# EVALUATION OF CABLE MATERIAL CONSUMTION OF TIES DEPENDING ON THE NODAL DISPLACEMENTS OF HIERARCHIC ROOF

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### Introduction

One of the most rational solutions concerning material consumption suggests the application of cable structures. Tensile stresses act at all cross-section points of the elements of cable structure, thus providing an efficient utilization of the cable material. It should be taken into account that these structures induce as well the application of new generation materials with increased strength.

In the previous work [7] the basic principles for modeling of hierarchic cable structures were defined. These structures consist of higher level part (top level cable structure and the bottom level tie net) and base standard elements - orthogonal anticlastic cable saddle (negative Gauss bend hyperbolic paraboloid saddle shape cable roof [1-5]). By using saddle shape structures with the flexible boundary cable as standard type elements for roofing of great areas, and by suspending their corners to higher level cable structure, a hierarchically intersubordinated cable roof is developed. The given span can even exceed 0,5 km. These structures inhere in the same advantages typical for separate saddle shape cable elements, however, with a better ratio between the roofed volume and the area.



*Fig.1* Simplified variant of hierarchic cable structures with vertical suspenders. \*Cable trusses for all standard elements have not been conventionally displayed

Fig.1 shows a simplified hierarchic cable roof variant with vertical suspenders with dimensions 80x80 m in plan. The dimension of each standard element is 20x20 m. Within the roof span, two top corners of each standard element are fastened to a top level cable structure, while the two bottom corners - to the bottom level tie net. Corners of the outer elements are supported by the bearings or columns, just like separate saddle shape roofing.

In accordance with the previous investigations [5], cable trusses between standard elements of the structure are used in order to decrease vertical displacements. For a better interpretation of the scheme, it should be noted that cable trusses near the standard elements conventionally are not shown for all elements in Fig.1. These systems with dimensions 80x80 m can be combined together to form wider roof areas.

## **Calculation of hierarchic cable structures**

Cable structures are characterized by large strain non-linearity. To solve such problems, the Finite Element Methods (FEM) software ANSYS [6] is used with utilization of the universal cable finite element, which has three degrees of freedom in each node with special bilinear stiffness matrix which defines that the element works on tension only.

To reduce the total amount and complexity of the calculation, and to avoid from the convergence problems, the so-called substructuring (subregion) method is used, which divides the structure into levels. In the first stage the displacements and reaction forces for one standard element - saddle shape roof with the compliant supporting contour - are calculated, thereby collecting information (boundary conditions) for the next stage calculations of the whole structure. At the second stage the calculation is made for the higher level cable structure according to the values of wind and accidental snow loads. The effect of interdependencies of separate structural elements and a higher level cable structure can be defined by using of iterative approximation method.

## **Results and discussion**

The previous investigations [7] showed, that the roof scheme with vertical pendants (Fig.2) is more preferable because the scheme with oblique suspenders has too long columns, which bring about too large displacements of top corners of the lower level standard elements, and too large horizontal forces due to the angle between the pendant and the vertical axis which may require the strengthening of the standard element. Resulting from the calculations, one can make sure that the structure has much smaller vertical displacements and internal forces, and horizontal forces in standard elements do not develop. Basing on this calculation, initial vertical displacements (uplifts) of the standard element supports can be obtained, and they are determined during the 2-nd stage iteration calculation. Such uplifts may be noticed during the assembling of structure in order to decrease deformations of the whole structure at full loading.



Fig.2. Scheme of top level cable structures with vertical suspenders. (----) - non-deformed position R - horizontal reaction,  $F_t$ - applied load from standard element, Uy - vertical displacements.

At the 3-d stage it is required to calculate the bottom level tie net. Since the method of structural uplifts cannot be applied for one plane net, i.e., it is impossible to entirely compensate vertical displacements of the bottom bearing nodes of standard elements. Investigations on determination of appropriate measures for decreasing eventual displacements as well as those on the maximum permissible displacements of the bearing nodes of standard elements were carried out, with the main limiting criterion being the presence of prestressing force in all stressing cables of standard elements (4th stage).

