Layered Slab Parameters Identification Using Movable Metal–Backing Method

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***Abstract* – In the paper the new approach of movable metal–backing method for layered slab's complex dielectric permittivity and thickness estimation is suggested. This method allows to identify considered environment as slab of homogenous layers, as environment with definable effective complex dielectric permittivity and as inhomogeneous environment with indefinable effective complex dielectric permittivity.**

***Keywords*: complex dielectric permittivity, inverse propagation problem, microwave propagation.**

I. Introduction

Nowadays the use of microwaves is getting increasingly more: in communications, in radar applications, in medicine, in civil engineering, hence the study of materials' dielectric properties as yet is claimed. In spite of the fact that detailed surveys of measurement techniques for materials characterization are available [3–5], the necessity of nondestructive, fast and precise techniques still exists.

In this paper authors suggest new application of free space movable metal–backing method for layered slab's parameters identification. In contrast to [2] the unique analytic solution for this problem is obtained. As well, solution for unknown slab's thickness is found.

II. Direct Propagation Problem

Wave propagation depends on objects' electromagnetic and spatial parameters. Environments' changes, especially discontinuous, lead to wave reflection, refraction and scattering. In this paper the interference of incident and reflected waves is considered.

The most convenient model for reflected power dependence on object parameters analysis is homogenous layered slab structure in case of normal wave incidence.

A. Model

Model consists of two slabs with dielectric parameters ε_s', ε_s'' (slab under investigation) and ε_3', ε_3'' (filling slab) with thicknesses l and d respectively, two semi–infinite mediums with dielectric parameters ε_1', ε_1'' and ε_4', ε_4'' and perfect conducting layer between filling slab and semi–infinite medium in $+z$ direction. Every medium can be characterized by complex wave impedance $\underline{Z}_1, \underline{Z}_s, \underline{Z}_3, \underline{Z}_4$ and complex wave number $\underline{k}_1, \underline{k}_s, \underline{k}_3, \underline{k}_4$ respectively.

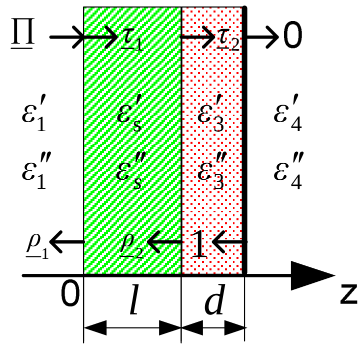

Fig. 1. One layer dielectric slab's structure with movable metal–backing.

The mathematical model of suggested structure is a system of equations, which can be easy obtained from boundary conditions for electrical and magnetic field intensity continuities on layers' boundaries [6].

Composed system of equations for structure exterior reflection coefficient $\underline{\rho}_1$ can be solved analytically or numerically. Also, $\underline{\rho}_1$ can be found using recursive multiplication algorithm or using infinite recursive sum [1]. Generally, $\underline{\rho}_1$ is a complex function of following arguments:

$$\underline{\rho}_1 = \underline{\rho}_1\left(\omega, \varepsilon_1', \varepsilon_1'', \varepsilon_s', \varepsilon_s'', \varepsilon_3', \varepsilon_3'', l, d\right)$$

Analysis of $\underline{\rho}_1$ (henceforward in this article $\underline{\rho}$) is selected, because measurement of reflected power is proportional to square of structure's exterior reflection coefficient. Measurement of reflected power can be realized by spectrum analyzer FSP–30 (manufactured by Rhode&Schwarz).

III. Inverse Propagation Problem

The solution of this problem is reconstruction of objects' spatial and electromagnetic parameters, which cause revealed changes in field distributions.

It is discovered, that for unambiguous numerical slab's parameters identification the number of different circumstances must be greater than unknown parameters number [6]. Commonly, to satisfy this necessitating, different incident wave signal frequencies are chosen. Instrumentally it is useful, but such approach is undesirable, because of causality (Kramers–Kronig relation).

Various circumstances can be realized by different incident wave angles, different slab thickness, and different additional conditions before or after slab.

Movable metal–backing method is considered as quickly implemented method, because of extra device minimal need and additionally measurable data simple estimation.

Usually, solution of inverse scattering problem is fined by two–dimensional simplex minimization routine for coincidence between the calculated and the measured values of reflection coefficients for different obstacles behind the slab [2]:

$$\chi(\tilde{\varepsilon}) = \sum_{j=1}^{n} \left(\underline{\rho}_m \left(d_j \right) - \underline{\rho}_c \left(d_j \right) \right)^2 \qquad (1)$$

In [2] it is suggested to detect such d – variable distance between the sample and the screen, which satisfies resonance conditions of $\underline{\rho}(d)$ dependence.

