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Shape Optimization of a Superelement of Hexagonal Lattice Structure

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Abstract. **Aim of the study was determination of a minimum mass - maximum static stiffness optimal shape for a hexagon-type lattice superelement. Additionally the correlation between the static and dynamic stiffness of the superelement was studied. The shape parameterization based optimization involved building of a geometric shell model according initial assumptions and defining appropriate optimality criteria. The optimality criteria were based on correlations between selected mechanical parameters of the superelement.**

The chosen optimality criteria provided similar geometries, which represent a certain geometric limit case under a maximum mass constraint. This allowed for the conclusion that the chosen criteria have yielded geometries which, except for minor differences, correspond to maximum static stiffness structures.

Keywords: **Hexagonal Lattice, Superelement, Shape Optimization, Optimization Criteria**

INTRODUCTION

According to previous findings [1], spatial hexagon-type lattices can be considered optimal from maximum static stiffness structure points of view. Due to this, the intended application of the lattice is civil and space structures. Considering multiscale application possibilities, another potential application field could be sandwich structures [2].

For mechanical design purposes, a tetrapod-shaped superelement had been partitioned off the lattice structures [3]. Determination of an optimal shape for the superelement was the aim of the study. The parametric shape optimization [4], which had been chosen for the task, involved building of a geometrically parameterized model and defining appropriate optimality criteria, based on correlations between chosen mechanical parameters of the superelement under the assumed boundary and loading conditions.

PROBLEM SETTINGS

Assumptions

For simplification, it was assumed that the model could be optimized in the range of linear stress-strain relations, allowing for the use of Hookean material law, and small deformations. For the study an isotropic lightweight material, corresponding to the properties of amorphous semi-transparent polyethylene terephthalate (density 1370 kg/m^3, Young's modulus 2800-3100 MPa), was chosen to model the structure.

Due to lightweight considerations, the superelement was modelled as a shell structure, based on a simplified geometric model (Fig. 1) [5], which was determined by a set of five independent shape parameters defining the superelement's midsurface geometry. For each of the parameters there were lower and upper geometric limits set. The shell thickness *tk* was chosen to be defined as a constant for the initial calculation with the estimate that a thickness distribution function, based on the initial results, in a parametric form *tk*(*x1*, *x2*) (Fig. 2-3), would be defined subsequently.

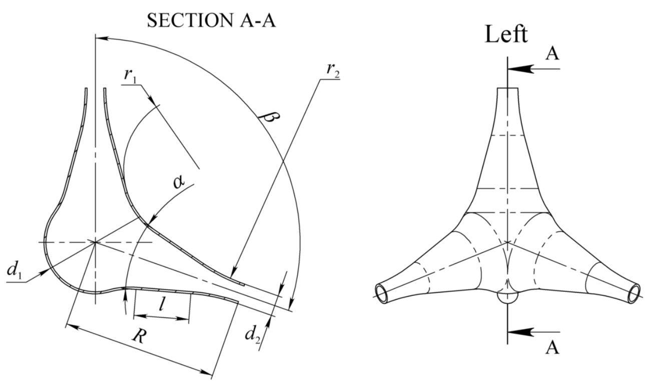


Fig.1. Parameterized model of tetrapod-shaped superelement

Input Parameters

For the sake of scale-independent superelement's geometric model definition, the values for each of linear independent parameter limit were defined with respect to the superelement's ray length *R*, which can be considered as a scaling factor. This can be justified by the finding, based on a simple calculation, that in the limit case of an infinite lattice scaling of the lattice causes no difference for the structural compliance. This is also in accordance with results of metallic foam, which can be regarded as a microstructural analogue to a structural lattice, experiments [6]. For the present study, a size of 75 mm was assumed for the ray length *R*. The rest of independent shape parameters [5] were consequently constrained as follows:

$$tk_{\max} < d_2 < \left(\tan(\beta/2)\cdot R - tk_{\max}/2\right)\cdot 2\,, \tag{1}$$

$$tk_{\max}/2 < r_{1,2} < R\,, \tag{2}$$

$$0 < \alpha < 90° - \beta/2\,, \tag{3}$$

$$0 < tk \leq tk_{\max};\ tk_{\max} = k \cdot R\,, \text{ where} \tag{4}$$

k – coefficient for shell-type geometry.

