Initial Implementation of Generalized Haar-Like Orthonormal Transforms into FPGA-Based Devices - Part II: Parametrical Decomposition-Reconstruction Filters

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***Abstract*—This paper deals with the aspects of practical implementation of parametrical decomposition-reconstruction filter(s) (DeReFs) based on generalized parametrical Haar transform (RA-HT) introduced before by the authors. The presented CORDIC-based RA-HT device is implemented as Altera's Cyclone II FPGA. A simplified block diagram and parameters of the filter module are provided. Two versions of the module are reviewed and the version comparison is provided. The developed real-time filter for selected Altera's chip can run at sampling frequencies up to 12.5 MHz. The filters can be successfully used for both extraction and rejection of pulse-like signals with an infinite diversity of shapes. Two examples of separation of single pulse-like signal by the filter are provided. Obtained results show a noticeable improvement of extraction/rejection in comparison to the classical Haar DeReF.**

***Index Terms*—CORDIC algorithm, FPGA, Generalization of Haar functions, Orthogonal filters**

I. INTRODUCTION

THE pioneer in introducing and using of rotation angle-based orthogonal filter(s) (OF) is P. P. Vaidyanathan [1]. In [2] we can find out several hundred papers issued by many authors and related to different aspects of OFs – from theory (e.g. [3], [4]) to practical applications [5]. Several dozen works in the previous two decades were produced by Vaidyanathan. But, the Vetterli's and Herley's tandem is very fruitful in the area of time-varying OFs. [6] and [7] are only the first two from the series of impressive papers. Most of the "orthogonal filters" papers from the two recent decades deal with wavelets decomposition-reconstruction filters (DeReFs). It seems that P. A. Regalia is initiator of adaptive orthogonal filtering [8]. P. Rieder with colleagues in series of papers (for example, [9]) discuss the CORDIC-based implementation of orthogonal wavelet transforms and DCT. Rieder is the first after H. C. Andrews [10] who emphasizes using of rotation angles (parameterization) in signal processing. Probably, [11] is one of the first work dealing with problems of using CORDIC and fixed-point arithmetic in OFs. The CORDIC algorithm is a main base for different categories of OFs (one of the recent examples is [12]).

Processing of medical and biomechanical signals may be an area to aim the applications of DeReFs presented in this paper. Presently, most authors use wavelets for such signal processing [13]-[15].

We avoid here a detailed overview of all available works on rotation angle approach because of the limited space of paper. Implementation examples of orthogonal filters presented in the paper we suppose are unknown before.

We introduced novel classes of generalization of Haar-like discrete orthonormal functions (RA-HT transforms) in [16]. Correspondingly, novel classes of orthogonal (orthonormal) Decomposition-Reconstruction Filter(s) (DeReFs) are presented in [17]. This paper supplements simulation results obtained in [17], and here are presented mainly experimental results. We suppose that the matters exposed in [18] and below are novel.

II. BASICS

The second section of [18] (the paper is submitted for this proceeding) is also introductory for the current paper. For better understanding of the content of this paper, please refer to [17] or [18]. We provide below a minimum of basic notions only.

A. Elementary Rotation

The elementary rotation is a keystone for building DeReFs:

$$\begin{bmatrix} y_m \\ y_{m+1} \end{bmatrix} = \begin{bmatrix} \sin\varphi_m & \cos\varphi_m \\ \cos\varphi_m & -\sin\varphi_m \end{bmatrix} \cdot \begin{bmatrix} x_m \\ x_{m+1} \end{bmatrix} \qquad (1)$$

where φ_m – rotation angle, x_m, x_{m+1} – input (signal) samples, y_m, y_{m+1} – output (spectrum) samples. (1) has been used in decomposition phase and performs the Given's rotation with permutation of samples after rotation. In the reconstruction phase the transposed rotation matrix is used. This matrix in the case of the used rotation is equal to the provided matrix.

B. Elementary Decomposition-Reconstruction

Further we will use terminology well-known from wavelets [2]. The first row of rotation matrix (1) corresponds to the **A**pproximation **D**ecomposition (AD) filter, but the second one to the **D**etails **D**ecomposition (DD) filter. The mentioned rows correspond also to the **A**pproximation **R**econstruction (AR) filter and the **D**etails **R**econstruction (DR) filter, respectively. We extend these concepts with an attributive word "parametrical", for example, **P**arametrical **D**etails

