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Conditional Linear Periodical Random Process as a Mathematical Model of Photoplethysmographic Signal

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***Abstract*: In this paper a new mathematical model of photoplethysmographic signal is proposed. This mathematical model is built as a conditional linear random process that allows representing main biophysical peculiarities both of the signal creation and of signal stochastic periodicity. It makes possible analyzing the photoplethysmographic signal with the help of the method of characteristic functions.**

Key words: conditional linear random process, mathematical model, periodical process, photoplethysmographic signal.

I. Introduction

Development and validation of a mathematical model is one of the most important steps in digital signal processing. An adequate model should provide data, which can be used in diagnostic systems in order to solve problems of informative features detection, problems of pathology identification, problems of decision making and computer modelling problems.

Photoplethysmographic signals (PPGS) are time rhythmic variations of light absorption in some organs of a human body caused by changes of their perfusion by blood. These signals characterize the functional state of live body tissues by indexes of blood filling dynamics on the level of the microcirculation system. This system has the most important role in the provision and support of the tissue homeostasis. Disorders of the microcirculation system functioning are sources of the development of practically all pathological processes.

It is difficult even to list all fields of photoplethysmography applications in medicine; they are physiology, therapy, surgery, dermatology, gynaecology, neuropathology, stomatology, paediatrics, etc. Clinicians can use photoplethysmography as an additional method both for disease diagnostics and for scientific researches. Besides, photoplethysmography has a subsidiary diagnostic and prognostic importance for study of many cardiovascular and neural diseases, which are now the most often cause of deaths and disabilities of young people. Photoplethysmography can be useful also for hygienists, for sports doctors as well as for doctors, specialized in the space medicine field.

Biophysical aspects of creation of photoplethysmographic signals and the current state of technical devices for their registration were described, in particular, in the papers [1], [2].

The PPGS nature is essentially random, with observation-to-observation changes. That’s why the stochastic approach is used for modelling and processing of photoplethysmographic signals. Also, it is necessary to consider the PPGS rhythmic structure caused by the cyclic heartbeat. An example of the PPGS realization is given in the Fig. 1, where it is easy to observe its cyclicity as well as the absence of a deterministic periodicity.

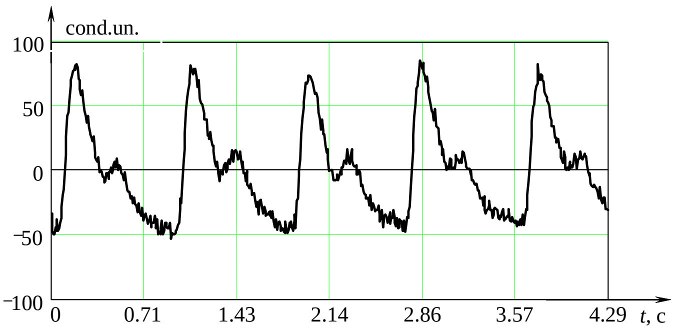

Fig. 1. Realization of the photoplethysmographic signal

The most widely used and the simplest method of the PPGS processing is a determination and statistical analysis of such characteristics as maximums, minimums, inflection points, area under curves, etc. [3], [4]. These diagnostic features are understandable and familiar for doctors, since they are used to process such data in a “manual” mode while making diagnosis in their everyday clinical practice.

Automated methods of the PPGS classification on the basis of the signal form analysis are considered in the work [1] that involves the anterior filtration [1], [5]. Some of correlative methods of PPGS analysis are described in the paper [2]. Also, authors of the paper [1] analyse possibilities of applications of fuzzy logics methods for processing of diagnostic data, whereas in the work [6] it is proposed to use neuron networks for the PPGS classification.

The structural scheme of the computer-information system for registration and analysis of photoplethysmographic signals is given in the Fig. 2. Such system is used for the medical diagnostics of a functional state of a vascular channel.