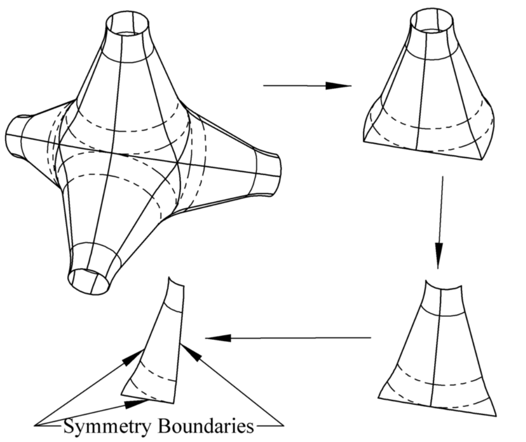


Fig. 2. Derivation of superelement's finite symmetric element

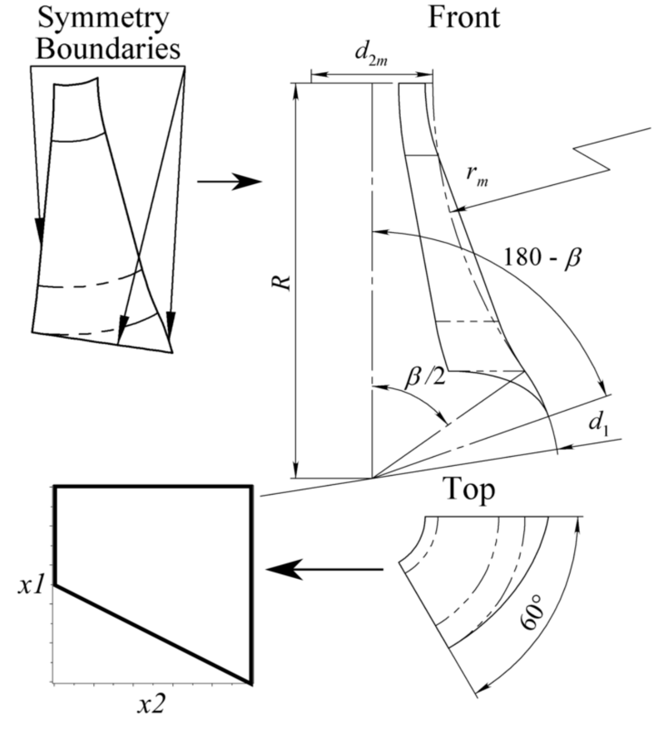


Fig.3. Descriptive parameterization scheme for superelement's thickness distribution function definition $tk(x1, x2)$, where d_{2m} and r_m are replacement dimensions for d_2 and r_1, r_2, respectively, due to additional approximation

Boundary Conditions

A supposed advantage of the spatial hexagon-type lattices is homogenous distribution of load, resulting in dominantly axial loading for all elements. Therefore it was chosen to define approximate boundary conditions, which would allow for the use of symmetries [7] and thereby reduce the computing time. Since the superelement itself is regular, for a symmetric loading case (Fig. 4) it was possible to partition off a quadrangle-shaped area (Fig. 2) as the finite symmetric element, which corresponds to 1/24 of the superelement and is defined by symmetry planes and a 1/6 portion of the loaded outer edge of a ray. The magnitude of the axially compressive force F (25 N) was chosen with the estimate that the superelement's deadweight effect should be at least an order lower than the axial force and thus could be excluded from the study as negligible. This implied an upper mass limit constraint of 250 g or volume limit of 1.82E+05 mm^3.

To account for the constant thickness simplification during the initial calculation, the superelement's outer edges were initially defined as rigid, which should account for thickening of the shell in superelement contact regions due to assumed stress concentration. For all subsequent calculation the edges were defined as lying on symmetry planes.