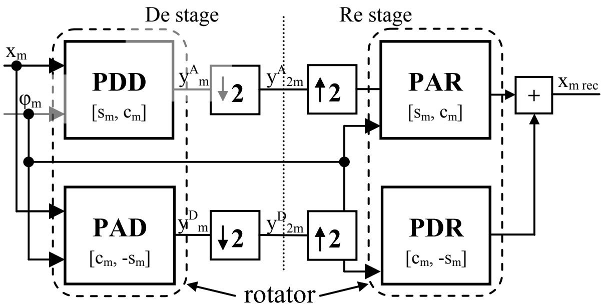

Fig. 1. Simplified block diagram of elementary decomposition-reconstruction filter with PAD, PDD, PAR, and PDR blocks

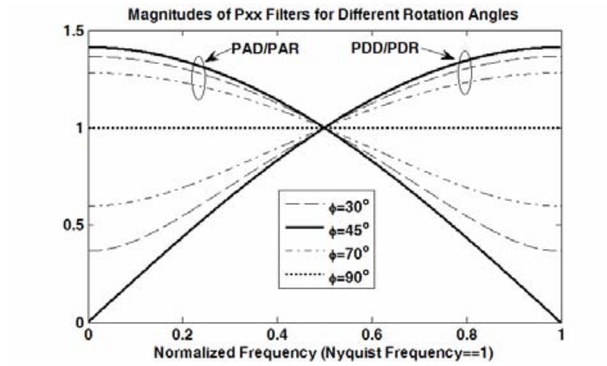

Fig. 2. The magnitudes of the PAD/PAR and PDD/PDR filters for different rotation angles

Decomposition (PDD) filter.

Fig. 1 shows a simplified block diagram of elementary decomposition-reconstruction filter. Shortcuts are used – s_m=sin(φ_m) and c_m=cos(φ_m). Blocks PAD, PDD etc. (further Pxx) differ by the values of impulse response (IR) that has the length equal to 2 and can be changed while the filter is running. The rotation angle (parameter) must be kept constant during incoming of each two samples at least. The decimation by 2 has been performed after filtering using PxD filters. Correspondingly, the up-sampling (by insertion of zero after each sample) is performed in PxR filters.

C. Comparison with Characteristics of Wavelet Filters

In the case when φ_m=const=45° we have the classical Haar filter.

- The length of IR of elementary filters (PAD, PDD, PAR, and PDR) is exactly equal to 2 in opposite to wavelet filters where the length can be also > 2.
- The characteristics (IR, magnitude) of the elementary filters are controllable using the parameters (angles).
- The principal difference is the existence of shelves in magnitudes for angles other than 45°. These shelves are located near the high frequencies for the PAD/PAR filter and near the low frequencies for the PDD/PDR filter. We cannot observe such shelves for the wavelet filters at all. This difference is caused by the presence of DC components in the basis function(s) (BFs) of RA-HT transforms, as opposed to wavelets lacking the DC components [17].

D. Basic Architecture of DeReF

The filter includes three main parts: the decomposition filter, the reconstruction filter and the signal processing module (see Fig. 3). The decomposition filter has a tree-like fan-out structure (see Fig. 4), bet the reconstruction filter – a fan-in structure.

There are infinite possibilities how to organize the signal processing module (SPM). The simplest SPM is described in [17] and called "Configuration Block" (CB). This block contains only programmable switches (one switch per fan-out signal wire). Each incoming signal can be fully rejected or passed through CB. In such a way, we can use DeReF for rejection or extraction of signals with shapes of RA-HT basis function(s) (BFs).

There are many possibilities how we can improve SPM. If we supplement CB switches, for example, by attenuators, we can perform more sophisticated signal processing.

III. Signal Processing Examples

We selected very simple and convincing examples. The used test scheme is shown in Fig. 6. The test pulse former is implemented as DeReF with a programmable CB.

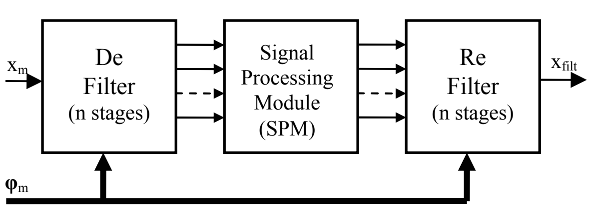

Fig. 3. Simplified block diagram of the DeReF with the processing module

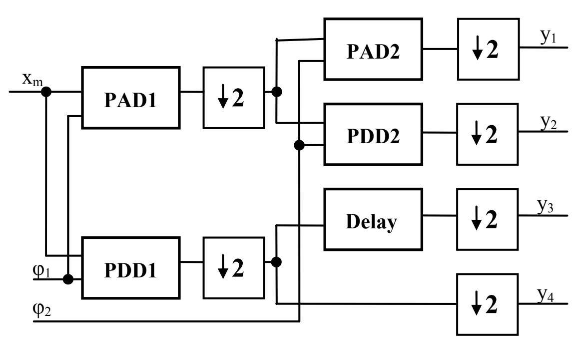

Fig. 4. Simplified block diagram of the decomposition filter (n=2)

