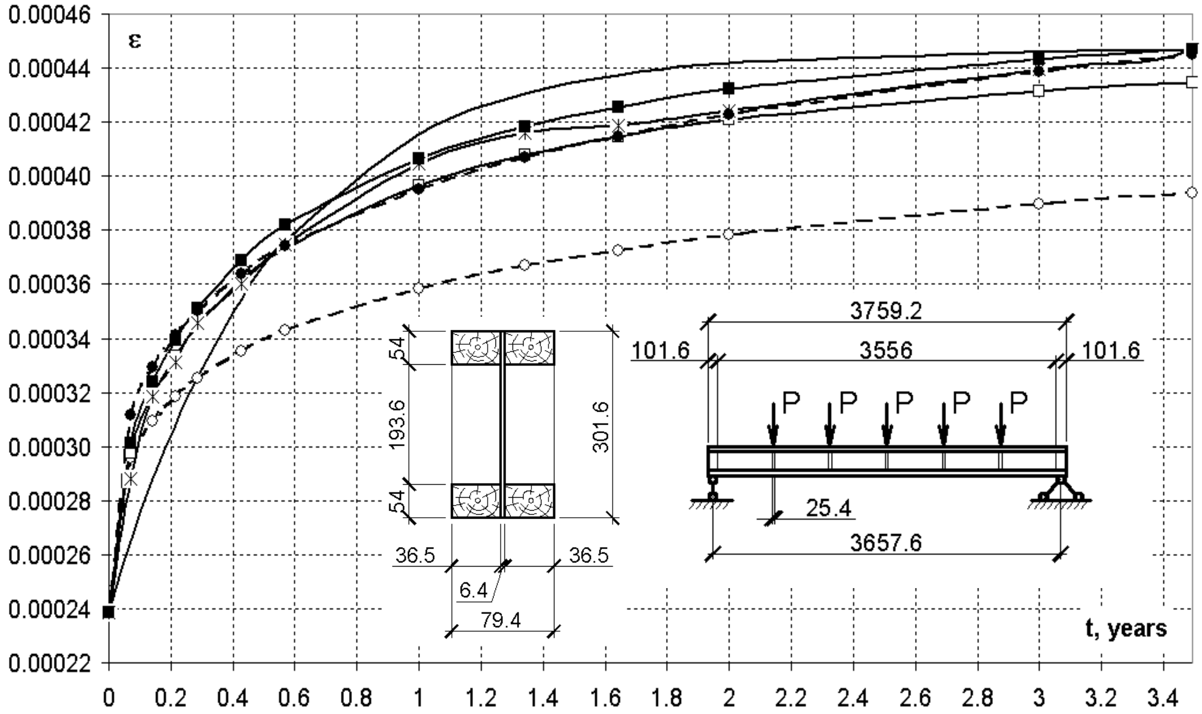

Fig.4. Loading scheme, cross-section and creep curves of composite wood I-beam-1, if load P=1.089 kN.

Patters applied in graphic:

