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STRESSES CALCULATION OF SLIDING FRICTION SURFACES

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One of the most important quantities characterizing a normal functioning of a movable joint is surface roughness which determines, to a large extent, the distribution of a normal and tangential forces within the contact area, the lubrication properties and wear resistance of the rubbing surfaces.
The main portion of rubbing surfaces is formed by surfaces of irregular nature of roughness which in the process of wear within some contact micro volumes tend to create stresses varying in time. The variable stresses brought about by load some points of the surface layer may exceed a limit beyond which, within the material under wear, a process of gradual accumulation of faults takes place, which results in the formation of micro cracks, their growth and the separation of wear particles. The starting of micro cracks in the final analysis, will occur in those points of the material's surface layer, which would satisfy a destruction criterion which takes into account the value of stresses caused by elastic and plastic deformation as well as the properties of the contacting material.

Analysis of the stress distribution under elliptical contact spot

On contact and subsequent sliding friction of two rough surfaces, the micro asperities undergo mainly a plastic deformation and the process itself of their flattening is usually followed by elastic shrinkage of the asperities. Thus, to elastic deformation are, as a rule, subject the asperities at the level of waviness or deviations of shape. Therefore, further on, the term asperity will imply peaks of waviness and deviation of form.
One of the objects of the work is to find the area of stresses appearing under each local contact of any two asperities in friction as well as to evaluate the condition of surface layer of the material under possible destruction (Fig. 1). The calculation has been done for a case of stabilized wear conditions (i.e. after run-in), with signs of fatigue phenomena on the friction surfaces, and when the contact stresses, as a rule, do not exceed the yield point of the surface material.
In the case when the asperities of rubbing surfaces are described by normal homogeneous random fields with a directed anisotropy conceding with the direction of frictional forces (e.g. run-in surfaces), the shape of peaks of asperities can be represented as a continuous function with a finite curvature in the form of an elliptical paraboloid [1]. In this case the actual area of contact between two asperities will have the shape of a drawn out ellipse.
Calculation of stress distribution at the points of compression region only under normal load within the ellipse of contact is dealt with in a number of works [2,3] (Fig. 2). The combined action of normal and tangential loads is discussed also in works [4,5] whose authors examines the stresses condition at points of elastic semi space, part of whose boundary is

under simultaneous action of two loads interrelated through Amontov-Coulomb law and distributed, in accordance with the Hertzian theory, by the elliptical law. The total stress area produced under the elastic contact spot is calculated in works [1,6]. However, the above mentioned works have not examined the distribution of stresses under the elliptical spot within the sub surface layer where the concentration of stresses, with certain friction properties and structure of the material, is most essential.

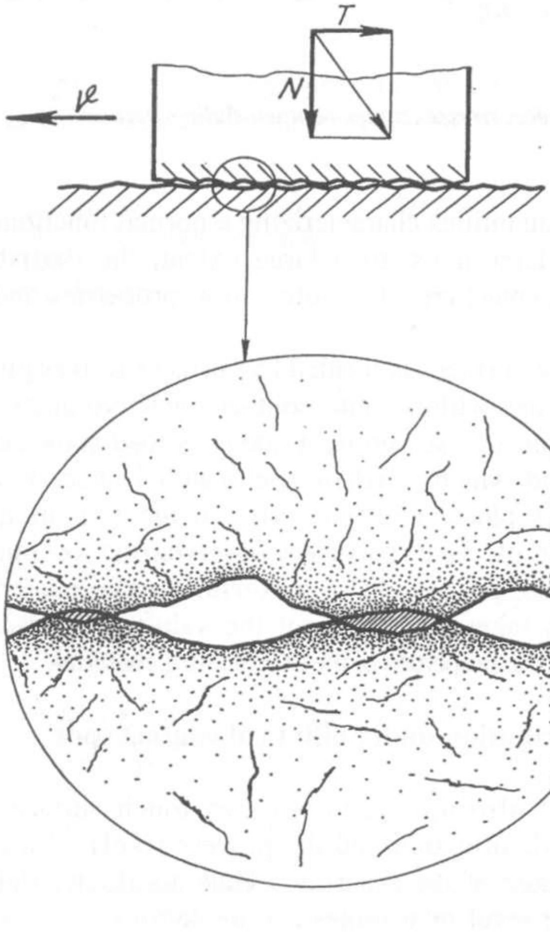

Fig.1. Contact of two surfaces in friction

The solution this problem needs to finding a stress field within an elastic half plane which is the contact surface of an individual asperity, i.e. with some approximation, representing the situation under each of the several loaded asperities which interact between a pair of large surfaces sliding against each other.
The strengths down axes x, y, z are the following:

