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# Solution of Electric Circuits by a 9-stage Hermite-Birkhoff-Taylor DAE Solver of Order 11

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***Abstract* – The ODE solver HBT(11)9 is expanded into a differential algebraic equation (DAE) solver, called HBT(11)9DAE, for nonstiff and moderately stiff systems of fully implicit DAEs. Pryce's structural pre-analysis and automatic differentiation for DAEs is adapted to the new solver. The stepsize is controlled by a local error estimator. HBT(11)9DAE uses only the first five derivatives of y as opposed to 11 with the Taylor series method of order 11. HBT(11)9DAE is applied to electric circuit problems, and, on the basis of the CPU time and the maximum global error, it is superior to Dormand – Prince's DP(8,7)13M adapted to solve DAEs.**

***Keywords* – General linear method for non-stiff DAE, Hermite-Birkhoff method, Pryce structural analysis for Taylor method, electric circuit.**

## I. INTRODUCTION

The ODE solver HBT(11)9[8] is expanded into a differential algebraic equation (DAE) solver, called HBT(11)9DAE, by the addition of a modification of Pryce's structural pre-analysis and automatic differentiation (AD) techniques to solve smooth nonstiff and moderately stiff systems of fully implicit DAEs of arbitrarily high fixed index of the form

$$f(t, y(t), y^{(1)}(t), y^{(2)}(t), \ldots) = 0, \tag{1}$$

with consistent initial conditions, where $y : \mathbb{R} \to \mathbb{R}^d$ and $f : \mathbb{R} \times \mathbb{R}^d \times \cdots \to \mathbb{R}^d$.

HBT(11)9DAE combines Taylor and Runge–Kutta methods. These are linked by predictors which use $y^{(1)}$ to $y^{(5)}$ at current points $t_n$, and only $y^{(1)}$ at the off-step points.

Recently, improved Taylor series methods have been developed for solving smooth nonlinear and fully implicit high index DAEs containing derivatives of any order [13], [1], [6], [7] with polynomial cost in the number of digits of accuracy [5]. The main cost in solving an ODE/DAE by a Taylor series method lies in the repeated evaluation of the Taylor coefficients of the functions involved. For smooth DAEs, Pryce [13] used automatic differentiation techniques with a structural analysis to compute the coefficients of Taylor series solutions about the current time-point. An adaptation of this procedure is implemented in [10] and [11] for HBT(10)9DAE and HBT(12)5DAE, respectively.

In this paper, increased efficiency is achieved by the addition of off-step points, where only $y^{(1)}$ is required, and the order of the necessary $y^{(j)}(t_n)$ is reduced to 5 as compared to 11 with the Taylor series method of order 11, denoted by T11. HBT(11)9DAE takes advantage of this fact and relies on the above modification of Pryce's procedure.

Forcing an expansion of the numerical solution to agree with a Taylor expansion of the true solution to order 11 leads to a combination of Taylor- and Runge–Kutta-type order conditions which are reorganized into Vandermondetype linear systems whose solutions are the coefficients of the method. These coefficients are obtained only once as solutions of these systems by means, say, of Gaussian elimination.

We call DP(8,7)13MDAE, the DAE solver obtained by expanding the Dormand–Prince ODE solver DP(8,7)13M [12]. On the basis of the CPU time (in seconds) and maximum global error, HBT(11)9DAE wins over DP(8,7)13MDAE in solving electric circuit problems.

Section II briefly sketches the ODE solver HBT(11)9 found in [8]. Section III summarizes Pryce's structural analysis for DAEs. Section IV considers the analytical solution of a linear circuit. Section V presents numerical methods and compares DAE methods.

## II. ONE-STEP HBT(11)9

The predictors $P_2$ to $P_9$, integration formula IF, order conditions, off-step points $\{c_i\}$, $i = 1, 2, \ldots, 9$, Vandermondetype formulation and region of absolute stability of HBT(11)9 found in [8] are the same for ODE and DAE methods. But for DAEs, we need to add a step control predictor, $P_{10}$, found in [9] to advance integration for HBT(11)9DAE.

## III. STRUCTURAL ANALYSIS FOR DAES

*Definition 1:* A *consistent point*, $P(t^*)$, of (1) is defined as a scalar $t^*$ together with a set of scalars $\eta_{j\ell}$, for $j = 1, 2, \ldots, d$ and $\ell = 1, 2, \ldots, L < \infty$, such that there exists a solution to (1) in a neighbourhood of $t = t^*$ with $y_j^{(\ell)}(t^*) = \eta_{j\ell}$ and the solution is unique.

