CHARACTERISATION OF ADVANCED COMPOSITE MATERIAL PROPERTIES BY INVERSE TECHNIQUE

MODERNO OGLEKĻŠĶIEDRAS KOMPOZĪTMATERIĀLA ELASTĪGO ĪPAŠĪBU NOTEIKŠANA IZMANTOJOT INVERSO METODI

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Introduction

Advanced composite laminates are being extensively used in aerospace industry, especially for the fabrication of high-performance structures. The determination of stiffness parameters for complex materials such as fiber-reinforced composites is much more complicated that for isotropic materials. A conventional way is testing the coupon specimens, which are manufactured by the technology similar to that used for the real large structures. In employing such a method, a question arises of whether the material properties obtained from the coupon tests are the same as those in the large structure. Therefore, the determination of realized material properties for composite laminates using non-destructive evaluation techniques has been widely investigated.

A number of various non-destructive evaluation techniques have been proposed for determination of the material properties of composite laminates [1-3]. In the present study, attention is focused on the identification of elastic properties of laminated plates using the vibration test data. The modal vibration testing is a rapid and inexpensive method for obtaining data for the identification of elastic properties [4]. There is a great deal of information in the literature on the identification of elastic constants of laminated plates employing the vibration test data [5-13]. The problem associated with the vibration testing is converting the measured modal frequencies to elastic constants. A standard method for solving this problem is the use of a numerical-experimental model and optimization techniques [5-6, 9-12]. The identification function represents the discrepancy between the numerical model response and the experimental one. This discrepancy should be minimized taking into account the side constraints on the design variables (elastic constants). The minimization problem is solved by using the non-linear mathematical programming techniques and sensitivity analysis [6, 9-12]. A similar identification function has been employed in [14, 15], but the minimization method was different. Instead of the direct minimization of the function, the experimental design and the response surface approach are employed for approximation of the numerical (finite element) model. Such an approach can reduce the computational efforts significantly.

In order to reduce the computational efforts, the methods based on approximation concepts were used in the structural optimization for the first time [16]. The development of approximation functions has become a separate problem in the optimum structural design [17]. Approximating models can be built in different ways. The empirical model building theory is discussed in [18]. To construct a more general model of the original function, the methods of experimental design [19, 20] and approximate model building [21-23] can be employed. A simplified model, called “metamodel” [24], is elaborated using results of the numerical experiment on sample points of the experimental design. The response analysis using the simplified model is computationally much cheaper than the solution employing the original model.

In the present study, the identification of elastic properties of laminated composite plates from the measured eigenfrequencies is carried out. Three unidirectionally reinforced and three multidirectionally reinforced plates were tested for vibration, measuring the eigenfrequencies and the corresponding eigenmodes. Using the measured vibration data the identification of material properties was performed.

Parameters of identification and criterion

The numerical-experimental method proposed in the present investigation consists of the following stages (see *Figure 1*). In the first stage the physical experiments have been performed. Also the parameters to be identified, the domain of search and criterion containing experimental data have been selected. In the second stage the finite element method has been used in order to model the frequency response of the structure. The results of the finite element solution of the eigenvalue problem have been employed as numerical experimental data. The finite element calculations have been performed in the reference points of the variables to be identified. The experiment design points have been determined using the method of design of experiments. In the third stage the numerical data obtained by the finite element solution in the reference points have been used in order to determine a simple functions using response surface method for calculation of the eigenfrequencies. In the fourth stage on the basis of the simple models and experimental data of the measured eigenfrequencies the identification of the material properties is performed. For this a corresponding functional is minimized using a conventional method of non-linear programming.