$$\begin{cases} N_Z(x;y)=p(x;y); \\ T_Y=0; \\ T_X=fp(x;y), \end{cases}$$

where N – normal strength; T_Y – tangential strength.

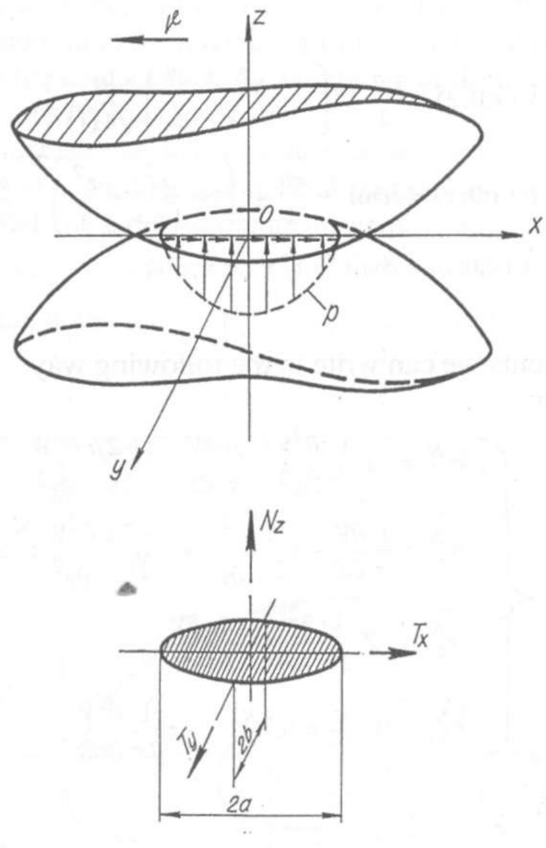

Fig.2. Elliptical contact region of two surface asperities

$$p(x,\ y) = p_o \sqrt{1 - \frac{x^2}{y^2} \cdot \frac{y^2}{b^2}} \quad \text{and} \quad p_o = \frac{3P}{2\pi ab}.$$

If we assume the asperities like eliptic paraboloids,

$$a = \frac{3P}{2} \cdot \frac{F - E}{Ae^2 \cdot \pi} \left(\frac{1 - \mu_1^2}{E_1} + \frac{1 - \mu_2^2}{E_2} \right) \text{ and } b = a\sqrt{1 - e^2}.$$

Parameter A we can find according following system of equations:

$$\begin{cases} f_1(x; y) + f_2(x; y) = Ax^2 + By^2; \\ \dfrac{(F - E)(1 - e^2)}{[E - (1 - e^2)F]} = \dfrac{A}{B}; \\ F = \displaystyle\int_0^{\pi/2} \frac{d\varphi}{\sqrt{1 - e^2 \sin^2 \varphi}}; \\ E = \displaystyle\int_0^{\pi/2} \sqrt{1 - e^2 \sin^2 \varphi}\, d\varphi. \end{cases}$$

For this scheme of contact we can write the expressions of Nuton, logarithmic and biharmonic pressure potentials:

$$V(x;0;z)=\frac{\pi\beta}{2}\int_t^{\infty}\left(1-\frac{x^2}{1+s}-\frac{z^2}{2}\right)\frac{ds}{\sqrt{\Delta(s)}};$$

$$U(x;0;z)=const-\frac{\pi\beta}{2}\int_t^{\infty}\left(1-\frac{x^2}{1+s}-\frac{z^2}{s}\right)^2\frac{sds}{\sqrt{\Delta(s)}};$$

$$W(x;0;z)=const-\int_0^{\infty}V(x;y;z)dz.$$

Then pressure components we can write in the following way:

- from normal strength:

$$\begin{cases}\sigma_X^N=-z\frac{1}{2\pi}\frac{\partial^2V}{\partial x^2}-\frac{\mu}{\pi}\frac{\partial V}{\partial x}-\frac{1-2\mu}{2\pi}\frac{\partial^2W}{\partial x^2};\\ \sigma_Y^N=\frac{\mu}{\pi}\frac{\partial V}{\partial Z}-z\frac{1}{2\pi}\frac{\partial^2V}{\partial y^2}-\frac{1-2\mu}{2\pi}\frac{\partial^2W}{\partial y^2};\\ \sigma_z^N=-Z\frac{1}{2\pi}\frac{\partial^{2V}}{\partial z^2}-\frac{1}{2\pi}\frac{\partial V}{\partial z};\\ \tau_{XY}^N=0,\tau_{YZ}^N=0,\tau_{XZ}^N=-z\frac{1}{2\pi}\frac{\partial^2V}{\partial x\partial z};\end{cases}$$

- from tangential strength:

$$\begin{cases}\sigma_x^T=f\left[-z\frac{1-2\mu}{2\pi}\frac{\partial^3W}{\partial x^3}-\frac{1+\mu}{\pi}\frac{\partial V}{\partial x}-\frac{\mu}{\pi}\frac{\partial^3U}{\partial^3x}\right];\\ \sigma_y^T=f\left[\frac{\mu}{\pi}\frac{\partial V}{\partial x}-z\frac{1-2\mu}{2\pi}\frac{\partial^3W}{\partial y^2\partial x}\frac{\mu}{\pi}\frac{\partial^3U}{\partial y^2\partial x}\right];\\ \sigma_z^T=f\left[-z\frac{1}{2\pi}\frac{\partial^2V}{\partial x\partial z}\right];\\ \tau_{xy}^T=0;\tau_{yz}^T=0;\tau_{XZ}^T=f\left[-z\frac{1}{2\pi}\frac{\partial^2W}{\partial X^2}+\frac{1}{2\pi}\frac{\partial V}{\partial z}\right],\end{cases}$$

where μ - poison's ratio of material.

The pressure components are summated:

$$\begin{cases}\sigma_i=\sigma_i^N+\sigma_i^T;\\ \tau_{ij}=\tau_{ij}^N+\tau_{ij}^T;\\ i,j=x;y;z; \qquad i\neq j.\end{cases}$$

According Mises field criterion deviator of stresses we can write in the following way:

$$J_2(D_\sigma)=\tau_{xy}^2+\tau_{xz}^2+\tau_{yz}^2+\frac{1}{6}\left[(\sigma_x-\sigma_z)^2+(\sigma_y-\sigma_z)^2+(\sigma_x-\sigma_y)^2\right].$$

The material destruction begins when

$$[J_2(D_2)]^{1/2}=T$$

is more than shear strength limit. The calculations of characteristic T are given in Fig. 3, 4, 5.

Analyzing the distribution of the most stressed points of the material under an elliptical contact spot in terms of the Mises yield criterion indicates that the most dangerous for destruction may be regions both in the surface layer of material, and on the contact spot itself, and if the more drawn out is the contact spot, the beginning of destruction is most probable on the elliptical contact spot or in a region close to it.

The equations obtained to calculate the fields of stresses are rather complicated for practical application and was conducted according to a special program on a computer and yielded an approximate relationship to calculate the maximum dangerous stress.

$$\sigma_{max} = (0{,}251 + 0{,}016\, p + 1{,}158\, f^{3})q_{max},$$

where q_{max} - maximum pressure in the centre of contact; β - ration of half-axes; f - coefficient of friction.

