EXTENDED ABSTRACT

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COPULA BASED NONPARAMETRIC REGRESSION ESTIMATION

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1 Introduction

The methods and algorithms of time series analysis play an important role in financial econometrics for identification and prediction of risk. The paper deals with the identification and prediction problems of the autoregressive models of nonlinear time series using nonparametric estimates of the conditional mean and conditional variance.

2 Copula based model

The reason of waiver of traditional linear models is no Gauss type of random values, describing the dynamics of the real models. We will remind that assumption about normal distribution of time series allows to calculate the conditional expected value of phase variable as linear functional of its past values$\{x_t,\ s \leq t\}$.We should deal with the estimation of unknown function in nonlinear difference equation of the first order with usual kind of information about the distribution law. In many applied problems of regression analysis for time series already in simplest case we deal with the equation

$$x_t = f(x_{t-1}) + h_t \tag{1}$$

where h_t are the uncorrelated tailings, on the average equal to the zero.

If we assume that x_t have integral distribution function of $\mathfrak{F}(x)$, than for copula we should done a substitution $u_n = \mathfrak{F}(\hat{x})$. Then we receive the pair (u_{n+1}, u_n), where $u_{n+1} = g(u_n, \xi_{n+1})$.

If we designate F^t minimum sigma-algebra, in relation to which random values $\{h_s,\ s \le t\}$ are measured, the needed function can be defined through the conditional expected value $f(x_{t-1}) = \mathrm{E}\{x_t|F^{t-1}\}$. To use the sequence of sigma-algebra and conditional dispersion $\sigma_t^2 = \mathrm{E}\{h_t^2|F^{t-1}\}$ tailings h_t can be present in form work of "white noise" $\{\xi_t, t \in Z\}$, (i.e. sequences of the independent identically distributed (i.i.d.) random values with zero mean and by single dispersion) and with conditional standard deviation: $h_t = \sigma_t \xi_t$.For searching for of function $f(x_n)$ we need to create separate discrete intervals of values and then on every interval we can use either least-squares or consider the model of phase space discretization and presentation of values in form eventual number of no splitting areas $\{S_k,\ k$ =1, ...,r$\}$ which can be examined as the states of some Markov chain.

This property of tailing's dispersion is called as conditional heteroskedasticity and can be modulate through linear difference equations with coefficients, linearly depending on white noise (GARCH (p,q) processes). For searching for the function $f(x)$ the set of values we can break up enough small length δ on intervals and then on every interval we can use either least-squares method or minimize specially built functional as an integral with the kernels of different form.

3 Connection with Markov chain

We will suppose that is observed random process of type

$$x_{n+1} = f(x_n) + \sigma_n \xi_{n+1}, \tag{2}$$

ξ_n is a random error of observations, (i.i.d.) . $\mathrm{E}\{\xi_n\} = 0$, $f(x_n)$ is a nonlinear function of the elements of chain.

Equation (2) can be interpreted so, that a random sequence depends on the "history".

Also we can write that the conditional expected value of random variable looks like

$$E\ \{x_{n+1}|F^n\} = E\ \{x_{n+1}|x_n\} = \sum_y p\,(x_n, y) \cdot y = f(x_n) \tag{3}$$

that determines non-linearity of functional dependence x_{n+1} from x_n. Until now the researches described the dynamics of chain $\{x_n\}$ due to find the functional dependence$f(x_n)$. For searching for of function $f(x_n)$ they have created separate discrete intervals of values and then on every interval have used either least-squares or minimize specially built functional as an integral with the kernels of different form.

Our assumption about the math. model

$$u_{n+1} = h(u_n) + g(u_n)\,\xi_{n+1}, \tag{4}$$

where

$h(u_n)$ - conditional mean;
$g^2(u_n)$– conditional variance.

Our nowadays aim is to create and investigate the Markov chain built on the equation (4). So we need to express the functions $h(u_n)$ and $g^2(u_n)$ through the transition probabilities. For this purpose the main task is to find the transition probabilities of Markov chain on the basis of observed values of the time series. So, it is shown that the Markov chain theory can be successfully applied to study nonlinear time series.

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