We assume that system (1) is sufficiently differentiable with respect to its independent variable $t \in \mathbb{R}$ and all its dependent variables $y_j^{(k)} \in \mathbb{R}$ in a neighborhood of a consistent point. The Taylor method for DAEs requires a pre-analysis which solves an assignment problem by Pryce's signature method [14].

Let the nonnegative integer $\kappa_i$, called **equation-offset**, be the number of times the $i$th equation $f_i = 0$ has to be

differentiated to be reduced to an ODE. Also, let the nonnegative integer $\delta_j$, called **variable-offset**, be the highest order to which derivatives of $y_j$ occurs in any of the *d* differentiated equations. We let

$$\rho_{ij} = \kappa_i - \delta_j. \tag{2}$$

The following definition and theorem [13], [14] are central pieces of Pryce's staggered solution of DAE's by Taylor series.

*Definition 2:* The $d \times d$ matrix $J = (J_{ij})$

$$J_{ij} = \begin{cases} \partial(f_i)/\partial\left(y_j^{(\rho_{ij})}\right), & \text{if } y_j^{(\rho_{ij})} \text{ is present in } f_i, \\ 0, & \text{if } y_j^{(\rho_{ij})} \text{ is absent from } f_i \\ & \text{or } \rho_{ij} < 0, \end{cases} \tag{3}$$

is called a *system Jacobian matrix* of equation (1).

*Theorem 1:* If the DAEs (1) are sufficiently smooth in a neighborhood of a consistent point P, then the Taylor series method applied at *P* succeeds if and only if the $d \times d$ system Jacobian matrix *J* is nonsingular at *P*.

We summarize Pryce's structural analysis and the corresponding algorithm as follows.

(a) Form a $d \times d$ *signature matrix* $\Sigma = (\sigma_{ij})$ with

$$\sigma_{ij} = \begin{cases} \text{highest order of the derivatives of } y_j \text{ present} \\ \quad \text{in the } i\text{th equation } f_i = 0, \\ -\infty \text{ if the variable } y_j \text{ is absent} \\ \quad \text{from the } i\text{th equation } f_i = 0. \end{cases}$$

(b) Solve an assignment problem to determine a *highest value transversal* (HVT), which is a subset of indices (*i*,*j*) describing just one element in each row and each column, such that $\sum \sigma_{ij}$ is maximized and finite. We assume that such an HVT exists.

(c) Determine the smallest offsets $\kappa_i$ and $\delta_j$ of the problem such that

$$\delta_j - \kappa_i \geq \sigma_{ij} \text{ for all } 1 \leq i \leq d, 1 \leq j \leq d,$$

with equality on HVT. The *structural index* is then defined as

$$\nu = \max_i \kappa_i + \begin{cases} 0, & \text{if all } \delta_j > 0, \\ 1, & \text{if some } \delta_j = 0. \end{cases}$$

(d) Form the *system Jacobian J* defined in (3).

(e) If *J* is non-singular at a consistent point *P* at **step point** $t^* = t_{n+1}$, compute the solution by Theorem 1, that is, substitute the expansion

$$y(t) = \sum_{\ell \geq 0} \frac{1}{\ell!} y^{(\ell)}(t^*)(t - t^*)^\ell$$

into equations (1) and expand in Taylor series to obtain

$$f(t, y, y^{(1)}, y^{(2)}, \ldots) = \sum_{q \geq 0} \frac{1}{q!} \frac{d^q f}{dt^q}(t^*)(t - t^*)^q = 0.$$

The system

$$\frac{d^q f}{dt^q}(t^*) = 0 \tag{4}$$

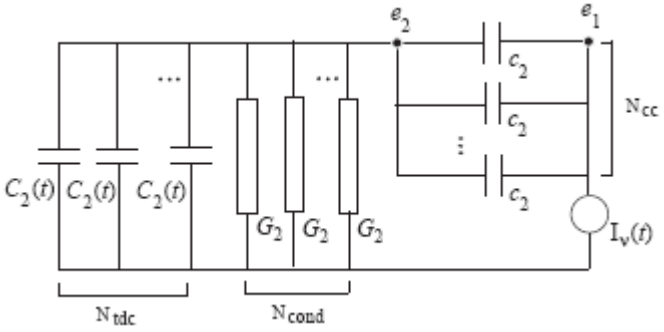


Fig. 1. Linear circuit with $N_{tdc}$ time-dependent capacitances, $N_{cond}$ constant conductances and $N_{cc}$ constant capacitances

is solved in a "staggered" way for $y^{(\ell)}(t^*)$ by means of the following "lockstep" solution scheme: solve