—✳— *Experimental results [7];*

decoding of patterns ——— , – ○ – , —□— , —■— , – ● – *see in fig.1.*

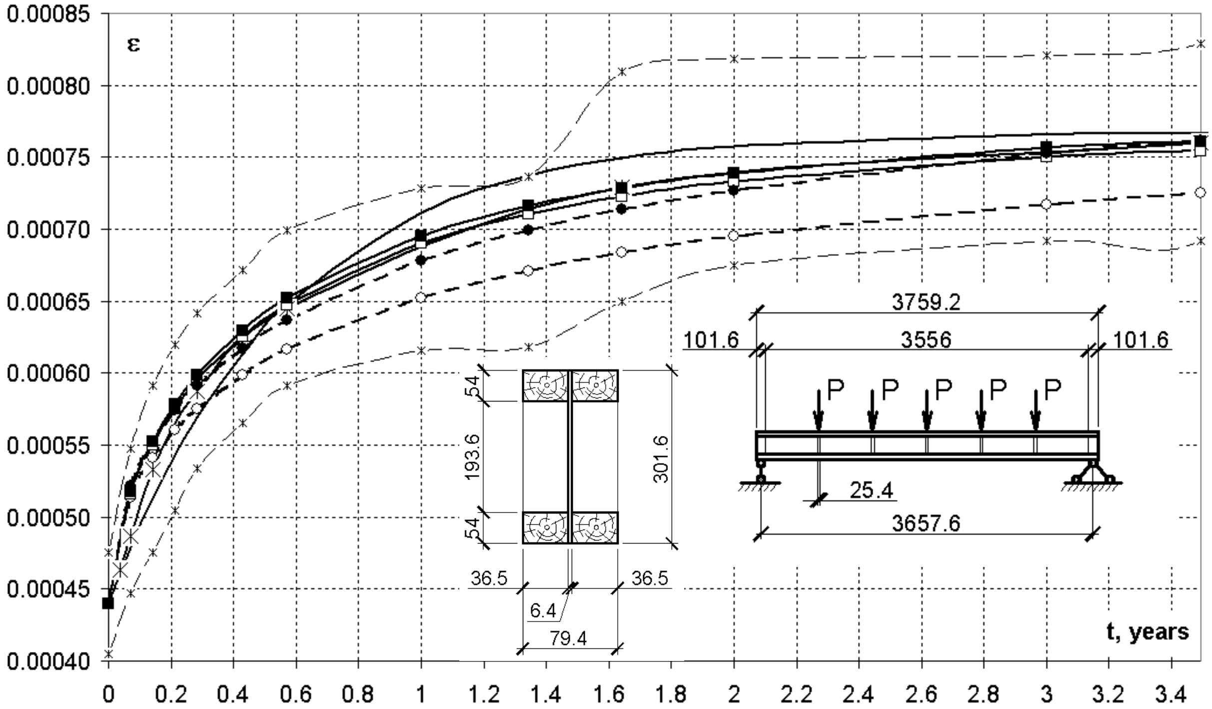

Fig.5. Creep curves of composite wood I-beam-2.

Patters applied in graphic:

—✳— *Experimental results [7];*

—⁎— *Dispersion borders of the experimental results [7];*

decoding of patterns ——— , – ○ – , —□— , —■— , – ● – *see in fig.1.*

2.table. Differences of the calculated creep curves from the experimentally obtained creep curve.

Time period of declared maximal difference:		*difference [%] in the time period* $t_0 \div {}^{t_B}\!/_{10}$			*difference [%] in the time period* ${}^{t_B}\!/_{10} \div t_B$		
Nr. p.k.	*Pattern of the calculation method** *Element, its material*	——	—○—	– ◆ –	——	—○—	– ◆ –
1.	*Wood beam*	*+ 23.81*	*+ 20.42*	*- 15.73*	*- 10.62*	*+ 27.67*	*- 8.26*
2.	*Chipboard plate*	*+ 42.14*	*+ 34.07*	*+ 10.15*	*+ 18.09*	*+ 31.80*	*- 17.56*
3.	*Plywood plate*	*- 4.05*	*- 7.35*	*- 4.80*	*- 2.89*	*+ 3.06*	*- 2.51*
4.	*"I-beam-1"*	*+ 9.50*	*+ 5.96*	*- 4.48*	*- 4.51*	*+ 11.86*	*- 2.43*
5.	*"I-beam-2"*	*+ 3.81*	*- 3.32*	*- 4.07*	*- 8.85*	*+ 6.78*	*- 5.73*

Patters applied in table:
—— *Calculation using rheological equation for typical viscous-elastic body;*
– ○ – *used "complemented rheological equation with coefficients* α *, n, H=*E_B*";*
– ◆ – *used "gradual approximation method".*

More precise creep curve is obtained when for the calculation of creep curve is used the "complemented rheological equation with coefficients α, n_j=f(H), H=E_{Bj}" not the "gradual approximation method". However, the advantage of developed "gradual approximation method" is in the determination of coefficient α , which is determined from four experimentally obtained longitudinal deformations at time moments t_0, $t_B/4$, $t_B/2$ and t_B. Only three longitudinal deformations are used when calculating coefficient α using [4], but two from them must be found with single condition - one of longitudinal deformations must be several times larger than other. Therefore using coefficient α calculated with previously developed dependence [4] there is need for complete experimentally obtained creep curve.