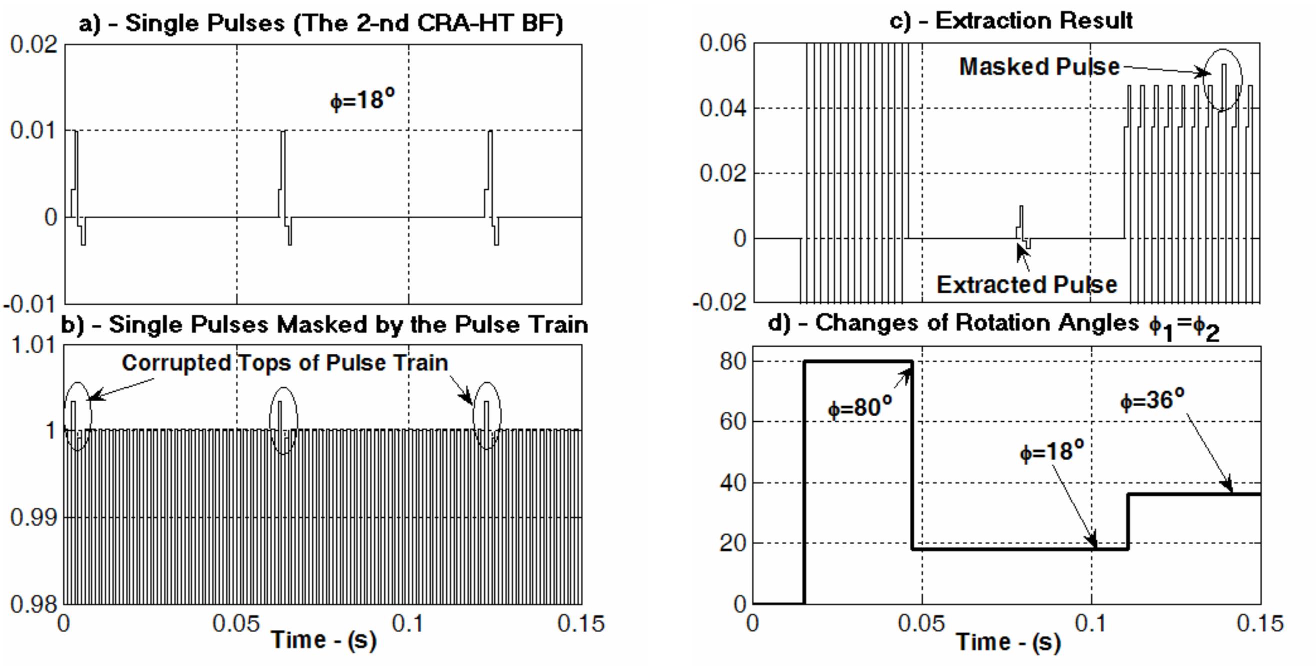

Fig. 5. Extraction of single pulses of 2-nd CRA-HT BF from the pulse train (SIMULINK simulation)

A. *Extraction of Pulse-Like Signal*

In [17] we demonstrated efficient extraction of CRA-HT (Constant Rotation Angle Haar Transform) BF from additive noise. In Fig. 5 we show how to perform a perfect extraction of single BF from the periodic pulse train. The amplitude of pulse does not exceed 1/100 of the amplitude of pulse train.

The same perfection can be achieved for almost any ratio of amplitudes (taking into account fixed-point error, of course). We see that the perfect extraction is possible when DeReF filter is tuned to the angle that has been used in the test pulse former. For other angles the filtered test pulse is corrupted by the damaged pulse train.

The ideal separation (extraction/rejection) of signals is possible for orthogonal signals. In this case the chosen signals are orthogonal, but pulses with angles ≠18° are not orthogonal to the selected pulse train.

B. *Rejection of Pulse-Like Signal*

The pulse former produces the fourth CRA-HT BF (N=16). The sine wave has been corrupted by the mentioned pulse shape. After filtering we have practically perfect sine. Fig. 8 shows the lower bound of Total Harmonic Distortion (THD) in dependence on the value of angle used for rejection of pulse. We see that the Haar filter gives **30000** (theoretically, ignoring fixed-point error) times worse result than a true filter (tuned to 38° for the example of corrupted sine in Fig. 7). The minimal value of lower bound is independent of the distortion amplitude ("shape resonance"). The THD bound values for other angles grows increasing the amplitude of distortion.