$$\frac{d^{k+\kappa_i} f_i}{dt^{k+\kappa_i}}(t^*) = 0 \text{ for } y_j^{(k+\delta_j)}(t^*), \quad i, j = 1, \ldots, d, \tag{5}$$

for stages $k = 0, 1, \ldots, 5$, where all other $y_j^{(\ell)}(t^*)$ occurring in (5) have already been found at earlier stages. In general, system (5) is linear in the corresponding unknowns and involves the same Jacobian *J* for $k = 1, 2, \ldots$, where *J* is computed in step (d). It is to be noted that, at the *initial stages* $k < 0$, the initial guesses $y_j^{(s)}, s = 0, 1, \ldots, \delta_j - 1$, are acquired by means of the components of $y_{n+1}$. The local error control will make these guesses close-to-consistent. The close-to-consistent point $y_{n+1}$ is then projected onto the enlarged DAE constraints system by a root finding process which is iterated to convergence to get the final value $y_{n+1}$ of the consistent point $P(t_{n+1})$.

(f) If *J* is non-singular at a consistent point *P* at **off-step point** $t^* = t_n + c_j h$, system (4) is similarly solved by means of (5) for stage $k = 0$ and (for certain problems) $k$=1, where all other $y_j^{(\ell)}(t^*)$ occurring in (5) have already been found at earlier stages or acquired by means of the components of $y_{n+c_j}$.

## IV. Analytical Solution of a Linear Circuit

One of the primary applications of the technique developed in this paper is in the simulation of electrical circuits. The challenge in simulating circuits with capacitors and resistors

comes from the oscillatory behavior of circuit potentials when subjected to an alternating voltage.

In this section, we consider the analytical solution of a modification of a linear circuit with a time-dependent capacitance considered in [4]. The modified nodal analysis applied to the circuit in hand shown in Fig. 1 leads directly to the following DAE system,

$$\begin{aligned} &N_{\mathrm{tdc}}[C_2(t)'e_2(t) + C_2(t)e_2'(t)] \\ &\quad + N_{\mathrm{cond}}G_2e_2(t) - N_{\mathrm{cc}}c_2[e_1'(t) - e_2'(t)] = 0, \\ &N_{\mathrm{cc}}c_2[e_1'(t) - e_2'(t)] - j(t) = 0, \\ &e_1(t) - \nu(t) = 0, \end{aligned} \tag{6}$$

where $C_2(t)$ is an arbitrary function, $\nu(t)$ is an input voltage, $c_2$ and $G_2$ are arbitrary constants and

- $N_{\mathrm{tdc}}$ is the number of capacitors with time-dependent capacitance $C_2(t)$;
- $N_{\mathrm{cond}}$ is the number of conductors with constant conductance $G_2$;
- $N_{\mathrm{cc}}$ is the number of capacitors with constant capacitance $c_2$.

This is a convenient test case because the numbers, $N_{\mathrm{tdc}}$, $N_{\mathrm{cond}}$ and $N_{\mathrm{cc}}$, of elements can be varied systematically and the input functions $e_2(t)$ and $C_2(t)$, and the constants $c_2$ and $G_2$ can be chosen arbitrarily to determine the oscillatory behavior of the potentials $e_1(t)$ and the current $j(t)$; moreover, the analytical solution is available.

*Algorithm 1:* **This algorithm solves system** (6) **for** $e_1(t)$ **and** $j(t)$.

Given arbitrary input functions $C_2(t)$ and $e_2(t)$ such that $C_2'(t)$ and $e_2'(t)$ exists, the following algorithm obtains the general solution $e_1(t)$, and $j(t)$ of (6).

(a) Compute

$$\begin{aligned} e_1'(t) = \frac{1}{N_{\mathrm{cc}}c_2}&[N_{\mathrm{tdc}}(C_2'(t)e_2(t) + C_2(t)e_2'(t)) \\ &+ N_{\mathrm{cond}}G_2e_2(t) - N_{\mathrm{cc}}c_2(-e_2'(t))], \end{aligned} \tag{7}$$

$$j(t) = N_{\mathrm{cc}}c_2[e_1'(t) - e_2'(t)]. \tag{8}$$

(b) If $e_1(t) = \int_0^t e_1'(\theta)d\theta$ does not exist analytically then an analytical solution to system (6), if it exists, cannot be found by mean of this algorithm.

(c) If $e_1(t)$ exists analytically, then $\nu(t) = e_1(t)$.

## V. Numerical results

The derivatives $y^{(1)}$ up to $y^{(5)}$ of the Taylor series are calculated at each integration step by Pryce's automatic differentiation technique.