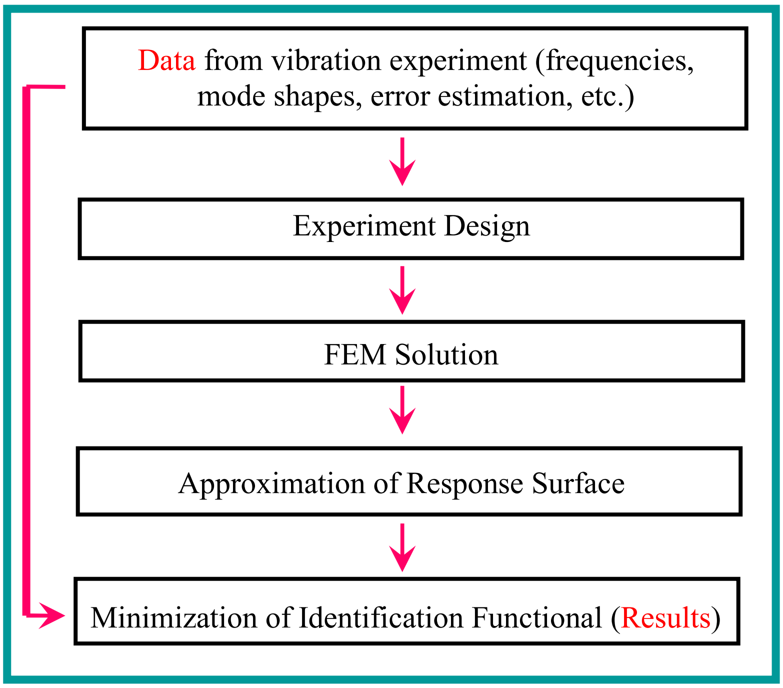

Fig.1. *Identification procedure*

Parameters of identification

The proposed numerical-experimental approach is used for identification of the elastic properties of laminated composite plates. For this the experimental data of the measured eigenfrequencies have been used. It is assumed that the plate dimensions (see *Figure 2*), plate mass and the layer stacking sequence is to be known. The plate is composed of transverse isotropic layers with principle material directions 1-2-3, where 1 is the fiber direction and 2,3 are the directions transverse to the fiber direction. In general the i^{th} layer of the laminated plate is oriented at an arbitrary angle β_i. The angles of the layers assumed to be fixed.

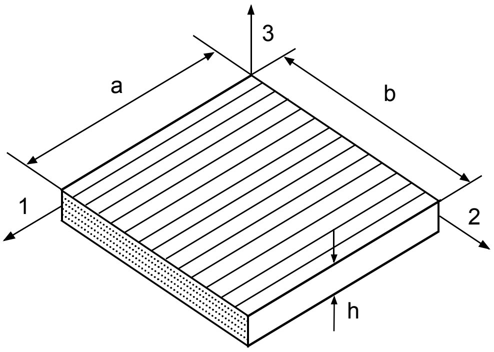

Fig.2. *Unidirectional reinforced laminated composite plate.*

The parameters to be identified are the elastic constants of a single layer in the laminated composite plate. These five parameters of a transverse isotropic layer are

E_1, $E_2 = E_3$	- two Young's modulus,
$G_{12} = G_{13}$	- shear modulus,
$\nu_{12} = \nu_{13}$	- Poisson's ratio,
G_{23}	- shear modulus in the plane of isotropy.

The vector of parameters $\boldsymbol{x}$ to be identified can be chosen in a different ways [15]. The major problem in parameter estimation is ill condition caused by unknown variables having substantially different order of magnitude. For example, Young's modulus and Poisson's ratio cannot be directly selected as components of vector $\boldsymbol{x}$ without proper scaling. In Frederiksen [18] the scaling by longitudinal modulus E_1 was employed and in addition a fixed scaling factor was chosen. Similar scaling and reparametrisation was employed in [15], where additional scaling by the first experimental frequency allows reducing the number of unknown variables from five to four. Thus, material parameters of single layer can be expressed in terms of dimensionless variables α_I [6]

$$\begin{aligned}
\alpha_2 &= 4 - 4(E_2/E_1) \\
\alpha_3 &= 1 + (E_2/E_1)(1 - 2\nu_{12}) - 4(G_{12}/E_1)\alpha_0 \\
\alpha_4 &= 1 + (E_2/E_1)(1 + 6\nu_{12}) - 4(G_{12}/E_1)\alpha_0 \\
\alpha_5 &= 4(G_{23} + G_{12})\alpha_0/E_1
\end{aligned} \tag{1}$$

Here

$$\alpha_0 = 1 - \nu_{12}^2 \frac{E_2}{E_1} \qquad (2)$$

Now the parameters to be identified $\boldsymbol{x}$ are defined through non-dimensional quantities α_i

$$\boldsymbol{x} = [x_1, x_2, x_3, x_4] = [\alpha_2, \alpha_3, \alpha_4, \alpha_5] \qquad (3)$$

Let the experimental eigenfrequencies be designated by $\overline{\varpi}_1, \overline{\varpi}_2, ..., \overline{\varpi}_I$, where I is the number of measured eigenfrequencies $\bar{f}_i$ ($\overline{\varpi}_i = 2\pi \bar{f}_i$). The value of I is typically taken between 7 and 15. The corresponding numerical eigenfrequencies f_i for the set of given material parameters α_i are represented by $\varpi_1, \varpi_2, ..., \varpi_I$. Let us consider the scaling parameter C, which is chosen through the relation

$$C = \frac{\overline{\varpi}_1^2}{\widetilde{\varpi}_1^2} \qquad (4)$$

where $\widetilde{\varpi}_1 = C\varpi_1 = 2\pi C f_1$ is the first numerical eigenfrequency calculated with the prior selected longitudinal Young's modulus E_1^0.