Extension of the cross-sections of cable A of the tie net with the corresponding maximum eventual increase of its force of prestressing is the main arrangement which enables to decrease displacements of nodes of the net from the plane (see Fig.3). To find the optimal bottom level net variant, three types of cables with main characteristics presented in Table 1 in accordance with the publications [8, 9] were calculated.

Table 1 Pro	perties of	variants	of the	e cables
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Properties	Unit	Steel cable	Hybrid	CFC cable
_			composite cable	BASF 1991
Specific gravity	kg/m <sup>3</sup>	7850	4040	1500
Tensile modulus	GPa	197	224	137
Tensile strength	MPa	1568	1613	1765
Design strength	MPa	813	896	1000



Fig.3 Scheme of bottom level tie net. (-----) - non-deformed position;  $F_b$  - applied load from standard element, Uy - vertical displacements.

To define the design values of tensile strength, safety factor for the material property which equals to 1,6 for steel, and 1,8 for CFC cables, due to insufficient investigation of the materials was applied in reserve of strength.

For hybrid composite cable, the design value of tensile stress was assumed resulting from ultimate elongation value  $\varepsilon = 0,4\%$ . Hybrid composite cable consists of one-way oriented carbon fibers with volume fraction amount of 0,6 of the cross-section of all the cable for producing low creep at tension, and 0,4 steel wire strands for increasing of breaking elongation. Volume fraction of each component was determined basing on the consumption, that in an emergency, when the strains exceed the limiting value for carbon fibers and these are disrupt, the steel wire must completely take care of tensile stresses, which are significantly decreased due to elongation of the cable.

Since values of tensile stresses  $\sigma_{sl}$  from the service load (bearing reactions directed upwards as concentrated forces of the standard element) make up a relatively small part of the summary stresses of the bottom level tie net (below 1,5%), and they tend to quickly decrease in accordance with the hyperbolic regularity of  $\sigma_{sl}=aA^{-l}$  type (when A > 0), we assume the value of prestressing  $\sigma_{istrain}=0.95R$  for all variants, i.e., by 5% less than the design value of tensile strength R of a cable in reserve of strength.



Fig.4 Dependence of maximum displacements of nodes of tie net on the material consumption per  $1m^2$  of the covered area

Results of the calculations which display the effect of changes of the cross-section of bottom level net cables (material consumption per  $1m^2$  of the covered area *C*) on the maximum displacements of bearing nodes of standard elements for the above-mentioned three variants of cables are shown in Fig.4.

The obtained results show that the application of hybrid cables enables to decrease maximum displacements of the bottom level tie net by more than two times, but that of CFCC - more than three times, in comparison with standard steel cable with identical material consumption per  $1 \text{ m}^2$  owing to a better strength to weight ratio. There was also defined the dependence of maximum displacements Uy on cross-section values of cables (material consumption per  $1 \text{ m}^2$  of the covered area C). It is hyperbolically shaped like  $Uy=bC^1$  (when C>0 and b - coefficient, which depends on the properties of the material and value of prestressing). This dependence makes it possible to define the trendline equation by two experimental points, and in this way considerably decrease the number of numerical experiments. The interpretation of these relations for both logarithmic axis scales is shaped as parallel straight lines (Fig.5).



Fig.5 Dependence of the material consumption per  $1m^2$  of the covered area on maximum displacements of nodes of tie net in logarithmic axis

It can be concluded that calculation results of the 4-th stage concerning the value of maximum permissible displacements of bearings of standard element must make up less than L/300 in the upward direction to the top level cable structure. The calculation was made by prestressing of all the cables of standard element by equal forces, and the absence of slack was the criterion in all stressing cables of standard element during simultaneous action of the design loads. It should be thereby observed that displacement of the bottom bearing nodes of standard elements causes in its turn a certain vertical displacement of the top bearing nodes to a higher level (upwards), which is the result of decrease of the values of bearing reactions due to approach of the bottom bearing nodes. It operates in reserve of load bearing capacity, and in the 1-st round of calculations was not taken into account. More precise, approximated to reality values of displacements can be obtained in the 2-nd round of iteration by using the corrected boundary conditions for all elements of the whole hierarchic cable structure.