In the scheme there are presented the following abbreviations:

- BO – Biological Object (section of the organism tissue, filled by blood vessels);

- OET – Optoelectronic Transformer, which consists of:
  - EB – Emitting Block of the light-diode type (the wave length of radiation spectral maximum is 0.94-0.96 mcm);
  - OC - Optic Channel, intended for directing of the infra-red radiation onto BO;
  - BR – Block of Reception of the reflected or diffused BO radiation;
- BRR – Block of Registration of photoplethysmographic signal Realizations;
- BEDP – Block of statistical Estimation of Diagnostic Parameters;
- BDS – Block of Diagnostic Space forming;
- BMD – Block of Making diagnostic Decisions.

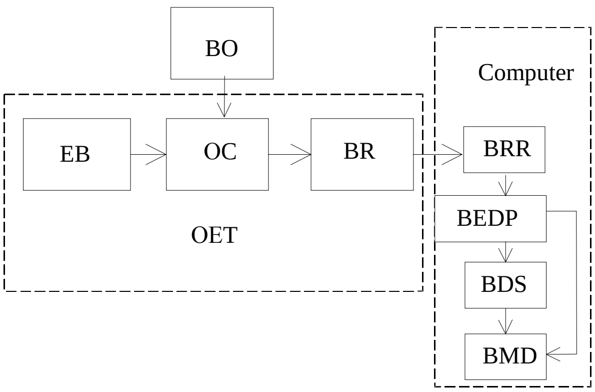

Fig. 2. The structural scheme of the computer-information system for the registration and analysis of photoplethysmographic signals

II. Problem Statement

At present, stochastic periodic random processes are often used for construction of new mathematical models of rhythmic signals. Some probabilistic characteristic of stochastic periodic random processes are periodic functions of time. These models allow to take into account the presence of cyclicity in the time structure of signals.

An additive composition of a weakly stationary random process and a deterministic periodic function is the well-known model of rhythmic biosignals being the simplest one [4]. This model is rather simplified and therefore it does not allow making a detailed description of signals needed in the modern diagnostics.

It should be also pointed out that this model is not based on some biophysical concepts. Instead, it was developed relying only on a posterior analysis of experimental results.

Signal models in the form of periodical correlative random processes (shortly PCRP) with periodical first two moment functions can also be referred to this type of models [7], [8]. Such models allow investigating signals only in the limits of the correlative theory. Higher moment functions and distribution functions cannot be calculated on the base of these models.

However, to determine the accuracy of measurements and statistical processing of photoplethysmographic signals even within the correlative theory, we need to know moments of the orders higher than the second one.

The mathematical model of a photoplethysmographic signal as a linear periodical random process is presented in papers [9], [10]. This model represents the essence of biophysical information about the origin of photoplethysmographic signals. It takes into account their stochastic and rhythmic character, and it allows to carry out signal probabilistic analysis with the help of the method of characteristic functions.

In this paper the mathematical model described in papers [9], [10] will be improved by means of considering the random character of kernel parameters of the corresponding stochastic integral transform. Also, probabilistic characteristics of the new model will be analyzed in this article.

III. Mathematical Model Justification

The photoplethysmographic signal is obtained by an infra-red optoelectronic transformer. Its operation is such that the investigated object is illuminated with the light flux from the light-emitting diode and then the reflected light flux is measured by the photo receiver (photodiode).

The radiation flux detected by the photo receiver consists of two components: a constant component and a variable one. It is known that the variable part is informative. The value of the variable part of light intensity is registered and investigated. It is determined by the amount of the intensity of the light diffused because of the reflection from erythrocytes [11]. It means that this amount is defined by the number and properties of erythrocytes. It is the number of erythrocytes in the light beam that essentially influences the blood light absorption, changing in time randomly and rhythmically according to the cyclic heartbeat.

Every k-th erythrocyte gets into a range explored by the light beam in the time moment $\tau_k, k \in \mathrm{Z}$. It reflects the light with intensity described by the function $\tilde{I}_k(\tau_k, t)$, where τ_k is the moment of the k-th erythrocyte entering into the light beam range; t is the observation moment. The time moments $\tau_k, k \in \mathrm{Z}$ are random.