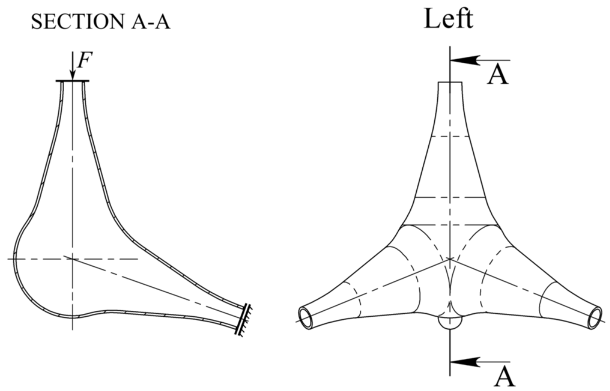


Fig. 4. Superelement's loading scheme, where F is axial loading force

Output Parameters

The output parameters chosen for defining the optimality criteria of superelement's structural optimization include total volume V and volumes V_i of each finite element after deformation, as well as equivalent (von Mises) stresses for each element $\tilde{\sigma}_i$, out of which the maximum equivalent stress $\tilde{\sigma}_{\max}$ can be picked out as a separate parameter. To provide some mechanical meaning for the two parameter sets V_i and $\tilde{\sigma}_i$, it was proposed to define an aggregate parameter as the weighted sum $V_i\tilde{\sigma}_i$ or, normalizing over the total volume V, $w_i\tilde{\sigma}_i$, such that $\sum w_i = 1$.

According to [8], a potential characteristic parameter for maximum global stiffness structures under elastic loading could be the total strain energy U, generally defined as:

$$U = \int_V \sigma_{ij}\varepsilon_{ij}dV\,, \tag{5}$$

where
ε_{ij} – strain tensor components;
σ_{ij} – stress tensor components.

An equivalent parameter to the strain energy U case could be the displacement u of ray's outer edge under the axial load F, which corresponds to the superelement's axial stiffness, defined as F/u, and also to the work of external forces. For simplicity, the displacement u was expressed with respect to the geometric centre of the superelement.

A separate parameter, relating to the dynamic stiffness of the superelement and being transferrable to a whole lattice, could be an eigenfrequency of the superelement with cyclically symmetric axial deformation shape. Specifically, the lowest value of eigenfrequencies – $f_{\min}$ – with the described shape could be maximized.

SOLUTION SCHEME

Design of Experiments

A further task for definition of the superelement's structural optimization criteria was to determine the correlations between the structural parameters given in section above. For calculation of the correlation coefficient values it was chosen to utilize the so-called space-filling experimental designs of Latin hypercube type. The experimental designs can be generated according to various criteria. For the particular task experimental designs were generated according to the Minimal Mean Squared Distance (MMSD) criterion [9], which, in combination with local quadratic approximation, has been found to provide results which can be better than those given by approximations using other space filling criteria and Kriging or Response surface methods [10]. The MMSD criterion is defined as

$$MSD = \sqrt{\left(\frac{1}{n}\right)\sum_{v=1}^{n}\min_{u=1\ldots N}\left[\sum_{i=1}^{s}\left(y_i^v - x_i^u\right)^2\right]}, \qquad (6)$$

where

n – number of mesh or training points;
N – number of points of the experimental design;
s – number of factors (shape parameters for the considered case);
y_i^v – points from a large sample of training points in design space R^s;
x_i^u – experimental design points.

The uniformity of the MMSD criterion could be verified by checking ratio of the number of geometrically feasible experimental design points against the total for different sizes of experimental design point sets. The ratio was found to remain approximately constant (≈ 0.6 for the range of (1)-(4)). For the calculation of correlation coefficient values a set of 500 experimental design points was chosen, out of which 319 points were geometrically consistent.

The experimental designs were generated with the help of optimization software tool EDAOpt, which was developed by the coworkers of Institute of Mechanics at Riga Technical University[1].

Parameter Correlations

It should be emphasized that for shape optimization aggregate parameters were searched for, which would provide potentially convex domains of solution, so that no active constraints on the output parameters would be required. For the specific task parameter couples describing a minimum mass – maximum static stiffness structure had to be found, which would have presumably negative correlation values and thus a potentially convex solution space. The correlations between the structural parameters were evaluated according to the values of Pearson's product-moment correlation coefficient.

The basic results (Table 1) show that all parameter couples having a negative correlation value – products $m\tilde{\sigma}_{\max}$, $mw_i\tilde{\sigma}_i$, mU and mu – are potential candidates for the definition of optimization criteria described above. It should be also noted that the parameters u and U possess a correlation value of 1, which agrees with the general assumption that the work of external forces in conservative systems under elastic deformation should be equal to the change of the strain energy U. Thus the two parameters are interchangeable and therefore henceforth only the parameter u out of both will be considered. Furthermore, the rest of correlations, having values above 0.9, suggest a strong link between parameters $\tilde{\sigma}_{\max}$, $w_i\tilde{\sigma}_i$ and u.