The **error** in a numerical solution at time $t_n$ is the norm $\|y_n - y(t_n)\|_\infty$ of the difference between the numerical solution $y_n$ and a reference solution $y(t_n)$ given by means of the analytical solution, or T12 or HBT(12)5DAE [11] at stringent tolerance, as appropriate.

The **maximum global error** (MGE) is the maximum of the errors over all the integration steps.

Using the data (MGE, CPU) we plot the curve $(-\log_{10}(\mathrm{MGE}), \log_{10}(\mathrm{CPU}))$, in a least-squares sense, by means of Matlab's polyfit. Then taking $-\log_{10}(\mathrm{MGE}) = j$ as an integer, we obtain $\log_{10}(\mathrm{CPU})$ from the curve and hence $\mathrm{CPU}_{1,ij}$ and $\mathrm{CPU}_{2,ij}$ for methods 1 and 2, respectively, associated with problem *i*.

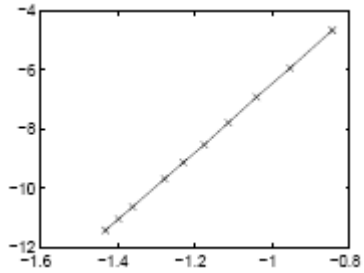


Fig. 2. $\log_{10}$(error) at $t_n = \pi$ (vertical axis) versus $\log_{10} h$ (horizontal axis) for constant stepsize HBT(11)9DAE applied to test problem (10) in the interval $t \in [0, \pi]$.

The **CPU time percentage efficiency gain** (CPU PEG) is defined by the formula (cf. Sharp [15]),

$$(\mathrm{CPU\ PEG})_i = 100\left[\frac{\sum_j \mathrm{CPU}_{2,ij}}{\sum_j \mathrm{CPU}_{1,ij}} - 1\right]. \tag{9}$$

*A. Numerical verification of the order of HBT(11)9DAE*

To show the relevance of the theoretical order of HBT(11)9DAE, we have applied the method with constant stepsize to the following test problem found in [2, pp. 98–99] and [3], over the interval $t \in [0, \pi]$ with the following initial conditions:

$$\begin{aligned} v_1' - tv_2' + v_1 - (1+t)v_2 = 0, &\qquad v_1(0) = 1, \\ v_2 - \sin t = 0, &\qquad v_2(0) = 0, \end{aligned} \tag{10}$$

and exact solution:

$$v_1(t) = e^{-t} + t\sin t, \quad v_2(t) = \sin t.$$

This problem is used to test several Runge–Kutta methods which are likely candidates for solving stiff or differential algebraic systems.

In Fig. 2, the error at $t_n = \pi$ is plotted for different stepsizes $h$ in a log-log scale so that the curve, which fits the data $(\log_{10} h, \log_{10}(\|y_n - y(t_n)\|_\infty))$ in a least-squares sense, appears as a straight line with slope $k$ whenever the leading term of the error is of order $k$, that is, $\|y_n - y(t_n)\|_\infty = O(h^k)$. For HBT(11)9DAE, we have a straight line of slope 11, thus confirming the order of the method.

*B. A linear circuit with time-dependent capacitances*

As an example of a DAE problem, we consider the linear circuit (6) with $N_{\mathrm{tdc}}$, $N_{\mathrm{cond}}$ and $N_{\mathrm{cc}}$ replaced by $N_{\mathrm{c}}$,

$$\begin{aligned} &N_{\mathrm{c}}[C_2'(t)e_2(t) + C_2(t)e_2'(t)] + N_{\mathrm{c}}G_2e_2(t) \\ &\quad - N_{\mathrm{c}}c_2[e_1'(t) - e_2'(t)] = 0, \\ &N_{\mathrm{c}}c_2[e_1'(t) - e_2'(t)] - j(t) = 0, \\ &e_1(t) - \nu(t) = 0. \end{aligned} \tag{11}$$

Given the input functions and parameters,

$$e_2(t) = \sin t + \cos t, \quad C_2(t) = 1 + 0.25(\sin t + \cos t), \quad c_2 = 1, \quad G_2 = 2, \quad (12)$$

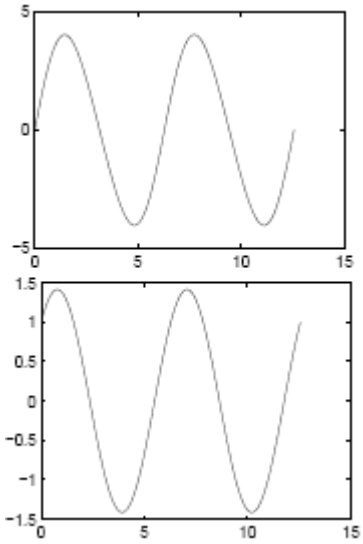

Fig. 3. Solution $e_1(t)$ (top) and $e_2(t)$ (bottom) of system (11)