Every k-th erythrocyte remains in the light beam during the finite time interval $[\tau_k, \tau_k + \tilde{l}_k(\tau_k)]$, where $\tilde{l}_k(\tau)$ >0 is the duration of the k-th erythrocyte presence in the light beam. $\tilde{l}_k(\tau)$ is a random function of τ, since the blood velocity (and, correspondingly, the time of the erythrocyte presence in the light beam) varies over the cardiac cycle, increasing in a systole and decreasing in a diastole.

Thus, $\tilde{I}_k(\tau_k, t)$, as a function of t, has an impulse character, i.e. $\tilde{I}_k(\tau_k, t) = 0$ at $t \notin [\tau_k, \tau_k + \tilde{l}_k(\tau_k)]$.

The size and shape of each erythrocyte as well as its location with respect to the incident radiation are of a random nature that makes pulses $\tilde{I}_k(\tau_k, t)$ random functions. The realization of such function is shown by a dotted line in the Fig. 3.

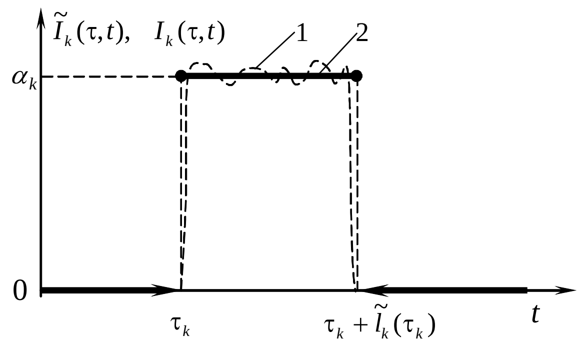

Fig. 3. Realizations of the intensity function: 1 – the reflected impulse ($\tilde{I}_k(\tau,t)$), 2 – the simulated impulse ($I_k(\tau,t)$).

Further simplifications made in the model of the photoplethysmographic signal are the following.

1. The function $I_k(\tau,t)$ is introduced instead of the function $\tilde{I}_k(\tau,t)$ (see Fig. 3):

$$I_k(\tau,t) = \alpha_k \phi\left(t-\tau; \tilde{l}_k(\tau)\right), \tag{1}$$

where $\alpha_k > 0, k \in \mathrm{Z}$ are random variables; $\phi(s;l)$ is a non-random function, which can be presented as:

$$\phi(s;l) = U(s)U(l-s), \; l > 0, \tag{2}$$

where $U(s) = \begin{cases} 0, & s < 0, \\ 1, & s \geq 0 \end{cases}$ is the Heaviside function.

The following assumptions are accepted regarding the process of erythrocytes entering into the light beam:

1. Erythrocytes get into a zone, explored by the light beam, in consecutive time moments $\{\cdots \tau_{-2} < \tau_{-1} < \tau_0 < \tau_1 < \tau_2 \cdots < \tau_k \cdots\}$, with values of intervals $\Delta\tau_k = \tau_k - \tau_{k-1}$, $k \in \mathrm{Z}$ between them being independent random variables.
2. During a rather small time interval $(\tau, \tau + \Delta\tau)$ the probability of appearance of one reflected impulse (erythrocyte entering into the light beam) is equal to $\lambda(\tau)\Delta\tau + o(\Delta\tau)$. Here $\lambda(\tau)$ is a deterministic function, which characterizes the intensity of the erythrocyte appearance in the light beam. It is a function of time τ, as it depends on the cardiac cycle phase, $\lambda(\tau) = \lambda(\tau + T)$, where T is the period of heartbeats. Probability of the appearance of more than one impulse during the small time interval $(\tau, \tau + \Delta\tau)$ has the order of $o(\Delta\tau)$.
3. $\alpha_k > 0, k \in \mathrm{Z}$ are independent similarly distributed random variables with the distribution function $F_\alpha(x) = \mathrm{P}(\alpha_k < x)$, where $F_\alpha(x) = 0$ at $x \leq 0$.
4. $\tilde{l}_k(\tau) > 0$, $k \in \mathrm{Z}$ are independent similarly distributed random processes, with one-dimensional distribution functions $F_l(x;\tau) = \mathrm{P}(\tilde{l}_k(\tau) < x)$ (here $F_l(x;\tau) = 0$ at $x \leq 0$) being T-periodical, i.e. $F_l(x;\tau) = F_l(x;\tau + T)$.
5. Random variables $\alpha_k, k \in \mathrm{Z}$, and processes $\tilde{l}_k(\tau)$, $k \in \mathrm{Z}$, are independent as well as they are not dependent regarding time moments $\tau_k, k \in \mathrm{Z}$.