Identification functional and minimization problem

The identification process is carried out through minimization of an error function that expresses the relative difference between the measured $\overline{\varpi}$ and numerically calculated $\widetilde{\varpi}$ frequencies [6].

$$\Phi(\boldsymbol{x}_{(1)}) = \sum_{i=2}^{I} w_i^{(1)} \frac{\left(\overline{\varpi}_i^2 - C[\widetilde{\varpi}_i(\boldsymbol{x})]^2\right)^2}{\overline{\varpi}_i^4} \qquad (6)$$

It is seen that criterion (6) is a non-linear function of the parameters of identification $\boldsymbol{x}$. The identification of the material properties $\boldsymbol{x}$ is performed on the basis of information obtained by the measurements of the I lowest frequencies. The identification problem is formulated as follows [6]

$$\min_{\boldsymbol{x}_{(1)}} \Phi(\boldsymbol{x}_{(1)}) \qquad (7)$$

Subject to

$$g_1(\boldsymbol{x}) = \alpha_2 > 0 \quad \text{or} \quad E_1 / E_2 > 1 \qquad (8)$$

$$g_2(\boldsymbol{x}) = \frac{(8 - \alpha_2 - 3\alpha_3 - \alpha_4)}{16\left\{1 - \left[\dfrac{\alpha_4 - \alpha_3}{8 - 2\alpha_2}\right]^2 \left(\dfrac{4 - \alpha_2}{4}\right)\right\}} > 0 \quad \text{or} \quad G_{12} / E_1 > 0 \qquad (9)$$

$$g_3(\boldsymbol{x}) = \frac{[2\alpha_5 - 1/2(8 - \alpha_2 - 3\alpha_3 - \alpha_4)]}{8\left\{1 - \left[\dfrac{\alpha_4 - \alpha_3}{8 - 2\alpha_2}\right]^2 \left(\dfrac{4 - \alpha_2}{4}\right)\right\}} > 0 \quad \text{or} \quad G_{23} / E_1 > 0 \qquad (10)$$

$$g_4(\boldsymbol{x}) = -\left|\frac{(\alpha_4 - \alpha_3)}{(8 - 2\alpha_2)}\right| + \sqrt{\frac{4}{4 - \alpha_2}} > 0 \quad \text{or} \quad \sqrt{E_1 / E_2} - |\nu_{12}| > 0 \qquad (11)$$

$$\alpha_i^{\min} \le \alpha_i \le \alpha_i^{\max}; \quad i = 2,3,4,5 \tag{12}$$

Here $\alpha_i^{\min}$, $\alpha_i^{\max}$ are the lower and upper side constraints, respectively. The upper and the lower limits of the parameters of identification are chosen different for each numerical example of identification.

After the evaluation of the optimum values for the non-dimensional material parameters α_i (i=2, 3, 4, 5) that satisfy equation (7) and constraints (8)-(12), the value of the Young's modulus of the layer in the fiber direction E_1 can be evaluated easily, since C and α_0 are known. The steps of this evaluation are shown below.

Finite element solution

The eigenvalue problem for the harmonic vibrations can be represented by

$$\mathbf{Ku} = \omega^2 \mathbf{Mu} \tag{13}$$

Here $\mathbf{K}$ is the stiffness matrix of the plate, $\mathbf{M}$ is the mass matrix of the plate and $\mathbf{u}$ is the displacement vector. The eigenvalue relation (13) for mode $\mathbf{u}_1$, which corresponds to the first experimental eigenfrequency $\overline{\omega}_1$ can be written in an equivalent form putting E_1 in evidence

$$E_1 \mathbf{K}^* \mathbf{u_1} = \overline{\omega}_1^2 \mathbf{Mu_1} \tag{14}$$

Here $E_1 \mathbf{K}^* = \mathbf{K}$ is the stiffness matrix. Taking into account the relation (4) this equation can be written as

$$C E_1^0 \mathbf{K}^* \mathbf{u_1} = C \widetilde{\omega}_1^2 \mathbf{Mu_1} \tag{15}$$

hence

$$E_1 = C E_1^0 \tag{16}$$

where E_1^0 is the initial guess value given to the Young's modulus in the fiber direction of the layer and E_1 is the corresponding identified mechanical property. After the evaluation of the optimum values of $\boldsymbol{x}$ the remaining mechanical properties are then calculated through the inverse relations (1).

Method of Experiment Design

For the direct identification of the elastic properties of the material it is necessary to perform multiply iterated finite element solutions. Such direct procedure has been used by Mota Soares *et al*. [6]. On each iterative stage the eigenvalue problem for the linear system has been solved by the finite element method and the non-linear programming algorithm was applied in order to minimize directly the identification functional (6). Such procedure takes very large computational efforts. Instead of direct minimization of the criterion (6) the method of planning of experiments can be used.

Initial information for elaboration of the experiment plan is the number of variables n and the number of experiments k. The main principles in the proposed approach are as follows [25]

1) the number of levels in the domain of experiments for each variable is equal to the number of the experiments and for each level only one experiment is performed;

2) the reference points (points of experiments) in the domain of experiments are distributed as regular as possible.