A further study (Table 2) reveals that there is positive correlation between the potential criteria $m\tilde{\sigma}_{\max}$, $mw_i\tilde{\sigma}_i$ and mu. However, the correlation between $mw_i\tilde{\sigma}_i$ and mu is notably stronger (0.92) than between $mw_i\tilde{\sigma}_i$ and $m\tilde{\sigma}_{\max}$ or mu and $m\tilde{\sigma}_{\max}$ (0.42 and 0.62, respectively).

Of a special interest during the correlation study was the relation between the lowest axial deformation eigenfrequency of the superelement and the maximum static stiffness criteria. The low correlation values (Table 3), especially for $mw_i\tilde{\sigma}_i$ and mu, appear to be related to the distinct convex regions in the respective diagrams (Fig. 5). These also convey the idea that shape parameter optimum regions for the maximum static stiffness criteria differ distinctly from those corresponding to the maximum value of $f_{\min}$.

TABLE 1

Correlation coefficient values of superelement's structural response parameters for constant thickness distribution function and rigid outer edges definition (significance level p < 1E-6)

	u	U	m	$\tilde{\sigma}_{\max}$
u		1.00	-0.27	0.94

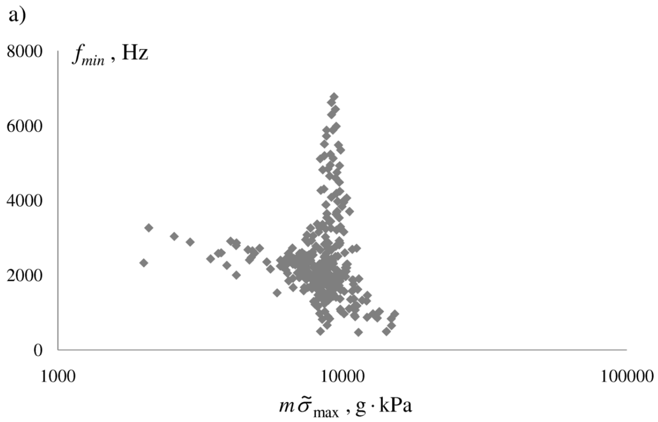
[1] Machine & Mechanism Dynamics Research Laboratory, Ezermalas Street 6, Riga LV-1006, Latvia, http://www.mmd.rtu.lv.

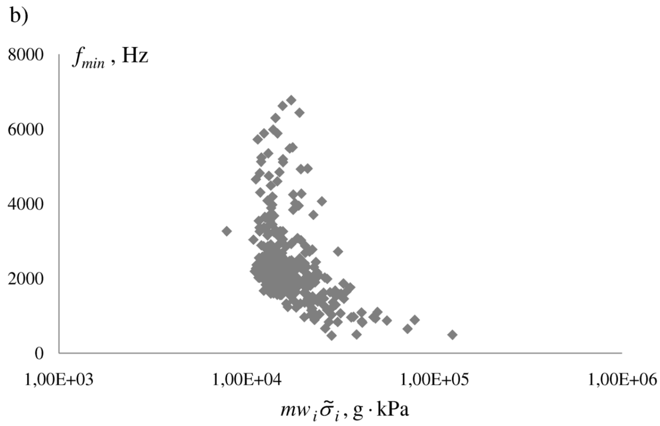
U	1.00		-0.27	0.94
m	-0.27	-0.27		-0.34
$\tilde{\sigma}_{\max}$	0.94	0.94	-0.34	
$w_i\tilde{\sigma}_i$	0.99	0.99	-0.32	0.95

TABLE 2

Correlation coefficient values of minimum mass - maximum static stiffness criteria for constant thickness distribution function and rigid outer edges definition (p < 1E-13)

	$m\tilde{\sigma}_{\max}$,	$mw_i\tilde{\sigma}_i$	mu
$m\tilde{\sigma}_{\max}$		0.42	0.62
$mw_i\tilde{\sigma}_i$	0.42		0.92
mu	0.62	0.92	