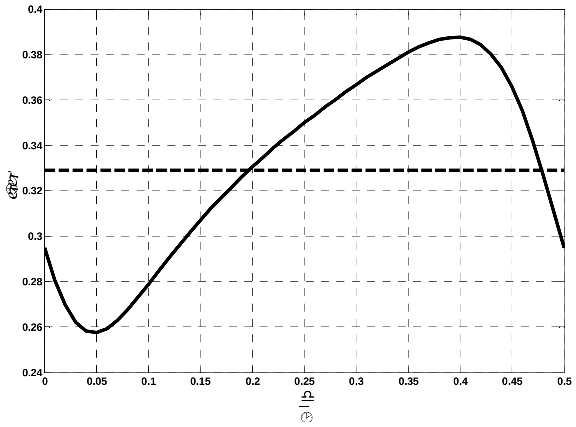

Fig. 2. $\underline{\rho}(d)$ resonance form, λ is an incident wave's length.

Fig.2. shows example of $\underline{\rho}(d)$ dependence, the dashed line corresponds to structure's general reflection coefficient in case of model without metal–backing.

For the first time it is find that one layer slab parameters can be recognized analytically. In comparison with in [2] proposed method no additional measurements and coincidence findings are needed.

A. Slab's parameters identification algorithm

For calculation simplification and without degradation of accuracy, it is considered that environment characteristics before and after investigated slab are known and are the equal:

$$\underline{Z}_1 = \underline{Z}_3 = \underline{Z}_a$$

$$\underline{k}_1 = \underline{k}_3 = \underline{k}_a$$

(index $_a$ means free space, that agrees with realizable measurement conditions).

In this case, $\underline{\rho}$ becomes a function of following variables:

$$\underline{\rho} = \underline{\rho}\left(\varepsilon_s', \varepsilon_s'', l, d\right)$$

where d is an known directly measurable argument.

For computation obviousness an additional intermediate variables are needed:

$$\underline{\varphi}_a = \underline{k}_a \cdot d$$

$$\underline{\varphi}_s = \underline{k}_s \cdot l$$

where $\underline{\varphi}_a$ is a wave phase incursion in medium between sample and screen, $\underline{\varphi}_s$ is a wave phase incursion in sample.

Using recursive formula for slab input impedance suggested in [1] and taking into consideration backing material's perfect conductivity it is possible to obtain compact $\underline{Z}_{in}^{(s)}$ expression:

$$\underline{Z}_{in}^{(s)} = \frac{-i \cdot \left(\underline{Z}_a \cdot tg\underline{\varphi}_a + \underline{Z}_s \cdot tg\underline{\varphi}_s \right)}{\underline{Z}_s - \underline{Z}_a \cdot tg\underline{\varphi}_a \cdot tg\underline{\varphi}_s} \cdot \underline{Z}_s \qquad (2)$$

Structure's general reflection coefficient can be represented in the following simple way:

$$\underline{\rho} = \frac{\underline{Z}_{in}^{(s)} - \underline{Z}_a}{\underline{Z}_{in}^{(s)} + \underline{Z}_a} \qquad (3)$$

from whence $\underline{Z}_{in}^{(s)}$ also can be obtained as:

$$\underline{Z}_{in}^{(s)} = \underline{Z}_a \cdot \frac{1+\underline{\rho}}{1-\underline{\rho}} \qquad (4)$$

Taking into consideration (4) and (2) $\underline{Z}_s$ can be expressed via recursive formula (5):

$$\underline{Z}_s = \underline{Z}_a \cdot \frac{1+\underline{\rho}}{1-\underline{\rho}} \cdot \frac{\underline{Z}_s - \underline{Z}_a \cdot tg\underline{\varphi}_a \cdot tg\underline{\varphi}_s}{-i \cdot \left(\underline{Z}_a \cdot tg\underline{\varphi}_a + \underline{Z}_s \cdot tg\underline{\varphi}_s \right)} \qquad (5)$$

For unique slab's parameters identification it is necessary to solve system of equations, see formula (6), where subindex d_j means different distances between sample and screen.