The plan of experiment is characterized by the matrix of the plan B_{ij}. Such matrices were calculated by the program PLANEX [25] for number of variables n = 2, 3, ..., 15 and for the number of experiments k = 2, 3, ..., 25. For example, the plan of experiment with the 36 reference points (k = 36) and two variables (n = 2) is presented in *Figure 3.*

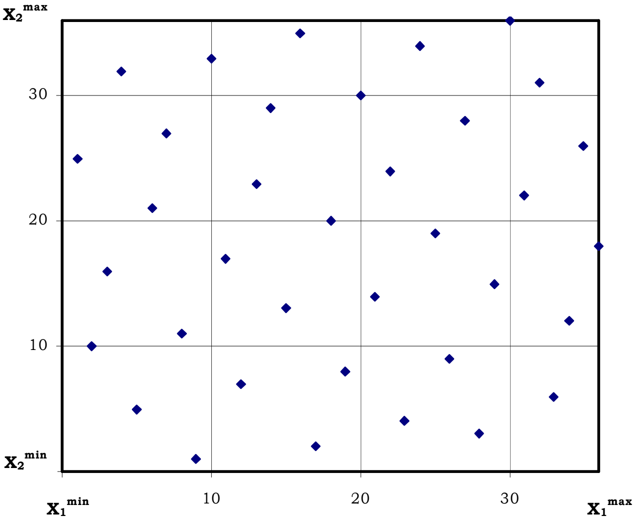

Fig.3. *Plan of experiment for two variables (n=2) and 36 reference points (k=36)*

The reference points are distributed in the domain of interest - $x_j \in \lfloor x_j^{min}; x_j^{max} \rfloor$, which is formed by the lower and upper limits of variables (side constraints). Limits for the variables are chosen by using the initial guess values of elastic constants. These values can be taken from the properties of a similar material or from the static test of the present material. Thus, in this domain the reference points, where the experiments must be performed, are calculated by the expression

$$x_j^{(i)} = x_j^{min} + \frac{1}{k-1}\left(x_j^{max} - x_j^{min}\right)\left(B_{ij} - 1\right) \qquad (17)$$

Here i = 1, 2,..., k and j = 1, 2,..., n.
The details of this procedure and corresponding program PLANEX were described in [25].

Approximation of Response Surface

Information about the behaviour of the object can be obtained by the physical experiment or by the computer solution in the reference points. This information can be represented as a table of data, where the response function $y(\mathbf{x})$ of the object is to be in relationship to the variables $x_1, x_2, ..., x_n$. The goal is by using the data of experiments (in our case data are obtained by the Finite Element solution in the reference points) to obtain the relation $y(\mathbf{x})$ in the mathematical form or so called equation of regression. The details of this procedure and corresponding program RESINT were described in [25].

Vibration Experiment

Plates were tested for vibration in order to measure the eigenfrequencies and the corresponding modes. The natural frequencies of the test plates were measured by a vibrograph [charge-coupled device (CCD) camera] using shearography technique. The shearography employs a single expanded beam of laser light which is reflected back from the specimen to the CCD camera. The camera includes an image shearing device, which brings two separate points of the object surface to meet in the image plane. The two overlapped portions of the sheared images interfere and produce a speckle pattern. When the object is deformed, the speckle pattern is slightly modified. A comparison of the two (stressed and unstressed) speckle patterns produces a fringe pattern which depicts the relative displacement of two neighbouring points. Since the magnitude of shearing is small, the fringe pattern approximately represents the first derivative of displacement with respect to the shearing direction, which may be either in-plane or out-of-plane. The experiments are performed under free boundary conditions on all edges of the plate so that to exclude the influence of boundary conditions on the results of identification. The specimens are hung by two corners using a band simulating free boundary conditions along the edges of the plate (see *Figure 4*). The plate is excited by a piezo-ceramic disc bonded to it. The excitation with small piezo-ceramic discs works via the radial expansion of the disc causing a bending moment to the plate surface. The piezo-ceramic disc is connected to an amplifier and the frequency is varied by a frequency generator. To enable a better scanning, the specimens are painted in white. A typical test procedure is as follows: the plate is excited continuously, and the laser measures its response. Then the experimental results are compared with the predicted frequencies, which are calculated by the finite element code employing the initial guess values of elastic constants. Such preliminary finite element calculations are necessary to be sure that all experimental frequencies are recorded in the range. Since not all the frequencies are observed experimentally, they are ranged according to the finite element solution.