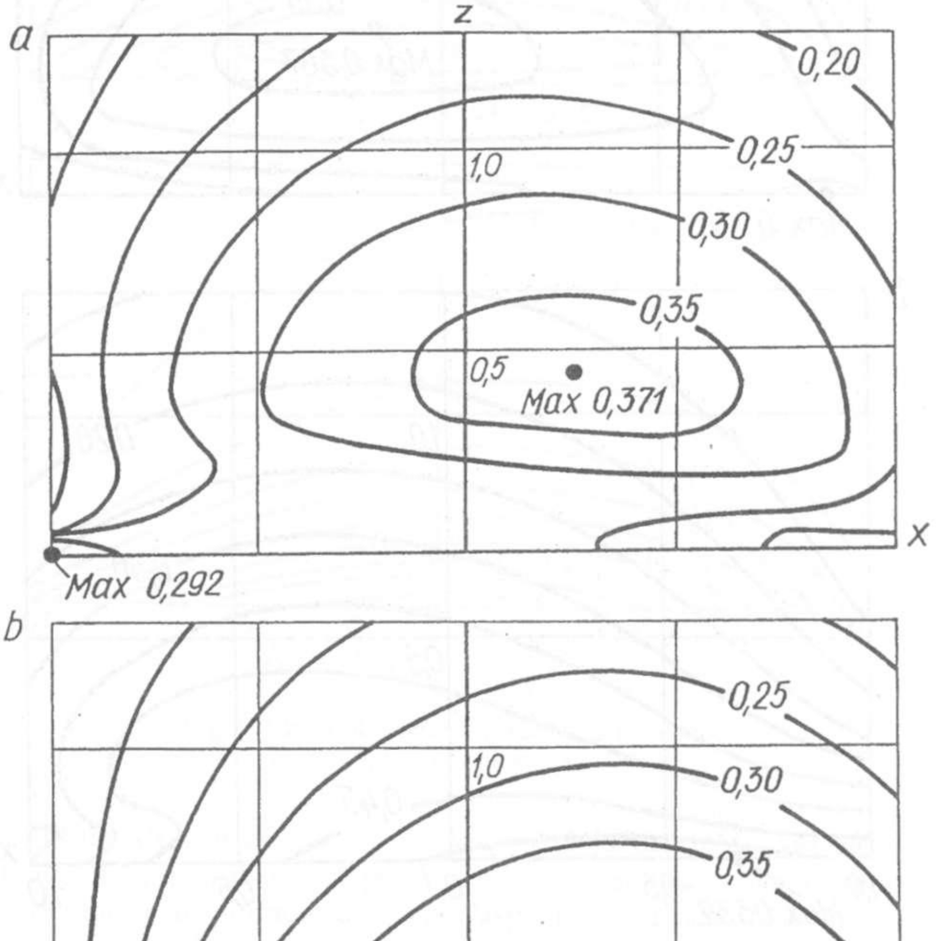

Fig.3. Lines of stresses T for round contact:
β=0,1; a) f=0,2; b) f=0,5

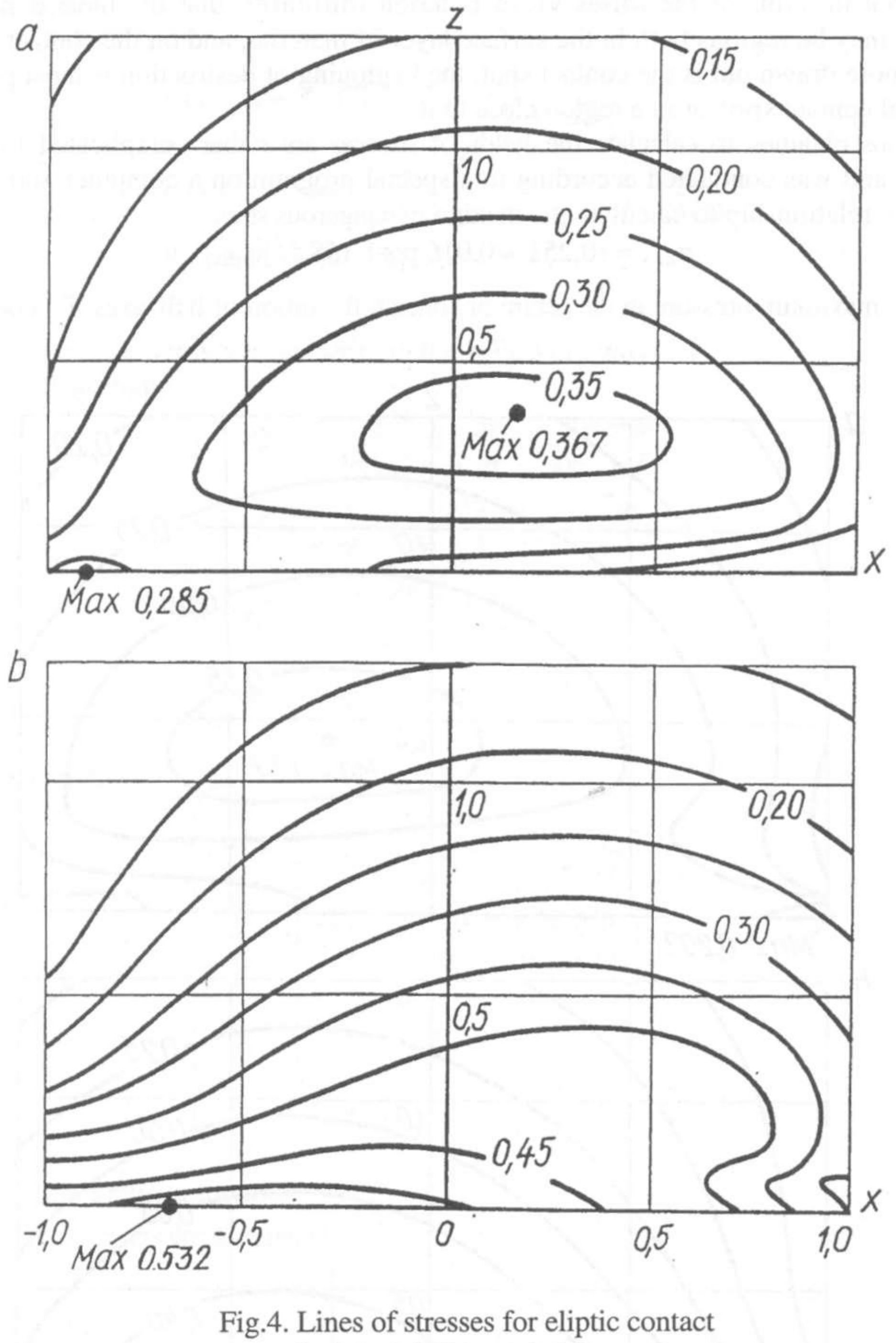

Fig.4. Lines of stresses for eliptic contact
β=0,1; a) f=0,2; b) f=0,5

From the relationship obtained one can see that the numbers containing parameters p and f will assume, in the case of boundary friction (f$\rightarrow$0,1) and a drawn out elliptical contact spot (p$\rightarrow$0.01) for engineering calculations, we can use the expression

$$\sigma_{max} \approx 0,251 q_{max} .$$

With larger values of the friction coefficient f and the ration of half-axes β, the importance of these parameters in the entire relationship increases too.