Based on p. 1, 2 we can conclude that the process of reflected impulses occurrence is a Poisson flow with the parameter $\lambda(\tau)$, $\tau \in (-\infty, \infty)$.

Environment of propagation of reflected light impulses has linear properties. Therefore, the intensity of the light obtained by the photo receiver (the total reflected signal), is a sum of intensities of light impulses reflected by each erythrocyte separately.

Thus, the variable term $\xi(t)$ of the photoplethysmographic signal can be presented as:

$$\xi(t) = \sum_{k=-\infty}^{\infty} I_k(\tau_k, t) = \sum_{k=-\infty}^{\infty} \alpha_k \phi(t - \tau_k; \tilde{l}_k(\tau_k)), \; t \in (-\infty, \infty). \tag{3}$$

The expression (3) can be presented in a form, which is more convenient for solving theoretical and practical tasks. For that let us introduce a non-uniform simple Poisson process $\pi(\tau)$, $\tau \in (-\infty,\infty)$, $\mathrm{P}(\pi(0)=0)=1$, which corresponds to a non-stationary Poisson flow with the parameter $\lambda(\tau)$.

The Poisson flow presented above characterizes a process of the appearance of reflected light impulses (i.e. jumps of the process $\pi(\tau)$ happen in the time moments $\tau_k, k \in \mathrm{Z}$). Besides, let $\pi_1(\tau)$, $\tau \in (-\infty,\infty)$, $\mathrm{P}(\pi_1(0)=0)=1$ be non-uniform generalized Poisson process, jumps of which happen at the same time moments $\tau_k, k \in \mathrm{Z}$ and the value of each jump in the time moment τ_k is equal to the random variable α_k.

In this case the process $\xi(t)$ can be written as the stochastic integral:

$$\xi(t) = \int_{-\infty}^{\infty} \phi\left(t - \tau; \tilde{l}_{\pi(\tau)}(\tau)\right) d\pi_1(\tau), \; t \in (-\infty, \infty). \tag{4}$$

The random process (4) is called a conditional linear random process. The function $\phi(s;l)$ is the kernel of the transform (4), which describes a shape of reflected impulses and their duration; $\pi_1(\tau)$ is the generating process, which characterizes moments of the impulses appearance as well as their amplitudes.

IV. Periodical Characteristic Function of the Signal Model

Let $f_\xi(u_1, u_2, ..., u_n; t_1, t_2, ..., t_n) = \mathrm{M}e^{i\sum_{k=1}^{n} u_k \xi(t_k)}$, $u_k, t_k \in (-\infty,\infty), k = \overline{1,n}, \; i = \sqrt{-1}$ be a characteristic function of a conditional linear random process (4).