Conclusions

- Unambiguous "gradual approximation method" for the determination of creep curve shape coefficient, relaxation time and long-term modulus is developed for the rheological equation for typical viscous-elastic body, which is complemented with creep curve shape coefficient;
- Creep curves of five different wood materials were calculated using the developed method and compared with creep deformations of bent elements - Douglas-fir beam, three-layer medium density particleboard plate, 3-ply plywood plate, composite hardboard-webbed I-beam with Douglass-fir flanges and composite plywood webbed I-beam with Douglass-fir flanges;
- Creep curves calculated using the rheological equation for typical viscous-elastic body, the rheological equation for typical viscous-elastic body, which is complemented with the creep curve shape coefficient, and "gradual approximation method" were compared with the experimentally obtained creep curves. The difference from the experimentally obtained deformations for mentioned wood material elements were smallest in all the cases when rheological coefficients for calculated creep curve were determined using proposed "gradual approximation method".

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Baikovs A., Rocēns K. Liektu koksnes materiālu elementu šļūdes deformāciju aproksimācijas metode.

Izstrādāta šļūdes līknes formas koeficienta, relaksācijas laika, ilglaicīgā elastības moduļa noteikšanas metodika tipiska viskozi- elastīga ķermeņa reoloģiskajam vienādojumam, kas papildināts ar šļūdes līknes formas koeficientu. Nosakot reoloģiskos koeficientus pēc izstrādātās metodikas tiek izmantoti piecos laika momentos eksperimentāli noteikti šļūdes deformāciju lielumi.

Izmantojot izstrādāto metodiku aprēķinātas piecu dažādu koksnes materiālu šļūdes līknes un salīdzinātas ar liektu elementu šļūdes deformācijām - duglāzijas egles koksnes sijai, trīsslāņu vidēja blīvuma skaidu plates plātnei, trīskārtu saplākšņa loksnes plātnei, saliktai dubult-T profila sijai ar skaidu plates sieniņu un duglāzijas egles koksnes brusu plauktiem un saliktai dubult-T profila sijai ar saplākšņa loksnes sieniņu un duglāzijas egles koksnes brusu plauktiem. Izmantojot reoloģiskos koeficientus, kas noteikti lietojot izstrādāto koeficientu noteikšanas metodiku atšķirības starp aprēķinātām un eksperimentāli noteiktām šļūdes līknēm ir mazākas kā gadījumos, kad aprēķinātajām šļūdes līknēm izmantoti reoloģiskie koeficienti, kas noteikti lietojot iepriekš izstrādātās metodikas.

Baikovs A., Rocens K. Creep approximation method for bent wood material elements.

Method for the determination of creep curve shape coefficient, relaxation time and long-term modulus is developed for the rheological equation for typical viscous-elastic body, which is complemented with creep curve shape coefficient. Experimentally obtained deformations in five time moments are used to determine the rheological coefficients with developed method.

Creep curves of five different wood materials were calculated using the developed method and compared with creep deformations of bent elements - Douglas-fir beam, three-layer medium density particleboard plate, 3-ply plywood plate, composite hardboard-webbed I-beam with Douglass-fir flanges and composite plywood webbed I-beam with Douglass-fir flanges. Using rheological coefficients calculated with developed method of coefficient determination, the difference between calculated and experimentally obtained creep curves are smaller than in the cases when for calculating of creep curves are used rheological coefficients determined using beforehand developed methods.

Баиковс А., Роценс К. Метод аппроксимации деформации ползучести гнутых елементов из древесины.

Разработанная методика определения коефициентов формы кривой ползучести, времени релаксации и длителного модуля упругости для реологического уравнения типического вязко-упругого тела, который дополнен коефициентом форми кривой ползучести. Определьая реологические коефиценти с выработанной методикой используетса значения експериментально определенных деформаций ползучести в пяти моментах времени.

Воспользуясь разработанной методикой, вычислены кривые ползучести пять разных древесных материалов и сравненны с деформациями ползучести гнутых елементов – балки из древесины ложнотсуговой ели, трехслойной стружечной плити средней плотности, плити из трехслойной фанери, составной двутавровой балки со стенкой из стружечной плити и полками из древесины ложнотсуговой ели и составной двутавровой балки со стенкой из фанеры и полками из древесины ложнотсуговой ели. Используя реологические коефициенты определенные с разработанной методикой определения коефициентов, разница между вычисленными и експериментально опредиленными кривыми ползучести менше чем в случаях когда для вычисления кривых ползучести изпользованни реологические коефициенти, которие определены исползуя прежде разработанные методики.