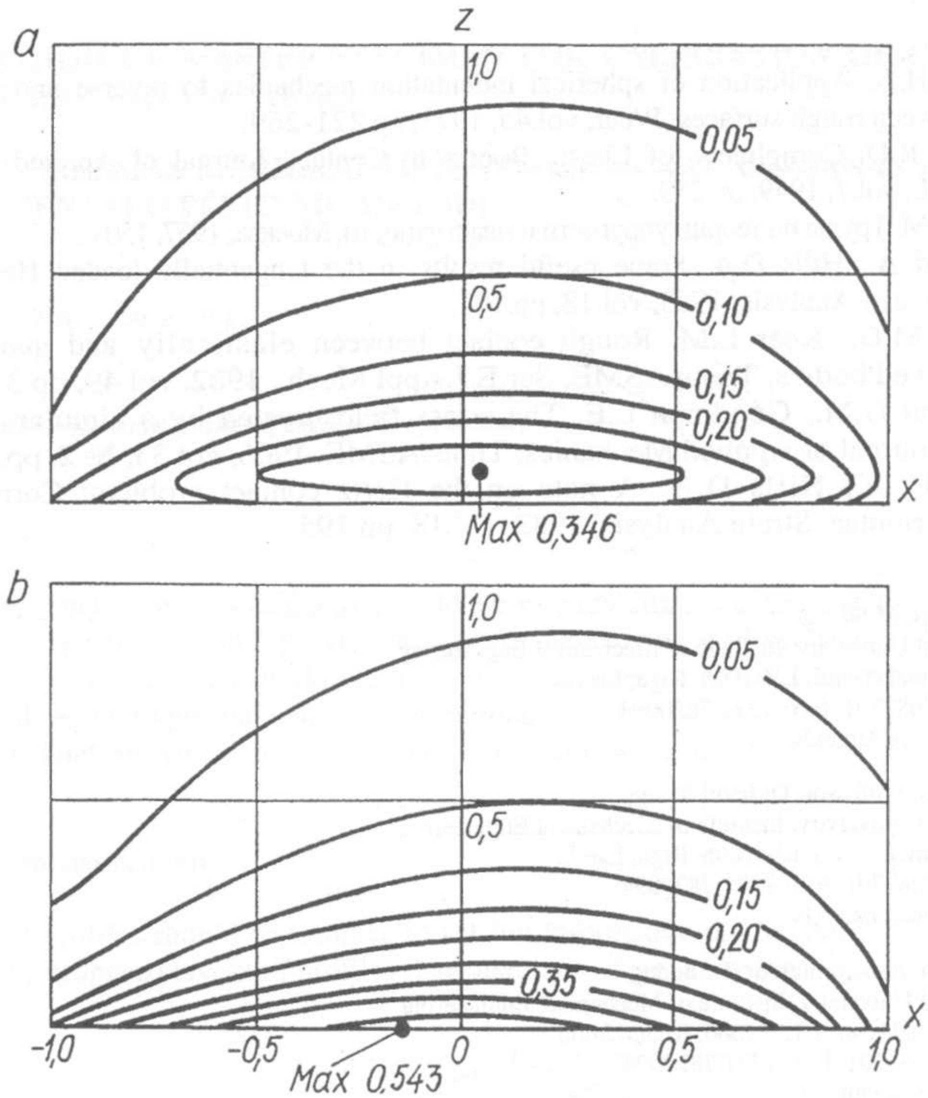

Fig.5. Lines of stresses T for elliptic contact
β=0,1; a) f=0,2; b) f=0,5

The analysis of the sliding contact mechanism for rough surfaces gives that within elastic contact the fatique is responsible for asperity destruction. So the surface waveforms are most response for durability of surface, the term "asperity" will determine the wave summit. Fatique failures are actual for steady - state wear process i.e., after running in, when the phenomena in surface layer are due to multicycle fatique. Under cyclic load stresses may exceed certain limit, beyond which surface layer starts accumulate failures, initiate cracks, their growth and, as a result, layer destructs.

In their relative motion all asperities of certain height have specified strain and corresponding stress field. Since asperities of moving surface have random height distribution, stress in interactive asperity changes randomly.

Within the given contact model (elastic contact, fatique stress accumulation, slight asperity slope e.t.c.) we have following destruction mechanisms. Under cyclic load of asperity summits in friction the cracks in sublayer will originate. Due to stress fields cracks join and grow up, wear particles spell from main material like flakes. According these considerations it is possible to calculate linear wear of rough sliding surfaces.

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Konrads G., Rudzītis J., Geriņš Ē., Torims T. Slīdes berzes virsmu spriegumu aprēķins.

Izstrādāts matemātiskais modelis slīdes berzes virsmu nodiluma aprēķinam detaļu piestrādes periodam. Iegūtās izteiksmes bāzējas uz kritisko spriegumu sadalījuma analīzi raupju virsmu atsevišķu negludumu kontakta zonā.

Konrads G,. Rudzitis J., Gerinsh E., Torims T. Stresses calculation of sliding friction surfaces.

The mathematical model for wear calculation of surfaces in sliding friction, concerning period, subsequent running in, has been developed. Obtained empirical expressions are based on the analysis of critical stress distribution within contact zone of separate asperities, located on two rough surfaces.

Конрадс Г., Рудзитис Я., Гериньш Э., Торимс Т. Расчет напряжений при трении скольжения поверхностей.

Разработана математическая модель расчета износа поверхностей при трении скольжения для периода приработки деталей. Полученные эмпирические выражения основаны на анализе распределения критических напряжений в зоне контакта отдельных неровностей двух шероховатых поверхностей.