We can show that its logarithm may be presented in the next form:

$$\ln f_{\xi}(u_1, u_2, ..., u_n; t_1, t_2, ..., t_n) = \\ = \int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\left(e^{ix\sum_{k=1}^{n}u_k\phi(t_k-\tau;\, y)}-1\right)dF_{\alpha}(x)dF_l(y;\tau)\lambda(\tau)d\tau \,. \qquad (5)$$

The characteristic function (5) is a *T*-periodical function of its time arguments, i.e.:

$$\ln f_{\xi}(u_1, u_2, ..., u_n; t_1, t_2, ..., t_n) = \\ = \ln f_{\xi}(u_1, u_2, ..., u_n; t_1+T, t_2+T, ..., t_n+T) \,. \qquad (6)$$

So, we have:

$$\ln f_{\xi}(u_1, u_2, ..., u_n; t_1+T, t_2+T, ..., t_n+T) = \\ = \int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\left(e^{ix\sum_{k=1}^{n}u_k\phi(t_k+T-\tau;\, y)}-1\right)dF_{\alpha}(x)dF_l(y;\tau)\lambda(\tau)d\tau \,. \qquad (7)$$

Having executed the change of variable $\tau - T = s$ in the formula (7), we get:

$$\ln f_{\xi}(u_1, u_2, ..., u_n; t_1+T, t_2+T, ..., t_n+T) = \\ = \int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\left(e^{ix\sum_{k=1}^{n}u_k\phi(t_k-s;\, y)}-1\right)dF_{\alpha}(x)dF_l(y;s+T)\lambda(s+T)ds \,. \qquad (8)$$

Taking into account that $\lambda(s+T)=\lambda(s)$, and $F_l(y;s+T)=F_l(y;s)$ we can see that the condition (6) is satisfied.

Random processes, the characteristic function of which satisfies (6), are called periodical by Sluckij processes [12], or cyclostationary processes [8]. Thus, the proposed mathematical model of a photoplethysmographic signal in the form (4) can be named a conditional linear periodical random process.

The periodicity of the characteristic function (6) of the random process $\xi(t)$ results in the periodicity of all its moment functions. It means that the mathematical expectation $m_{\xi}(t) = \mathrm{M}\xi(t)$ and the correlative function $R_{\xi}(t_1,t_2) = \mathrm{M}\left(\left(\xi(t_1)-m_{\xi}(t_1)\right)\left(\xi(t_2)-m_{\xi}(t_2)\right)\right)$ of the random process $\xi(t)$ are periodical, i.e.:

$$m_{\xi}(t) = m_{\xi}(t+T), \\ R_{\xi}(t_1,t_2) = R_{\xi}(t_1+T, t_2+T) \,.$$

Thus, the process $\xi(t)$ is also a periodical correlative one.

Periodicity of probabilistic characteristics of the random process $\xi(t)$, $t \in (-\infty,\infty)$, causes the weakly stationarity of the random sequence $\xi(t_1+kT)$ for any $t_1 \in [0,T)$, $k \in \mathrm{Z}$. Therefore, it is possible to realize a joint statistical processing of any $s \geq 1$ stationary and weakly stationary dependent random sequences:

$$\xi(t_1+kT),\ \xi(t_2+kT),\ \ldots,\ \xi(t_s+kT), \\ t_1, t_2, ..., t_s \in [0,T),\ k \in \mathrm{Z} \,.$$

For that, well-known methods of statistical analysis and prognosis of stationary time series may be used. The last statement is a fundamental principle of statistical processing of random signals with periodical probabilistic characteristics.

V. Conclusions

1.The new mathematical model of a photoplethysmographic signal in the form of a conditional linear periodical random process is developed based on biophysical information about the photoplethysmographic signal origin. In contrast to the results obtained in the papers [9, 10], the new model takes into account that erythrocytes stay in a light beam during a random time.

2.The characteristic function of the mathematical model of the photoplethysmographic signal is derived. Its periodicity is proved.

3.The periodicity of probabilistic characteristics of the photoplethysmographic signal mathematical model results in a possibility to realize signal statistical processing based on the well-known methods of cyclostationary processes statistical inference.