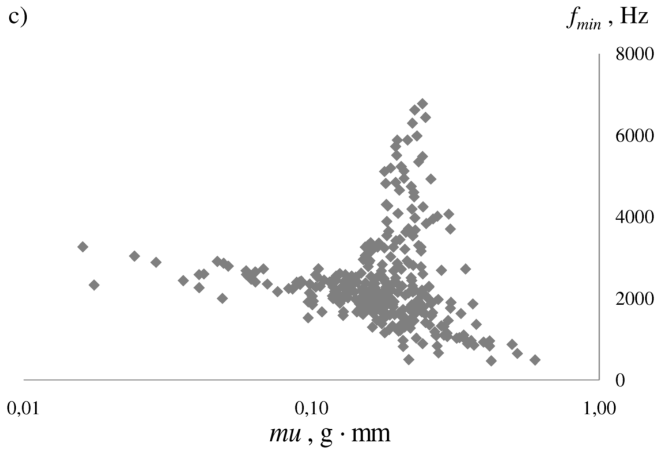


Fig.5. Distribution of 319 space-filling geometrically consistent design points, obtained with constant thickness distribution function and rigid outer edges definition, in the space of minimum mass - maximum static stiffness criteria: $mw_i\tilde{\sigma}_i$ or mass - weighted equivalent stress sum product (a); $m\tilde{\sigma}_{\max}$ or mass - maximum equivalent stress product (b); *mu* or mass - displacement product; and the lowest axial eigenfrequency of the superelement f_{min} (c)

TABLE 3

Correlation coefficient values of minimum mass - maximum static stiffness criteria and superelement's structural response parameters for constant thickness distribution function and rigid outer edges definition (p < 1E-4, for exceptions see footnotes)

	m	u	$\tilde{\sigma}_{\max}$	$w_i\tilde{\sigma}_i$	$f_{\min}$
$m\tilde{\sigma}_{\max}$	-0.46	0.28	0.51	0.29	0.39
$mw_i\tilde{\sigma}_i$	-0.55	0.22	0.25	0.23	0.11[a]
mu	-0.63	0.38	0.45	0.39	0.14[b]

[a] $p < 0.05$

[b] $p < 0.01$

Parametric Optimization

Since the initial limits of shape parameters included most of the geometrically feasible combinations, the metamodelling approach [9], also termed subproblem approximation [11], was chosen as the main optimization method. The method is generally classifiable as a zero-order method, since it requires only the values of the dependent variables (objective function and state variables) and not their derivatives. A particular implementation of the method is available as a module of the ANSYS software package (version 11.0), which was used for the study.

For the study the objective function was approximated by a quadratic fit function with cross-terms, whereas the state variables were approximated by a quadratic fit function. For the least squares fitting weights of the design sets were defined as triple products of distance in design space, objective function values, and feasibility/infeasibility. The approximations were updated by inclusion of the newly calculated solution set after every optimization loop.

A specific task for the implementation of the subproblem method is the definition of design variable sets, required for the definition of approximation functions. As the optimization module of ANSYS allows for a maximum number of 130 design points being stored at a time in the optimization database, initial sets of between 100-130 geometrically consistent design points, generated by the optimization software tool EDAOpt according to the MMSD criterion, were used during the study. The chosen number of design points corresponds to the recommended range for 10 parameters according to [9], defined as:

$$N = k \cdot (d+1) \cdot (d+2) / 2 \,, \tag{7}$$

where

N – recommended number of design points for quadratic approximation,

d – number of dimensions,

k – empiric coefficient (k = 1.5…2).

The subproblem approximation method can be applied repeatedly after reducing the parameter limits according to the results obtained, thereby obtaining a refined result. Alternatively, a further local refinement could be obtained with the first-order method [11], which makes use of derivative information.

Topology Optimization

The built-in homogenization method based [12] topology optimization module of ANSYS software package was tested for the given loading scheme (Fig. 4). The method is applicable to shells and solids.

Within the task setting of the study, the most appropriate topology optimization criterion from the available was that of minimizing the energy of structural static compliance (5) of the superelement. The minimization is subject to a given constraint of minimum volume reduction, which was set to 90 percent during the study.

The topology optimization for solids was carried out for the maximum volume available for a single superelement (Fig. 6). The volume was derived from geometric compatibility conditions, which provided a structure of regular truncated tetrahedron combined with smaller tetrahedrons, which were placed on the truncated triangular surfaces of the larger tetrahedron.

The shell optimization option allowed only for 2-dimensional optimization on a given shell surface geometry. Consequently, it could be applied for definition of thickness variations, including cuts, on surfaces obtained by optimization of constant thickness geometry models.