TABLE I
CPU PEG OF HBT(11)9DAE OVER DP(8,7)13MDAE FOR PROBLEM (11).

| $N_c$ | CPU PEG of HBT(11)9DAE over DP(8,7)13MDAE |
|---|---|
| 1 | 173% |
| 400 | 158% |
| 800 | 172% |

Algorithm 1 obtains $\nu(t) = 4\sin t + 0.25\sin 2t$ and the exact solution of (11) which, by the choice $c_2 = 1$ and $G_2 = 2$, simplifies to

$$\begin{aligned} e_1(t) &= 4\sin t + 0.25\sin 2t, \\ e_2(t) &= \sin t + \cos t, \\ j(t) &= (3\cos t + 0.5\cos 2t + \sin t)N_c. \end{aligned}$$

Then the initial conditions are

$$e_1(0) = 0, \qquad e_2(0) = 1, \qquad j(0) = 3.5N_c.$$

For system (11) over the interval $t \in [0, 4\pi]$, Fig. 3 displays the voltages $e_1(t)$ and $e_2(t)$ , and, for $N_c$ = 1, $N_c$ = 400 and $N_c$ = 800, Fig. 4 displays the current $j(t)$.

Figure 5 displays the CPU time (CPU) (horizontal axis) versus $\log_{10}(\text{MGE})$ (vertical axis). It is seen from the figure that HBT(11)9DAE compares favorably with DP(8,7)13MDAE on the basis of CPU versus MGE and also from the CPU PEG listed in Table I. This shows the efficiency of HBT(11)9DAE for problems of this type with increasing $N_c$ and similar response characteristics.

*C. A linear circuit with time-dependent capacitances and increasing number of conductances*

As a second example, we consider a particular case of the linear circuit with time-dependent capacitances considered in section IV and which is a representative of circuits with oscillatory responses.

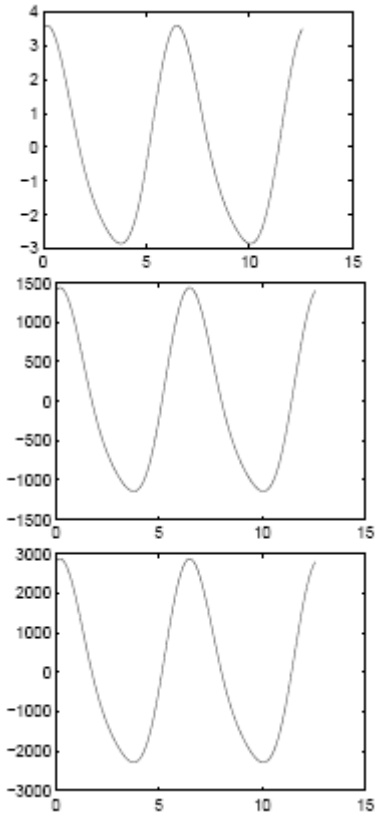

Fig. 4. Solution $j(t)$ of system (11) for $N_c$ = 1, $N_c$ = 400 and $N_c$ = 800 (from top to bottom)

Given the input functions and parameters,

$$e_2(t) = \sin(t\omega), \quad C_2(t) = 0.25\sin(t\omega), \quad c_2 = 1, \quad G_2 = 2, \quad (13)$$

Algorithm 1 obtains $\nu(t) = e_1(t)$ and the exact solution of problem (6),

$$\begin{aligned} e_1(t) &= -\frac{1}{N_{cc}c_2}\left[\frac{1}{8}N_{tdc}\cos(2t\omega) + N_{cond}\frac{G_2}{\omega}\cos(t\omega) - N_{cc}c_2\sin(t\omega)\right], \\ e_2(t) &= \sin(t\omega), \\ j(t) &= \frac{1}{4}N_{tdc}\omega\sin(2t\omega) + N_{cond}G_2\sin(t\omega). \end{aligned} \quad (14)$$

To apply HBT(11)9DAE, we rewrite system (6) in the form

$$\begin{aligned} &N_{tdc}[C_2'(t)e_2(t) + C_2(t)e_2'(t)] + N_{cond}G_2e_2(t) \\ &\quad - N_{cc}c_2[e_1'(t) - e_2'(t)] = 0, \\ &N_{cc}c_2[e_1'(t) - e_2'(t)] - j(t) = 0, \\ &e_1(t) - \nu(t) = 0, \end{aligned} \quad (15)$$