C. *Experimental Results*

The test scheme shown in Fig. 6 is implemented in Altera's FPGA (details see below). We use a RAM-based sine wave (16 samples) generator for the generation of basic signal and

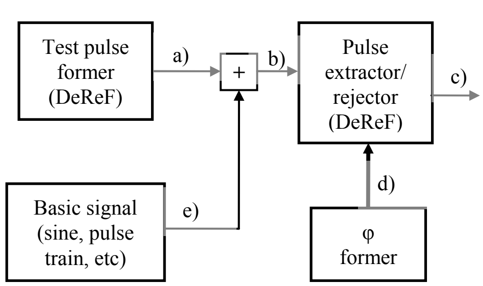

Fig. 6. Test scheme

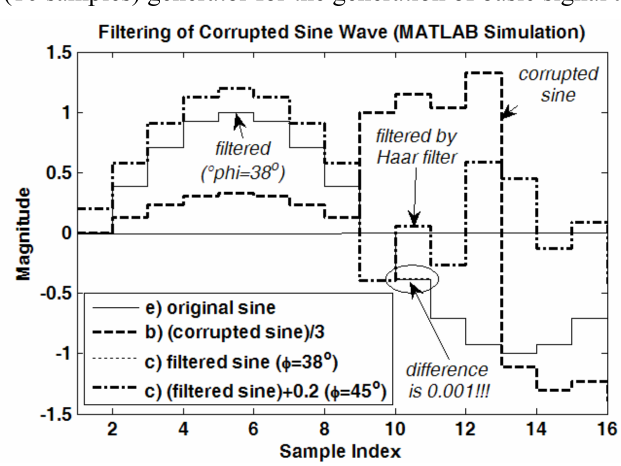

Fig. 7. Filtering of corrupted sine wave (16 samples). Simulation results

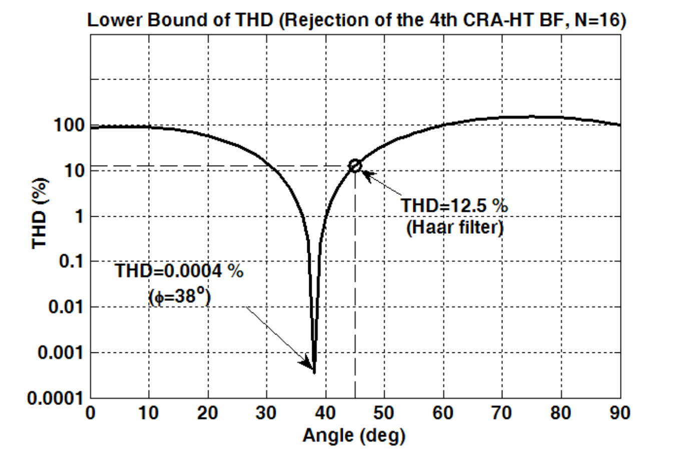

Fig. 8. Lower bound of THD for the filtered sine wave

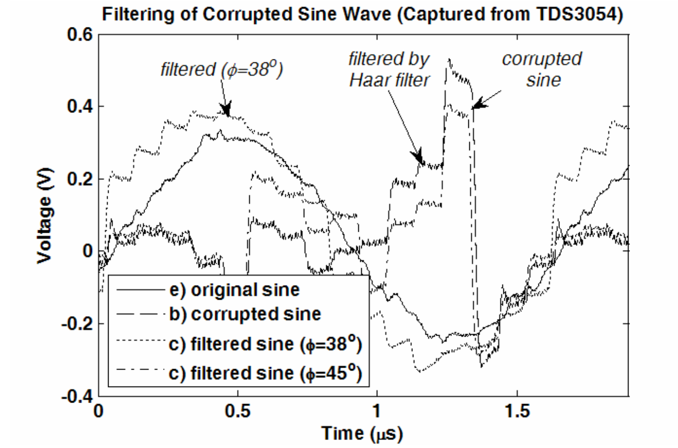

Fig. 9. Filtering of corrupted sine wave (16 samples). Experimental results

for the pulse-like signal former. In Fig. 9 we see dramatically corrupted sine wave. A 4-th CRA-HT BF (φ=38° in provided example) is used for distortion. After filtering we obtain quite acceptable result, but it is not perfect. It is mainly because of the sharpness of "shape resonance" and side effects (e.g., transformer on the output) of DAC. The filtering is very weak when the filter is tuned on Haar BF (φ=38°).