$$\begin{cases} \frac{1+\underline{\rho}_{d_1}}{1-\underline{\rho}_{d_1}} \cdot \frac{\underline{Z}_s - \underline{Z}_a \cdot tg\underline{\varphi}_{a_{d_1}} \cdot tg\underline{\varphi}_s}{\underline{Z}_a \cdot tg\underline{\varphi}_{a_{d_1}} + \underline{Z}_s \cdot tg\underline{\varphi}_s} = \frac{1+\underline{\rho}_{d_2}}{1-\underline{\rho}_{d_2}} \cdot \frac{\underline{Z}_s - \underline{Z}_a \cdot tg\underline{\varphi}_{a_{d_2}} \cdot tg\underline{\varphi}_s}{\underline{Z}_a \cdot tg\underline{\varphi}_{a_{d_2}} + \underline{Z}_s \cdot tg\underline{\varphi}_s} \\ \frac{1+\underline{\rho}_{d_2}}{1-\underline{\rho}_{d_2}} \cdot \frac{\underline{Z}_s - \underline{Z}_a \cdot tg\underline{\varphi}_{a_{d_2}} \cdot tg\underline{\varphi}_s}{\underline{Z}_a \cdot tg\underline{\varphi}_{a_{d_2}} + \underline{Z}_s \cdot tg\underline{\varphi}_s} = \frac{1+\underline{\rho}_{d_3}}{1-\underline{\rho}_{d_3}} \cdot \frac{\underline{Z}_s - \underline{Z}_a \cdot tg\underline{\varphi}_{a_{d_3}} \cdot tg\underline{\varphi}_s}{\underline{Z}_a \cdot tg\underline{\varphi}_{a_{d_3}} + \underline{Z}_s \cdot tg\underline{\varphi}_s} \end{cases} \quad (6)$$

B. Identification algorithm's Matlab code fragment

```
% Z_s -> x1, tg(fi_s) -> x2
% Z_a -> a, tg(fi_a) -> b, ro -> c

syms a b1 b2 b3 c1 c2 c3 x1 x2

y1=(a*b1+x1*x2)/(x1-a*b1*x2);          %(2)
[n1, d1] = numden(y1);
y2=(a*b2+x1*x2)/(x1-a*b2*x2);          %(2)
[n2, d2] = numden(y2);

c1p=(1+c1)/(1-c1);                     %(4)
c2p=(1+c2)/(1-c2);                     %(4)
c3p=(1+c3)/(1-c3);                     %(4)

y1_=simplify(c1p*d1*n2-c2p*d2*n1);    %(6)
y1=collect(y1_,x1);
x_1=solve(y1,x1);

for n=1:length(x_1)
  x1=x_1(n);
  y2=(a*b2+x1*x2)/(x1-a*b2*x2);        %(2)
  [n2, d2] = numden(y2);
  y3=(a*b3+x1*x2)/(x1-a*b3*x2);        %(2)
  [n3, d3] = numden(y3);
  y2_=simplify(c2p*d2*n3-c3p*d3*n2);  %(6)
  y2=collect(y2_,x2);
  x_2=solve(y2,x2);                 %tg(fi_s)
end

x11_=[];
x12_=[];
for n=1:length(x2_)
      x2=x_2(n);
      x11_=[x11_, eval(x_1(1))];      %Z_s
      x12_=[x12_, eval(x_1(2))];      %Z_s
end
```

IV. Results

Acquired system of equations (6) has set of trivial and nontrivial solutions one of which is proper to desired $\underline{Z}_s$ and $tg\underline{\varphi}_s$.

The contribution of possible measurement errors in the results of slab recognition and of distance choice also is inspected.

Suggested solution numerical results are stable for distance between sample and screen.

A. Numerical result examples for different distances between sample and screen

```
Selected distances d/λ: 0.00, 0.24, 0.48
Expected Zs value 1.9684e+002+2.4635e+001i
Expected φs value 0.1222-1.0855i
Obtained Zs value 1.9684e+002+2.4635e+001i
Obtained φs value 0.1222-1.0855i

Selected distances d/λ: 0.11, 0.21, 0.31
Expected Zs value 1.9684e+002+2.4635e+001i
Expected φs value 0.1222-1.0855i
Obtained Zs value 1.9684e+002+2.4635e+001i
Obtained φs value 0.1222-1.0855i
```

Suggested solution numerical results are stable for different distance measurement errors.

B. Numerical result examples for different distance measurement errors

```
Measurement error 0%
Expected Zs value 1.4927e+002+2.8981e+001i
Expected φs value -0.0003-1.0006i
Obtained Zs value 1.4927e+002+2.8981e+001i
Obtained φs value -0.0003-1.0006i

Measurement error 5%
Expected Zs value 1.4927e+002+2.8981e+001i
Expected φs value -0.0003-1.0006i
Obtained Zs value 1.5194e+002+2.6797e+001i
Obtained φs value -0.0003-1.0006i
```

For the moment, suggested $tg\underline{\varphi}_s$ solution numerical results are stable for different reflected power measurement errors, but $\underline{Z}_s$ aren't.