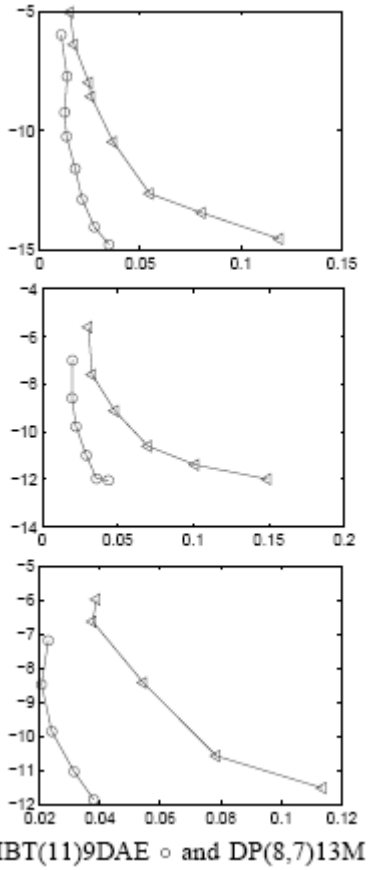


Fig. 5. CPU time in seconds (horizontal axis) is plotted versus log10 (MGE) (vertical axis) for problem (11) with $N_c = 1$, $N_c = 400$ and $N_c = 800$ (from top to bottom) over the interval $t \in [0, 4\pi]$

where $C_2(t) = 0.25\sin(t\omega)$, $c_2 = 1$ and $G_2 = 2$. The initial conditions are

$$e_1(0) = -\frac{1}{N_{cc}c_2}\left[\frac{1}{8}N_{tdc} + N_{cond}\frac{G_2}{\omega}\right],$$
$$e_2(0) = 0,$$
$$j(0) = 0.$$

and the exact solution (14) is used as a comparison for the numerical solution.

Let $N_{cond}$ denote the number of conductors in parallel. In our test case, $N_{cond}$ will take the *increasing values* 1, 400, 800, 1200. We consider system (15) *over the interval* $t \in [0, 0.2]$ with $N_{cond}$ conductors in parallel and fixed parameter values $N_{tdc} = 1$, $N_{cc} = 1$ and large $\omega = 200\pi$. This test case shows the efficiency of HBT(11)9DAE for problems of this type with increasing $N_{cond}$ and large ω leading to similar response characteristics. The response changes with increasing $N_{cond}$, making the circuit more challenging to simulate.

For system (15), Fig. 6 displays the voltage $e_2(t)$, Figs. 7 and 8 display the voltage $e_1(t)$ and the current $j(t)$, respectively, for increasing $N_{cond}$. The response changes slowly with increasing $N_{cond}$, making the already oscillatory equations stiffer and harder to solve increasing $N_{cond}$, making the already oscillatory equations stiffer and harder to solve.

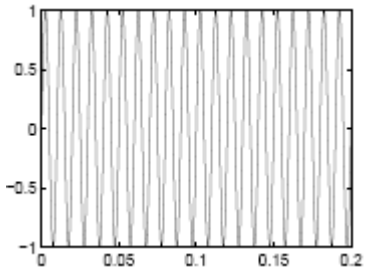

Fig. 6. Solution $e_2(t)$ of system (15).

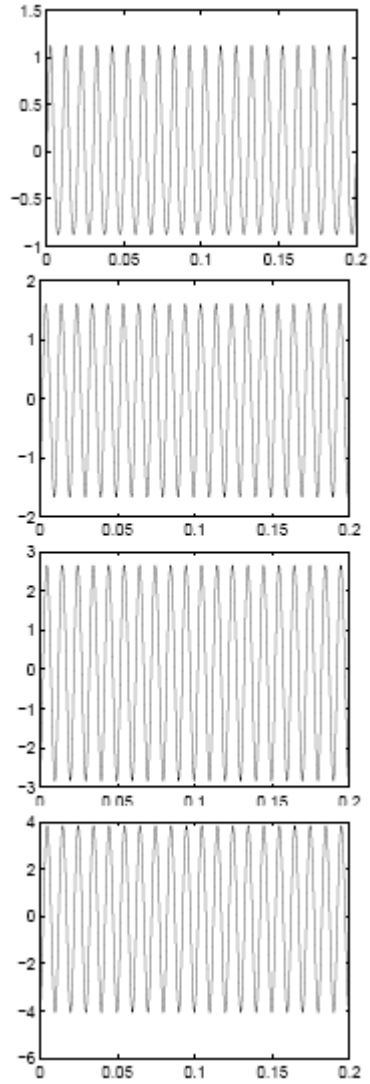

Fig. 7. Solution $e_1(t)$ of system (15) for Ncond = 1, Ncond = 400, Ncond = 800 and Ncond = 1200 (from top to bottom)