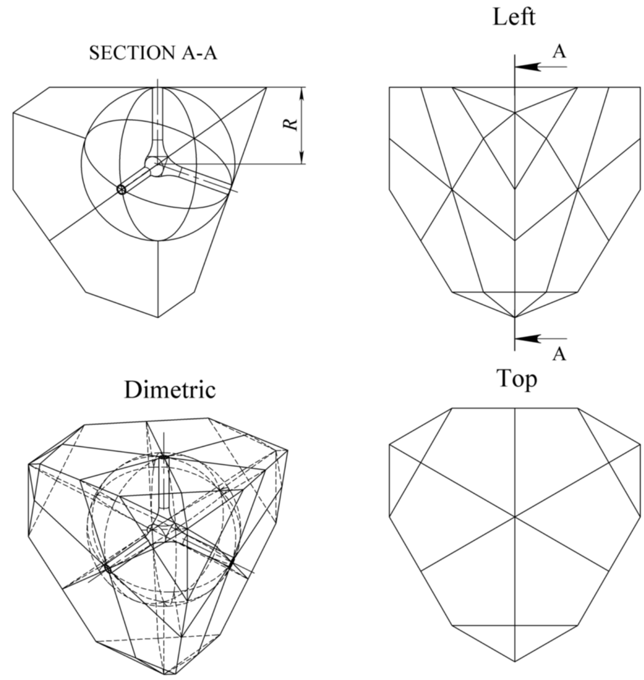


Fig.6. Design space volume for the superelement’s topology optimization showing the position of a topologically equivalent lattice element inside

RESULTS AND DISCUSSION

Constant Thickness Parametric Optimization Results

The optimization was carried out using the subproblem approximation method, followed by refinement using the first-order method. The mass constraint was not considered at this stage.

Numeric examination (Table 4) of the shape parameters, obtained according to the three minimum mass - maximum static stiffness criteria, shows that the sets are close to each other. Geometrically the sets have converged to the limit case of maximum diameter central sphere having narrow transitional segments, formed by the two transition fillets, which have close to minimum radius values (Fig. 7 (a)). Correspondingly, the conic surface has a negligible length of its generatrix. This type of geometry bears topological and geometric resemblance to the open-cell metallic foams with relative density of about 25%, which have been modelled by analogous tetrahedral unit cells [13] and are known as porous functional materials [14].

For the criteria $mw_i\tilde{\sigma}_i$ and mu local optima were obtained without additional constraints, whereas for the criterion $m\tilde{\sigma}_{\max}$ repeated maximum thickness limit changes up to $R/2$ revealed that the shell thickness tended to exceed assumptions of a shell-type geometry. The latter difference could be explained by the fact that the ratio of criterion’s $m\tilde{\sigma}_{\max}$ component correlations with the criterion itself,

$\mathrm{r}(\tilde{\sigma}_{\max}, m\tilde{\sigma}_{\max})/\mathrm{r}(m, m\tilde{\sigma}_{\max})$, was about two times greater than the respective ratio for the other two criteria (Table 3).

TABLE 4

Values of shape parameters, output parameters and objective functions, corresponding to optimal values of minimum mass - maximum static stiffness criteria, obtained with constant thickness distribution function and rigid outer edges definition

		Optimization Criteria		
		mu	$m\tilde{\sigma}_{\max}$	$mw_i\tilde{\sigma}_i$
Shape parameters	r_1 (mm)	13.8	18.7	2.9
	r_2 (mm)	3.4	18.7	1.1
	d_2 (mm)	203.5	173.3	202.7
	α (deg)	64.5	65.4	69.7
	tk (mm)	6.5	37.1	2.2
	d_1[a] (mm)	251.5	221.2	251.6
	l[a] (mm)	0.0	0.4	2.1
Output par.	u (mm)	2.23E-05	1.09E-05	6.59E-05
	m (g)	334	2550	116
	U (µJ)	4.65E-02	2.27E-02	1.37E-01
	$\tilde{\sigma}_{\max}$ (kPa)	19.7	2.0	70.0
Obj. F-ns	mu (g·mm)	**7.46E-03**	2.78E-02	7.66E-03
	$m\tilde{\sigma}_{\max}$ (g·kPa)	6597	**5081**	8125
	$mw_i\tilde{\sigma}_i$ (g·kPa)	1.19E+03	2.71E+03	**1.18E+03**

[a] dependent parameters

Accordingly, the criterion could be discarded for further study.