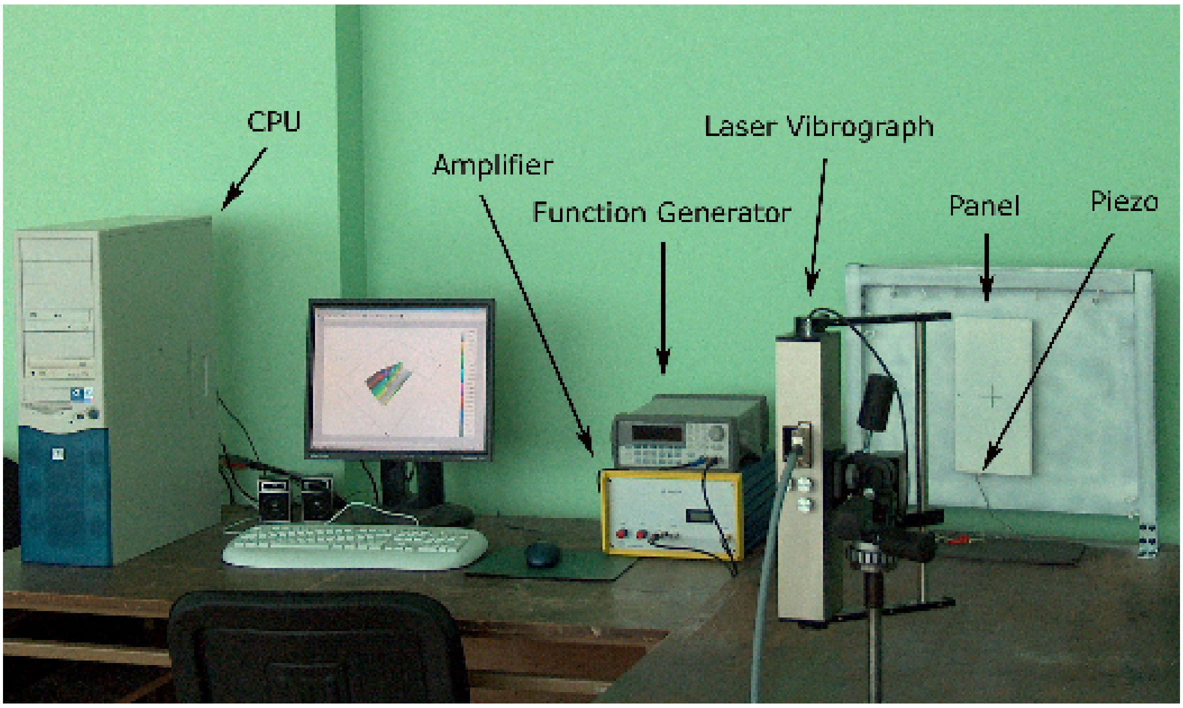

Fig. 4. *Vibration experiment of a one-stringer curved plate*

Identification examples

Example 1

In order to verify identification procedure it was proposed to test it on aluminium plates. For the identification two isotropic aluminium plates were tested. Geometric dimensions and density ρ of all plates are presented in *Table 1.*

Table 1.

Geometric dimensions and density of plates for vibration tests

Specimen	**SP1**	**SP2**
***a*, mm**	300	300
***b*, mm**	300	200
***h*, mm**	2,3	2,0
***ρ*, kg/m3**	2800	2685

The experimentally measured frequencies $f_i^{\exp}$ for plates are presented in *Table 2*. Since all frequencies experimentally were not observed, frequencies are ranged according to the finite element solution employing the initial guess values of elastic constants obtained from the static tests.

Table 2.

Experimental frequencies for the plates (Hz)

№	**SP1**	**SP2**
1	83	103
2	118	111
3	154	239
4	212	266
5	215	299
6	379	361
7	384	446
8	390	519
9	428	640
10	486	703
11	636	
12	655	
13	734	
14	761	
15	818	
16	285	
17	948	

The experiment design with two variables and 35 reference points was selected. The upper and lower bounds (domain of interest) of elastic constants for the plates were taken as follows

Table 3.

The domain of interest for identification

Property	***min***	***max***
E, GPa	60	80
G, GPa	22	30

Using the plans of experiment with 35 reference points for each plate in these reference points the finite element solution for about 20 first frequencies was obtained. Employing these numerical values of original functions the approximating functions (response surfaces) for all frequencies were obtained using the program RESINT [25]. Minimization of identification functional has been performed employing the standard programs for constrained minimization by using the *Visual Fortran Optimisation Toolbox*. By minimizing functional (6), elastic constants **x** are obtained. The identification results are given in *Table 4*.

Table 4.

Results of identification for aluminum plates

Property	**SP1**	**SP2**
E_1, GPa	70.85	64.26
E_2, GPa	26.64	22.16

Table 5.