C. Numerical result examples for different reflected power measurement errors

```
Measurement error 0%
Expected Zs value 2.3198e+002+3.1425e+001i
Expected φs value 0.1987-1.2103i
Obtained Zs value 2.3198e+002+3.1425e+001i
Obtained φs value 0.1987-1.2103i

Measurement error 1%
Expected Zs value 2.3198e+002+3.1425e+001i
Expected φs value 0.1987-1.2103i
Obtained Zs value 2.2866e+002+3.0121e+001i
Obtained φs value 0.1953-1.2156i
```

V. SUMMARY

In the paper the new approach of movable metal–backing method for layered slab's complex dielectric permittivity and thickness estimation is suggested. The advantage of this method is, that no additional tools and measurement conditions in comparison with standard method is needed. The fundamental difference from previous method, that while searching such variable distance between the sample and the screen, which satisfies resonance conditions of $\underline{\rho}(d)$ dependence, all measured reflection coefficients and corresponding distances can be taken into account for slab recognition, without simplex minimization routine for coincidence between the calculated and the measured values of reflection coefficients.

According to authors' opinion this method allows to identify considered environment as slab of homogenous layers, as environment with definable effective complex dielectric permittivity and as inhomogeneous environment with indefinable effective complex dielectric permittivity.

For the moment, $tg\underline{\varphi}_s$ suggested solution numerical results are stable for different reflected power measurement errors, but $\underline{Z}_s$ aren't. Hypothetically it is concerned with analytic solution form, which at the moment consists of 15000 symbols.

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Tatjana Solovjova, Jānis Semeņako, Dmitrijs Puriševs. Slāņainās plāksnes parametru noteikšana ar bīdāma metāliskā ekrāna metodi.
Publikācijā ir piedāvāts jauns bīdāma metāliskā ekrāna metodes pielietojums slāņaino plākšņu kompleksās dielektriskās caurlaidības un biezuma noteikšanā. Metodes priekšrocība ir raksturojama ar to, ka salīdzinājumā ar standarta metodi papildus mēriekārtas un mērījumu nosacījumi nav nepieciešami. Metodes fundamentāla atšķirība no zināmiem līdzīgiem pielietojumiem ir tāda, ka, kamēr tiek meklēts tāds attālums starp metālisko ekrānu un pētāmo objektu, pie kura izpildās rezonanses nosacījums sistēmas atstarošanas koeficientam, visi izmērītie atstarošanas koeficienti un atbilstošie attālumi var tikt ņemti vērā veicot pētāma objekta struktūras un sastāva atpazīšanu, neizmantojot simplekso minimizācijas rutīnu algoritmu aprēķināto un izmērīto atstarošanas koeficientu sakritību meklēšanai. Saskaņā ar autoru viedokli, piedāvātā metode ļauj aprakstīt pētāmo vidi kā homogēno slāņu plāksni, kā vidi ar definējamo efektīvo komplekso dielektrisko caurlaidību un kā nehomogēno vidi, kurai efektīvo komplekso dielektrisko caurlaidību definēt nav iespējams. Uz šo brīdi, ir novērots, ka viļņa fāzes uzskrējiena vidē risinājuma skaitliskais rezultāts ir stabils attiecībā pret atstarotās jaudas mērījumu kļūdu, bet vides viļņu pretestības skaitliskais rezultāts nav stabils. Hipotētiski tās ir saistīts ar analītiskā atrisinājuma formulu, kura uz šo brīdi sastāv no 15000 simboliem.

Татьяна Соловьёва, Янис Семеняко, Дмитрий Пурышев. Определение параметров слоистой пластины при помощи метода подвижного металлического экрана.
В публикации предложено новое применение метода подвижного металлического экрана для определения комплексной диэлектрической проницаемости и толщины слоистой пластины. Преимущество метода заключается в том, что для распознавания не требуются дополнительные измерительные средства и условия измерений. Фундаментальное отличие метода от уже существующих похожих методов заключается в том, что, пока происходит поиск такого расстояния между металлическим экраном и исследуемым объектом, которое удовлетворяет резонансным условиям для коэффициента отражения, все значения коэффициента отражения и соответствующие расстояния могут быть учтены при распознавании объекта. Не используя при этом алгоритм рутинной минимизации для измеренных и вычисленных значений коэффициентов отражения. По мнению авторов, рассматриваемый метод позволяет распознавать исследуемую среду как пластину, состоящую из однородных слоёв, как среду, определяемую эффективной комплексной диэлектрической проницаемостью и как среду, для которой определение эффективной комплексной диэлектрической проницаемости невозможно. На данный момент проверено, что численное значение решения для тангенса набега фазы волны в среде стабильно по отношению к ошибке измерений отражённой мощности, в свою очередь численное значение решения для сопротивления среды нестабильно по отношению к этому входному параметру. Предположительно это может быть объяснено неоптимизированностью формулы аналитического решения, которая на данный момент содержит 15000 символов.