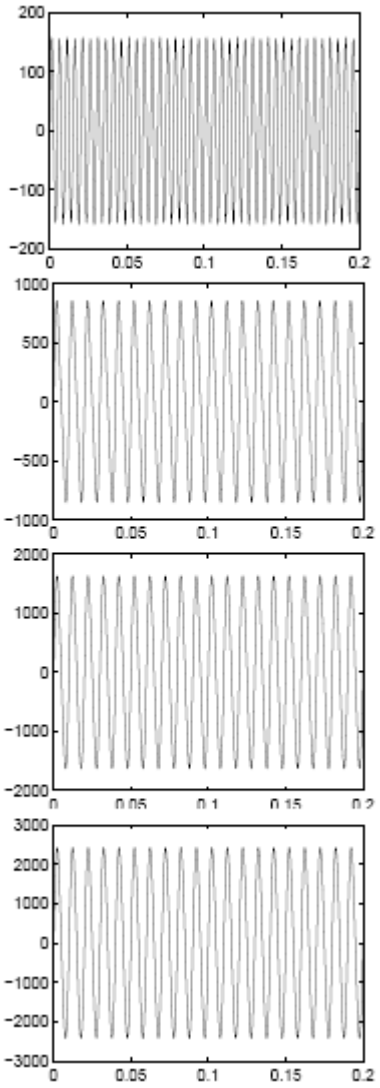

Fig. 8. Solution $j(t)$ of system (15) for $N_{cond} = 1$, $N_{cond} = 400$, $N_{cond} = 800$ and $N_{cond} = 1200$ (from top to bottom)

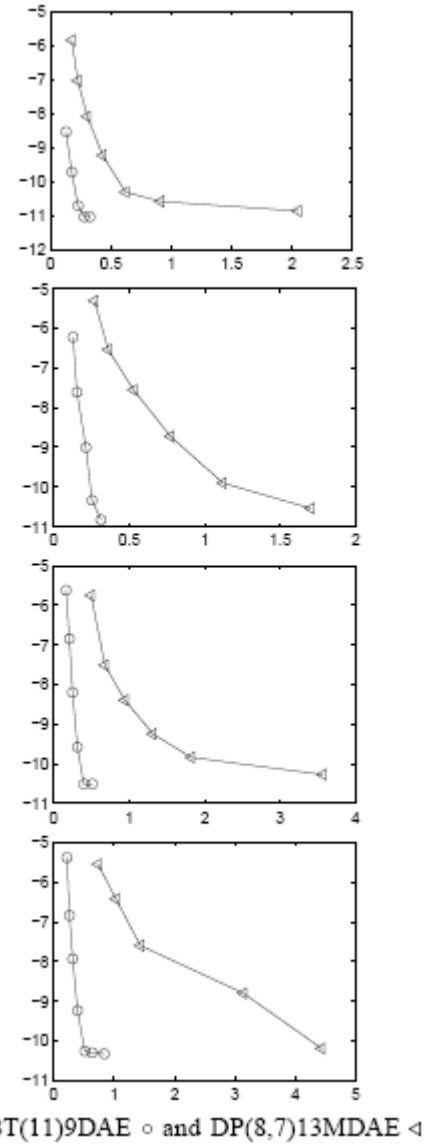


Fig. 9. CPU time in seconds (horizontal axis) is plotted versus $\log_{10}$ (MGE) (vertical axis) for the DAE problem (15) for $N_{cond} = 1$, $N_{cond} = 400$, $N_{cond} = 800$ and $N_{cond} = 1200$ (from top to bottom)

TABLE II
CPU PEG OF HBT(11)9DAE OVER DP(8,7)13MDAE FOR PROBLEM (15) WITH INCREASING NCOND .

| $N_{cond}$ | CPU PEG of HBT(11)9DAE over DP(8,7)13MDAE |
|---|---|
| 1 | 307% |
| 400 | 327% |
| 800 | 309% |
| 1200 | 494% |

In Fig. 9, the CPU time (CPU) (horizontal axis) is plotted versus $\log_{10}(\mathrm{MGE})$ (vertical axis) for problem (15) with increasing $N_{cond}$. It is seen from the figure that HBT(11)9DAE becomes more efficient with increasing $N_{cond}$, compared with DP(8,7)13MDAE on the basis of CPU versus MGE and also from the CPU PEG listed in Table II. These numerical results demonstrate the efficiency of our HBT(11)9DAE for problems of this type with increasing $N_{cond}$ and large $\omega = 200\pi$ and also show that the numerical solution of HBT(11)9DAE approximates very well the exact solution.