IV. FPGA Implementation of Filter

This first trial of real implementation of RA-HT DeReFs continues our work on the implementation of RA-HT devices. We expect that these filters will be used for the prototyping of next generation of filters.

A. Development Environment

The used environment is similar to the environment used for the development of the RA-HT signal spectrum analyzer-synthesizer module [18]. The *Altera's Quartus II v6.0* has been used as software design environment for the filters. The low-level design of the devices is coded in VHDL. We implemented the filter into *Altera's Cyclone II* FPGA. The *Altera's DSP Cyclone Development Kit* and the *DE2 Kit* have been used to develop hardware for the current versions of filters. For the digital-to-analog conversion of spectrum/signal has been used the *Texas Instrument* DAC *TIDAC904E* (on the *DSP Cyclone* kit board). We captured the DAC output signal using the *Tektronix TDS3054* oscilloscope. We used also *MATLAB* to import the captured shapes of signals from *TDS3054* to *MATLAB*.

B. Architecture of the DeReF

The architecture of DeReFs has been described above (see section II). Now comes the matters not mentioned before. The CORDIC rotator is the heart of Pxx blocks. One rotator is used per the pair PAD+PDD (one of decomposition stage). The reconstruction stage (PAR+PDR) needs also only one rotator.

The control of the filter is realized by simple clocks. For example, each next stage of decomposition filter uses the clock with frequency that is twice less than the frequency in the previous stage. In reconstruction stages frequencies are reversed – the lowest in the first stage, but the highest in the last stage.

We keep detailed description of angle control for next publications (mostly patents).

C. Comparison of Filter Versions

Two versions only will be viewed in this paper. In the first version of DeReF we used the following number of rotators per stage:

$$n_D = 2^m, \quad n_R = 2^{n-m+1}, \; m \in [1, n], \qquad n = \log_2(N) \tag{2}$$

where n_D – the number of rotators in m-th decomposition stage, n_R – the number of rotators in m-th reconstruction stage, N – the size of orthogonal matrix. This version of filter is extendable to other types of filters (not limited to RA-HT filters only. For example, CRAIMOT [19]).

For the simplest version one rotator only is used per filter stage. This version dominates in our experiments with different kind of RA-HT transforms. However, it has a low capability for further development of devices which does not use this particular kind of rotation angle-based transform.

See [18] for details. The simplest transform is a Constant Rotation Angle HT (CRA-HT) that uses only one angle. Then the filter can be controlled by this one angle only. But we have relatively poor diversity of pulse shapes for separation. A good compromise is the CRAIM-HT (Constant Rotation Angle in Matrix) filter that operates with one angle per stage.

D. Parameters of Filter

The filter module is mostly VHDL coded. The code for the simplest version of module contains approximately 1500 VHDL strings.

The table above summarizes some parameters of the module and consumption of hardware resources. The speed of the implemented module depends on the speed of CORDIC rotator. For the *Cyclone II EP2C35F672C6* chip the execution of 9-bit rotation requires approximately 70 ns. This time is practically independent of the clock because of the execution of rotation in hardware (alike for the SANSYN module [18]). The maximal sampling frequency for input/output signal does not exceed 12.5 MHz.

TABLE I
THE LIST OF PARAMETERS OF DERE FILTER

Parameter		*Value*
External clock used		f_c = 50 (100) MHz
Wordlength for sample values		w = 10 bits, Q1.x FPA
Sample time (processing time per stage		T_s= 70 ns
Maximal number of stages	version 1	$n = 3$
	version 2	$n=9$
Maximal number of angles	version 1	$n_\varphi=n*N/2=12$
	version 2	$n_\varphi=(2^{n+1}-2)=1022*$
	v.2, CRA-HT	$n_\varphi=1$
	v.2, CRAIM-HT	$n_\varphi=n$
	v.2, RSA-HT	$n_\varphi=2^{n-1}$
Number of logic elements	version 1	$700*(2^{n+2}-4)$
	version 2	$1400*n+15*(2^{n+1}-2)$

The number of maximal angles (in the RA-HT case) equal to 1022 (see table) is more theoretical than practical (because of complexity of service). The maximal number of stages for the decomposition/reconstruction part of filters is limited by the number of logic elements in the used chip.

At this moment the filter is tested with the number of stages ≤5 (N≤32).

V. CONCLUSIONS

- The present filter can be used as a very efficient device for the real-time extraction/rejection of pulse-like signals.
- The filter module can be adapted for a wide range of pulse shapes by simple change of parameters (angles).
- The module is tested mainly for CRA-HT and CRAIM-HT.
- We suppose that the CRAIM-HT is a trade-off between the diversity of pulse shapes (for separation) and the complexity of control parameters (angles).

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