The equivalent stress distribution over the converged geometry (Fig. 8) is mostly uniform having concentration regions over the transition surfaces, which reach maximum values at the narrowest locations of the truncated sphere.

Sensitivity tests for the obtained geometries (Table 5) show that the ray transition diameter d_2 had by far the greatest influence on the optimization criteria, which could be related both to the particular geometric limit case and the applied rigid edge definitions.

An approximate limit case is likewise represented by the optimal design set corresponding to the maximized lowest axial eigenfrequency $f_{\min}$ (Fig. 7 (b) and Table 6). Specifically, the two transition fillets are having their maximum radius values, whereas the angle, corresponding to the conical surface, has a value which is close to minimum. These, together with the close to minimum value for the ray's outer edge diameter, result in a geometry which approximately describes a single fillet transition for the central sphere.

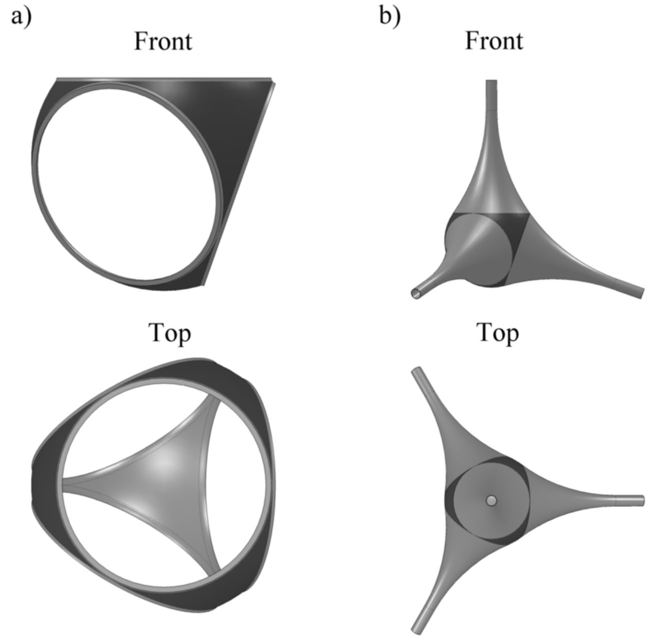


Fig.7. Superelement shapes corresponding to optimized minimum mass - maximum static stiffness (a) and the lowest axial eigenfrequency (b) criteria values, obtained with constant thickness distribution function and rigid outer edges definition

TABLE 5

Change of the minimum mass - maximum static stiffness criteria values due to +1 % change in design variable values with respect to the range of each considered variable, the reference values corresponding to the values of the respective criteria, optimized with constant thickness distribution function and rigid outer edges definition

	$\Delta m\tilde{\sigma}_{\max}$ (g·kPa)	$\Delta mw_i\tilde{\sigma}_i$ (g·kPa)	Δmu (g·mm)
r_1	2.6	-0.3	-5.06E-09
r_2	6.9	0.3	-4.60E-05
d_2	-53.2	-150.3	-1.12E-03
α	-1.4	-9.3	-8.51E-04
tk	-15.7	0.5	-4.18E-06

It should be noted that the objective function, defined as the negative of the lowest axial eigenfrequency ($-f_{\min}$), provided a diverging response, whereas the inverse of the lowest axial eigenfrequency ($1/f_{\min}$) proved successful. The diverging behaviour most likely could be explained by changing of the mode shapes corresponding to the lowest axial eigenfrequency, which is possible due to constant change of the surface geometry during the optimization procedure. Finally it may be concluded that the maximized lowest axial eigenfrequency has resulted in geometry which is not compatible with a maximum static stiffness design, as it was predicted during the correlation study, and could be excluded from further study.