## VI. CONCLUSION

A one-step 9-stage Hermite–Birkhoff–Taylor (HBT) method of order 11, called HBT(11)9 constructed in [8] for solving ODEs, is coupled with a modification of Pryce's pre-structural analysis and automatic differentiation to solve

DAE's. The new method is named HBT(11)9DAE. The stepsize is controlled by a local error estimator. HBT methods with five derivatives, $y^{(1)}$ to $y^{(5)}$, appear to be promising for linear circuits with time dependent capacitances with increasing sizes modeled as DAEs in the light of the numerical results since methods of high order can be derived and implemented efficiently. Furthermore, since these methods use a small number of derivatives, they may be useful for high dimensional problems.

## ACKNOWLEDGMENT

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**Truongs Nguen – Ba, Hemza Jagubs, Hans Hao, Remi Vajenkurs. Elektrisko ķēžu modelēšana ar 11. kārtas deviņsoļa Ermita- Birkhofa – Teilora metodi diferenciāli – algebrisko vienādojumu risināšanai**
Algoritms HBT11(9) parasto diferenciālvienādojumu risināšanai ir modificēts un pārveidots par algoritmu HBT11(9)DAE nestingru un mēreni cietu pilnīgi aizklātu diferenciāli- algebrisko vienādojumu (DAV) risināšanai. Jaunajā algoritmā izmantota Praisa struktūranalīze un DAV automātiskā diferencēšana . Soļa kontrolei izmanto lokālo kļūdas novērtējumu. Metodē HBT11 (9) DAE ir izmantoti tikai pirmie pieci funkcijas atvasinājumi atšķirībā no 11 atvasinājumiem, kurus lieto Teilora rindas metodē ar kārtu 11. Nesen piedāvātas uzlabotas Teilora rindas metodes, lai atrisinātu nelineārus un pilnīgi aizklātus DAV ar augstu indeksu, kas satur jebkuras kārtas atvasinājumus. Galvenais laika patēriņš DAV risināšanai ar Teilora metodi saistīts ar nepieciešamību vairākas reizes izskaitļot Teilora rindas koeficientus. Šajā rakstā prasība saskaņot skaitlisko atrisinājumu ar precīzo atrisinājumu izvirzījumu Teilora rindā (pat līdz 11 kārtas locekļiem) noved pie Vandermonda tipa lineārām sistēmām, kuru atrisinājumi ir metodes koeficienti. Šie koeficienti tiek izskaitļoti tikai vienu reizi ar Gausa izslēgšanas metodi. Metodi HBT11(9)DAE izmanto elektrisko ķēžu teorijas uzdevumu analīzei. Ir parādīts, ka metode HBT11(9)DAE ir pārāka par Dormanda – Princa DP(8,7) 13 M metodi, kas adaptēta DAV risinājumam, attiecībā pret tādiem parametriem kā procesoru laiks un maksimālā globālā kļūda.

**Труонг Нгуен-Ба, Хемза Ягуб, Хан Хао, Реми Вайенкур. Моделирование электрических цепей с помощью девятишагового метода Эрмита-Биркхофа-Тейлора порядка 11 для решения дифференциально-алгебраических уравнений**

Алгоритм HBT11(9) для решения обыкновенных дифференциальных уравнений модифицирован и преобразован в алгоритм HBT11(9)DAE для решения нежестких и умеренно жестких систем полностью неявных дифференциально-алгебраических уравнений (ДАУ) . В новом алгоритме использован структурный анализ Прайса и автоматическое дифференцирование ДАУ. Для контроля шага используется локальная оценка ошибки. Метод HBT11(9)DAE использует только первые пять производных функции у в отличие от 11 производных, используемых в методе рядов Тейлора порядка 11. Недавно были предложены улучшенные методы рядов Тейлора для решения гладких нелинейных и полностью неявных ДАУ с высоким индексом, содержащие производные любого порядка. Основные затраты при решении ДАУ с помощью метода рядов Тейлора связаны с необходимостью многократного вычисления коэффициентов рядов Тейлора. В настоящей статье требование согласования численного решения с разложением в ряд Тейлора точного решения (вплоть до членов порядка 11) приводит к линейным системам типа Вандермонда, решением которых являются коэффициенты метода. Эти коэффициенты вычисляются только один раз с помощью метода исключения Гаусса. Метод HBT11(9)DAE применен для анализа задач теории электрических цепей. Показано, что метод HBT11(9)DAE превосходит метод DP(8,7)13M Дорманда-Принса, адаптированный для решения ДАУ, по таким параметрам, как время ЦПУ и максимальная глобальная ошибка.