It should be mentioned that the limiting values of displacements of the bottom bearings of standard elements can be increased, and, consequently, can decrease the material consumption for the bottom level net by replacing of steel stressing cables of standard elements for polymer made

cables with higher than steel values of ultimate elongation (within the limits of 15 to 20%), such as high performance polyethylene fibers.

# Conclusions

Hierarchic cable structures are suitable for long span roofs in areas with insignificant snow load, as well as for completely or partially dismountable provisional coverings.

Application of the available software enables to calculate forces and displacements for complicated intersubordinated hierarchic cable structures, and to define the effect of separate structural elements on the remaining structural units.

In accordance with the results of calculation, it is evident that during the erection of the given type of structure it is necessary to develop the algorithm of erection and prestressing of the cables. Basically, the greatest attention should be concentrated on the preciseness of the internal force values of prestressing.

Also was defined the hyperbolically shaped dependence like  $Uy=bC^{-1}$  of maximum displacements Uy on material consumption per 1 m<sup>2</sup> of the covered area C. The obtained results show that the application of hybrid cables enables to decrease maximum displacements of the bottom level tie net by more than two times, but that of CFCC - more than three times, in comparison with standard steel cable with identical material consumption per 1m<sup>2</sup>.

The calculations of nodal displacements demonstrate that erection of the given type of structure will be complicated by using of steel cables. There is a requirement that each part of the structure should be made of cables with specific properties, which can only be provided by advanced composite materials.

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# Pakrastiņš L., Rocēns K. Savilču materiāla patēriņa novērtēšana atkarībā no mezglu pārvietojumiem hierarhiskā vanšu pārsegumā

Aplākoti hierarhisko vanšu pārsegumu veidošanas un aprēķina pamatprincipi. Šīm konstrukcijām piemīt visas priekšrocības, kas ir atsevišķam sedlveida vanšu elementam, bet ir labāka attiecība starp pārsegto tilpumu un laukumu. Lai samazinātu kopējo aprēķina apjomu un komplicētību, kā arī izvairīties no uzdevuma konverģences problēmām, pārvietojumu un piepūlu aprēķinam izmanto apakšreģionu metodi. Konstrukcijas elementu savstarpējas pakārtotības efektu ievēro izmantojot pakāpeniskās tuvināšanas metodi.

# Pakrastinsh L., Rocens K. Evaluation of Cable Material Consumption of Ties depending on the Nodal Displacements of Hierarchic Roof.

The creation and calculation principles of the hierarchic cable roofs have been considered. For these structures all advantages typical for a saddle shape cable roof can be applied, however, with a better correlation between the covered volume and the area. To reduce the complexity and the amount of calculation, and to avoid the calculation matrix convergence problem, calculation of stress and displacements of complicated hierarchically subordinated structures can be made by the method of substructuring. The effect of interdependencies of structural elements can be defined by application of iterative approximation method.

#### Пакрастиньш Л., Роценс К. Оценка расхода материала вант затяжек в зависимости от перемешений узлов иерархического вантового покрытия.

Рассматриваются принципы образования и расчета иерархических вантовых покрытий. Для подобных конструкций характерны те же преимущества, которые характерны для седловидных вантовых конструкций, однако, с лучшим соотношением между перекрываемыми объемом и площадью. Для того, чтобы уменьшить объем задачи и предотвратить проблемы сходимости матрицы вычисления, расчет усилий и перемещений в сложной иерархически подчиненной конструкции может быть выполнен с использованием метода подструктурирования. Эффект взаимозависимости элементов конструкции может быть определен используя метод последовательных приближений.