Experimental, numerical frequencies and residuals for the plates **SP1** and **SP2**

No.	**SP1**			**SP2**		
	Exp (Hz)	FEM (Hz)	Δ_I (%)	Exp (Hz)	FEM (Hz)	Δ_I (%)
1	83*	83	0.00	103*	103	0,0
2	120*	120	0.00	111*	112	-0,9
3	154*	153	0.65	239*	240	-0,4
4	214	215	-0.33	266*	265	0,4
5	216	215	0.60	299*	299	0,0
6	381	382	-0.14	361*	358	0,8
7	383	382	0.39	446*	447	-0,2
8	394*	394	0.00	519*	515	0,8
9	428*	428	0.00	640*	639	0,2
10	485*	482	0.62	703*	711	-1,1
11	649	654	-0.72			
12	657	654	0.50			
13	721*	730	-1.25			
14	764*	767	-0.39			
15	808	820	-1.44			
16	821	820	0.17			
17	953*	949	0.42			

* these frequencies were taken into account in identification functional

The results obtained were verified by comparing the experimentally measured eigenfrequencies with the numerical ones obtained by FEM at the point of optima (using the identified elastic properties). The residuals Δ_i are calculated by the expression

$$\Delta_i = \frac{\left| f_i^{FEM}(\boldsymbol{x}^*) - f_i^{\exp} \right|}{f_i^{\exp}} \times 100 \tag{18}$$

Here **x*** is vector of identified parameters.
Results of verification for the plates are shown in *Table 5.*

It is seen from the comparison of the results presented in *Table 5* that the frequencies calculated by the finite element method using the elastic properties obtained trough identification procedure are in good agreement with the experiment. The difference in terms of residuals is mainly less than 1%.

Example 2

Identification of elastic constants of carbon/epoxy laminated composite plates from the measured eigenfrequencies was carried out. For the identification six carbon/epoxy plates, three uni-directionally [**ud**] reinforced and three multi-directionally [**md**] reinforced plates, were manufactured and tested. Fiber volume content of the present carbon/epoxy laminates is about 60%. Geometric dimensions and density ρ of all plates are presented in *Table 1.*

Table 1.

Geometric dimensions and density of plates for vibration tests

Plate No.	***a*=*b*, mm**	***h*, mm**	**ρ, kg/m^3**
1. **ud1** $(0)_8$	210	1.0	1621
2. **ud2** $(0)_8$	200	1.0	1620
3. **ud3** $(0)_{18}$	200	2.25	1638
4. **md1** $(0/90/45/-45)_{s2}$	210	2.0	1619
5. **md2** $(0/90/45/-45)_s$	210	1.0	1621
6. **md3** $(0/90/45/-45)_s$	210	1.0	1614

The experimentally measured frequencies $f_i^{\exp}$ for all 6 plates are presented in *Table 2*. Since all frequencies experimentally were not observed, frequencies are ranged according to the finite element solution employing the initial guess values of elastic constants obtained from the static tests.
The experiment design with four variables and 35 reference points was selected. The upper and lower bounds (domain of interest) of elastic constants for the plates were taken as follows

Table 2.

Experimental frequencies for the plates (Hz)

No	**ud1**	**ud2**	**ud3**	**md1**	**md2**	**md3**
1	37.8	54.8	113.4	164.0	58.4	59.0
2	52.5	58.4	139.0	257.0	125.0	125.0
3	94.6	123.4	272.5	326.0	174.0	175.0
4	146.6	162.5		421.0	176.0	178.0
5	184.2	231.0	524.5	453.0	204.0	205.0
6	218.7	238.0	556.0		303.0	306.0
7	233.6	256.6	595.0	797.0	360.0	363.0
8	280.8	308.0		848.0	408.0	412.0
9		328.0	780.0	904.0	483.0	484.0
10	334.4	389.6	899.0	1021.0	506.0	509.0
11				1325.0	517.0	521.0
12		525.0	1252.0		614.0	619.0
13				1530.0	699.0	702.0
14	520.0		1396.0			
15	604.7	665.0		1692.0	795.0	
16	622.3	683.0	1574.0	1810.0		
17		745.0	1689.0	1878.0		
18	675.0				939.0	943.0

Table 3.

The domain of interest for identification

	ud1		**ud2**		**ud3**		**md1**		**md2**		**md3**	
	min	*max*	*min*	*max*	*min*	*max*	*min*	*max*	*min*	*max*	*min*	*max*
E_1^0	145		140		150		150		135		135	
E_2	8.0	10.0	8.0	10.0	8.5	10.5	7.5	9.5	5.0	5.0	5.0	7.0
G_{12}	3.5	4.5	6.5	7.5	5.5	6.5	5.0	6.5	5.5	5.5	5.5	6.8
G_{23}	4.0	8.0	4.0	8.0	4.0	8.0	5.0	10.0	5.0	5.0	5.0	9.0
ν_{12}	0.25	0.45	0.25	0.45	0.25	0.45	0.25	0.45	0.25	0.25	0.25	0.45

Using the plans of experiment with 35 reference points for each plate in these reference points the finite element solution for 20 first frequencies was obtained. Employing these numerical values of original functions the approximating functions (response surfaces) for all frequencies were obtained using the program RESINT [25]. For example, the approximating function (correlation c=98.0%) for the third frequency of the plate ud1 is given by