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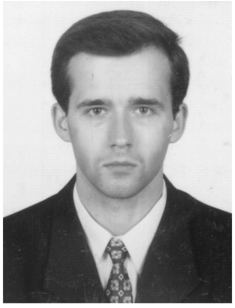

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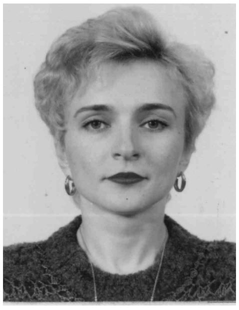

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Scientific-research interests deal with mathematical and simulation modeling of rhythmic signals as well as with their prognosis and statistical analysis. Area of practical applications of scientific interests is gas consumption in Ukraine, which is known as one of the most important fields of Ukrainian economy. Some scientific works are devoted to making decisions in medicine using statistical criterions.

Mihails Frizs, Bogdana Mlinko, Eļena Mul, Natalija Zagorodnaja. Nosacīts lineārs periodisks gadījuma process kā fotopletizmogrāfiska signāla modelis
Šajā darbā pamatota adekvāta fotopletizmogrāfiskā signāla matemātiskā modeļa izveidošanas aktualitāte, kā galvenais nosacījums tālākai statistiskās analīzes metožu izstrādāšanai, kas pamatojas uz šo modeli. Autori īsi apraksta zināmās fotopletizmogrāfiskā signāla matemātiskās modelēšanas metodes, kā arī analizē to priekšrocības un trūkumus. Darbā formulētas galvenās prasības jaunam matemātiskam modelim: iespēja ievērot stohastiskā un periodiskā signāla raksturu, iespēja veikt signāla modelēšanu un tā veidošanas uzskaiti, ievērojot atbilstību starp reālo signālu un matemātisko modeli, kam vienlaikus ir jābūt relatīvi vienkāršam un ērtam. Autori piedāvā jaunu fotopletizmogrāfiskā signāla matemātisko modeli kā nosacītu periodisku lineāru gadījuma procesu, kurā atspoguļotas pētāmā signāla rašanās biofiziskās īpatnības, ievērots tā stohastiskais un vienlaikus cikliskais raksturs. Piedāvātā modeļa īpašības ļauj analizēt fotopletizmogrāfisko signālu ar raksturīgās funkcijas metodi. Darbā arī pierādīts, ka nosacīta periodiska lineāra procesa raksturīgā funkcija ir periodiska, kas ļauj šo procesu aplūkot kā periodisku Slucka gadījuma procesu vai ciklostacionāru procesu. Gadījuma procesa raksturīgās funkcijas periodiskuma sekas ir tā momentānu funkciju periodiskums. Šis fakts ļauj uzskatīt gadījuma procesu par periodiski korelējošu.

Михаил Фрыз, Богдана Млынко, Елена Муль, Наталия Загородная. Условный линейный периодический случайный процесс как математическая модель фотоплетизмосигнала
В данной работе обоснована актуальность построения адекватной математической модели фотоплетизмосигнала, как главного условия дальнейшей разработки методов статистического анализа на ее основе. Авторами кратко описаны известные подходы к математическому моделированию фотоплетизмосигнала, а также проанализированы их преимущества и недостатки. В работе приведены основные требования к новой математической модели, а именно: возможность учитывать стохастический и периодический характер сигнала, возможность проведения имитационного моделирования сигнала и учета механизма его формирования, соответствие реального сигнала и его математической модели при ее относительной простоте и удобстве. Авторами предложена новая математическая модель фотоплетизмосигнала в виде условного периодического линейного случайного процесса, в которой отображены биофизические особенности происхождения исследуемого сигнала, учтены его стохастическая и одновременно циклическая природа. Свойства предложенной модели позволяют анализировать фотоплетизмосигнал методом характеристических функций. В работе также доказано, что характеристическая функция условного периодического линейного процесса является периодической, что позволяет рассматривать его как периодический по Слуцкому случайный процесс. Следствием периодичности характеристической функции случайного процесса является периодичность его моментных функций. Этот факт также позволяет считать случайный процесс периодически коррелированным.