$f_3(x)= +94.15-4.296^* z_2-2.822^* z_1-3.084^* z_3+.2410^* z_4$

where normalized variables z_i were introduced

$z_1 =-189.00+50.000^*x_1$
$z_2 =-46.399+50.891^*x_2$
$z_3 =-22.922+21.692^* x_3$
$z_4 =-2.7126+10.764^* x_4$

Similar approximating functions were obtained also for other frequencies. These approximating functions are employed in identification functional instead of original functions (calculation by FEM). Minimization of identification functional has been performed employing the standard programs for constrained minimization by using the *Visual Fortran Optimisation Toolbox.* By minimizing functional (6), elastic constants **x** are obtained. It should be noted that the number of frequencies, which are selected for identification, is different for each specimen. The experimentally measured frequencies, presented in *Table 2*, can be used for identification in any combination. A cross validation for all sample points was performed so that to achieve a better approximation of the original function and to select the most important (most sensitive to elastic constants) and reliable frequencies. The identification results are given in *Table 4.*

Table 4.

Results of identification for plates

Properties	**ud1**	**ud2**	**ud3**	**md1**	**md2**	**md3**	**Average**
E_1, GPa	144.80	139.75	152.15	153.17	136.70	137.90	144.08
E_2, GPa	8.36	8.31	9.72	8.16	5.90	5.97	7.74
G_{12}=G_{13}, GPa	4.02	7.14	6.04	5.79	6.14	6.33	5.91
G_{23}, GPa	5.69	5.71	6.46	7.84*	7.25*	7.43*	5.95
ν_{12}	0.39*	0.33	0.29	0.27	0.25**	0.25**	0.297

* upper limit of the preliminary selected domain of interest
** lower limit of the selected domain of interest

The results obtained were verified by comparing the experimentally measured eigenfrequencies with the numerical ones obtained by FEM at the point of optima (using the identified elastic properties). The residuals Δ_i are calculated by the expression

$$\Delta_i = \frac{\left| f_i^{FEM}(\boldsymbol{x}^*) - f_i^{\exp} \right|}{f_i^{\exp}} \times 100 \qquad (18)$$

Here **x*** is vector of identified parameters.
Results of verification for the plates are shown in *Tables 5-6.*

Table 5.

Experimental, numerical frequencies and residuals for the plate **ud1** and **ud2**

No.	ud1			ud2		
	Exp (Hz)	FEM (Hz)	Δ_I (%)	Exp (Hz)	FEM (Hz)	Δ_I (%)
1	37.8*	37.8	0.0	54.8*	54.8	0.0
2	52.5*	52.8	0.6	58.4*	58.1	0.5
3	94.6*	94.2	0.4	123.4*	126.6	2.6
4	146.6*	145.6	0.7	162.5*	160.2	1.4
5	184.2*	188.1	2.1	231.0*	234.0	1.3
6	218.7*	219.9	0.5	238.0*	238.5	0.2
7	233.6*	232.9	0.3	256.6	263.0	2.5
8	280.8	275.3	2.0	308.0*	314.4	2.1
9		287.5		328.0	335.3	2.2
10	334.4*	328.6	1.7	389.6*	389.4	0.1
11		360.4			451.5	
12		475.1		525.0*	522.8	0.4
13		493.1			596.8	
14	520.0*	518.8	0.2		617.3	
15	604.7*	604.5	0.0	665.0	656.4	1.3
16	622.3*	615.9	1.0	683.0	678.0	0.7
17		652.8		745.0*	742.8	0.3
18	675.0*	679.9	0.7		784.6	
19		713.1		829.0*	831.4	0.3
20		726.1			857.9	

* these frequencies were taken into account in identification functional

It is seen that for uni-directionally reinforced composite plates residuals are small, even for frequencies which were not used in identification. For multi-directionally reinforced composite plates residuals are greater which could be due to deviations from the nominal values (layer thickness, layer angle, fracture volume).

In order to validate results obtained from the vibration tests through identification it is necessary to compare elastic properties obtained from an independent test. Conventional static test was selected as independent test. Static tests were performed according to ASTM guidelines Test results are presented in *Table 7*. In parentheses the values obtained by the static compression test are given. In general good agreement of the results is observed. However, it is open whether the transverse shear modulus G_{23} could be reliably determined from the present vibration test.

Table 6.

Experimental, numerical frequencies and residuals for the plate **ud3** and **md1**

No.	**ud3**			**md1**		
	Exp (Hz)	FEM (Hz)	Δ_I (%)	Exp (Hz)	FEM (Hz)	Δ_I (%)
1	113.4*	113.4	0.0	164.0*	164.0	0.0
2	139.0*	140.9	1.4	257.0*	268.5	4.5
3	272.5*	271.1	0.5	326.0*	322.4	1.1
4		388.1		421.0	433.4	2.9
5	524.5*	520.4	0.8	453.0	449.6	0.8
6	556.0*	555.8	0.0		786.1	
7	595.0*	600.8	1.0	797.0	791.7	0.7
8		733.5		848.0	847.4	0.1
9	780.0*	767.1	1.7	904.0*	899.4	0.5
10	899.0*	887.9	1.2	1021.0*	1003.0	1.8
11		977.0		1325.0*	1309.0	1.2
12	1252.0*	1262.0	0.8		1343.0	
13		1330.0		1530.0*	1529.0	0.1
14	1396.0*	1380.0	1.1		1661.0	
15		1521.0		1692.0*	1668.0	1.4
16	1574.0*	1560.0	0.9	1810.0	1777.0	1.8
17	1689.0	1674.0	0.9	1878.0	1906.0	1.5
18		1808.0			2040.0	
19		1885.0		2175.0*	2162.0	0.6
20	1897.0	1888.0	0.5	2534.0	2529.0	0.2

* these frequencies were taken into account in identification functional

Table 7.

Comparison of elastic constants obtained from static and vibration tests

Properties	**Static test**	**Vibration test**
E_1, GPa	168 (145)	144
E_2, GPa	9.1 (8.9)	7.7
G_{12}=G_{13}, GPa	5.6	5.9
G_{23}, GPa	-	6.0
ν_{12}	0.33	0.3

Conclusions

The present method of identification based on planning of experiments can predict the major elastic properties of the laminated composite plate specimens. Some discrepancies have been found in the identification of the transverse shear modulus, which is not very sensitive to the eigenfrequencies of the thin plates. This parameter must be identified from the experiment on the thick plates. The main advantage of the method of identification proposed in the present paper is significant reduction of the computational efforts in order to calculate the numerical frequencies, which are presented in the functional to be minimized. Another advantage of the identification method used is that the elastic constants are determined only from vibration test using a plate sample. The proposed method can be used as a non-destructive method in order to determine the elastic constants of unidirectional laminated plates.

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E.Skukis, S.Ručevskis, E.Barkanov and A.Chate Moderno oglekļšķiedras kompozītmateriāla elastīgo īpašību noteikšana izmantojot inverso metodi

Rakstā tiek apskatīta metode kompozīto materiālu mehānisko īpašību noteikšanai. Lai spētu nodrošināt konstrukcijas augsta standarta drošību, materiālu īpašībām ir jābūt precīzi noteiktām. Darbā tiek aprakstīta skaitliskā eksperimenta plānošanas identifikācijas metode kompozīto materiālu elastīgo īpašību noteikšanai. Metode ir balstīta uz kompozīto materiālu plātņu dinamiskajiem eksperimentiem un Galīgo Elementu metodes aprēķiniem. Šajā darbā tiešās minimizācijas metode elastīgo īpašību noteikšanai ir aizstāta ar skaitliskā eksperimenta plānošanas metodi. Galvenā šīs metodes priekšrocība ir ievērojama nepieciešamo aprēķinu samazināšana. Galīgo Elementu metodes aprēķini tiek veikti tikai skaitliskā eksperimenta plānošanas metodes iegūtajos punktos. Minimizācijas funkcionāls apraksta starpību starp eksperimentāli iegūtajiem un skaitliski aprēķinātajiem parametriem. Materiāla elastīgās īpašības tiek iegūtas minimizējot šo funkcionāli.

E.Skukis, S.Ručevskis, E.Barkanov and A.Chate Characterisation of advanced composite material properties by inverse technique

This paper describes the method for the determination of the elastic constants of composite materials from the experimental results for eigenfrequencies of the plates. In this work it is proposed to use the numerical-experimental identification method. The identification procedure is based on the method of experiment design, the response surface approach and the finite element method. The main advantage of this method is significant reduction of the number of computations of the criterion. The finite element modeling is performed only in the reference points. Therefore, a significant reduction in calculations of the identification functional can be achieved in comparison with the conventional methods of minimization The functional to be minimized describes the difference between the experimentally measured and numerically calculated eigenfrequencies, which are dependent on parameters to be identified. The identification parameters are five elastic constants of material. By minimizing the functional the identification parameters are obtained. The method is employed to identify the elastic properties of angle-play laminates.

Э.Скукис, С.Ручевскис, Ж.Барканов и А.Чате Идентификация механических свойств углеродного композита используя инверстный метод.

В данной работе рассматривается метод определения механических характеристик композитного материала. Для того чтобы обеспечить высокую надежность конструкции, соответствующую стандартам, механические свойства материала должны быть определены наиболее точно. В работе описывается приминение экспериментально – числового метода идентификации. Метод основан на динамическом эксперименте пластины из композитного материала и применением метода конечных элементов. Вместо прямой минимизации критерия в работе используется метод планирования экспериментов. Главное достоинство данного метода – значителное сокращение количества вычислений критерия. Расчёты методом конечных элементов производятся только для точек полученных методом планирования экспериментов. Функционал минимизаций описывается разницей между эксперименталными данными и данными, полученными путём вычислений. Идентификацонные параметры находятся путём минимизации функционала.