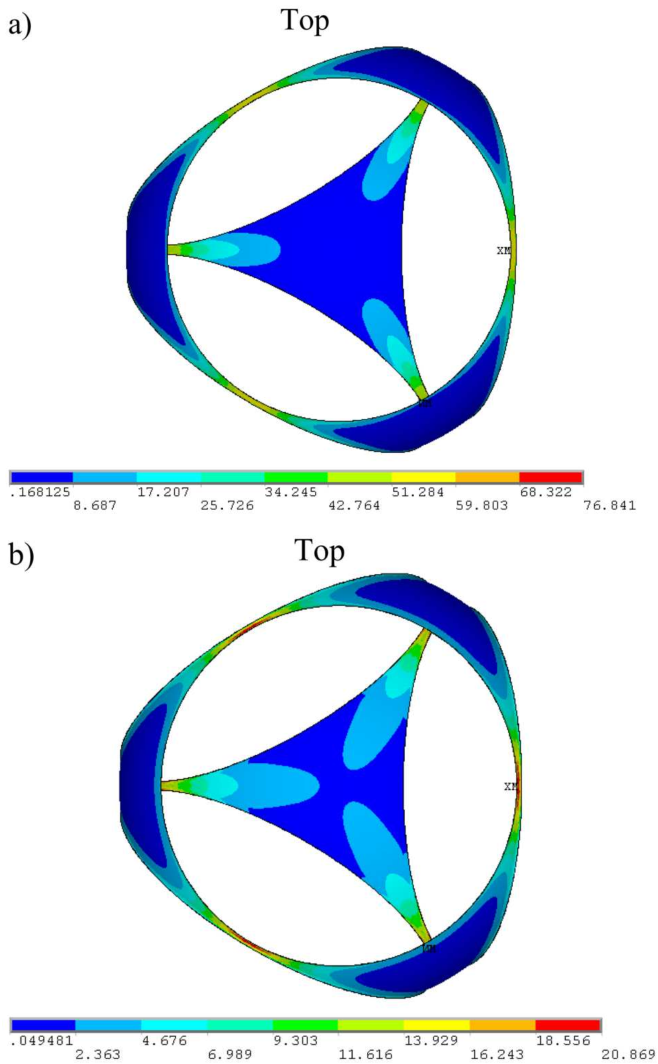


Fig. 8. Equivalent stress distribution (kPa) for superelement geometry corresponding to optimized minimum mass - maximum static stiffness criteria $mw_i\tilde{\sigma}_i$ or mass - weighted equivalent stress sum product (a) and *mu* or mass - displacement product (b) values, obtained with constant thickness distribution function and rigid outer edges definition

Topology optimization for solids provided geometries having a lattice-like structure with two regions of distinct pseudodensities (Fig. 9), which could be interpreted as two materials having stiffness properties in a ratio similar to the ratio of pseudodensities of the two regions, the region of highest density being located close to the outer boundary of the design space. Since the structure obtained could not be classified as a shell structure, the only applicable conclusion which could be drawn from it was that for a shell structure implementation the material should be distributed close to the outermost locations of the volume available.

TABLE 6

Values of shape parameters and objective function, corresponding to optimal values of maximum lowest axial eigenfrequency criterion, obtained with constant thickness distribution function and rigid outer edges definition

Shape parameters	r_1 (mm)	84.4
	r_2 (mm)	75.0
	d_2 (mm)	4.9
	α (deg)	0.26
	tk (mm)	4.9
	d_1^a (mm)	44.1
	l^a (mm)	13.6
Output parameters	*u* (mm)	4.31E-03
	m (g)	80
	U (μJ)	8.97
	$\tilde{\sigma}_{max}$ (kPa)	333
Obj. F-n	f_{min}, Hz	7576

[a] dependent parameters

Topology Optimization Results

This corresponds principally to the geometry yielded by the shape optimization (Fig. 7 (a)).

It should be remarked that with this method symmetric resulting density distributions could be obtained only when using the finite symmetry element (derivation analogous to Fig. 2) of the superelement.

In order to define a thickness distribution function for further study, topology optimization for the shell geometry obtained with the criterion *mu* was carried out. The choice of geometry was based on the estimate that criterion *mu* was the closest to the structural static compliance criterion, employed by the topology optimization method, since it had yielded the smallest deformation energy compared to criterion $mw_i\tilde{\sigma}_i$. The results displayed main material concentration at superelement's outer edges, corresponding to sphere transition fillets, and in beam-like connections between the edges, forming hexagons around the centres of the spherical surface portions (Fig. 10).

Based on the results of shell topology optimization (Fig. 10), a piecewise defined thickness distribution function of 6 parameters was defined, having three thickness parameters *TK1-TK3*, subject to (4), and three relative parameters *GC*, *Y2C* and *Y3* having ranges of 0…1. The respective meanings of the new parameters are explained in Fig. 11. Subsequent optimization according to minimum mass - maximum static stiffness criteria – *mu* and $mw_i\tilde{\sigma}_i$ – using the subproblem approximation method was carried out. The mass constraint was included for the calculation.

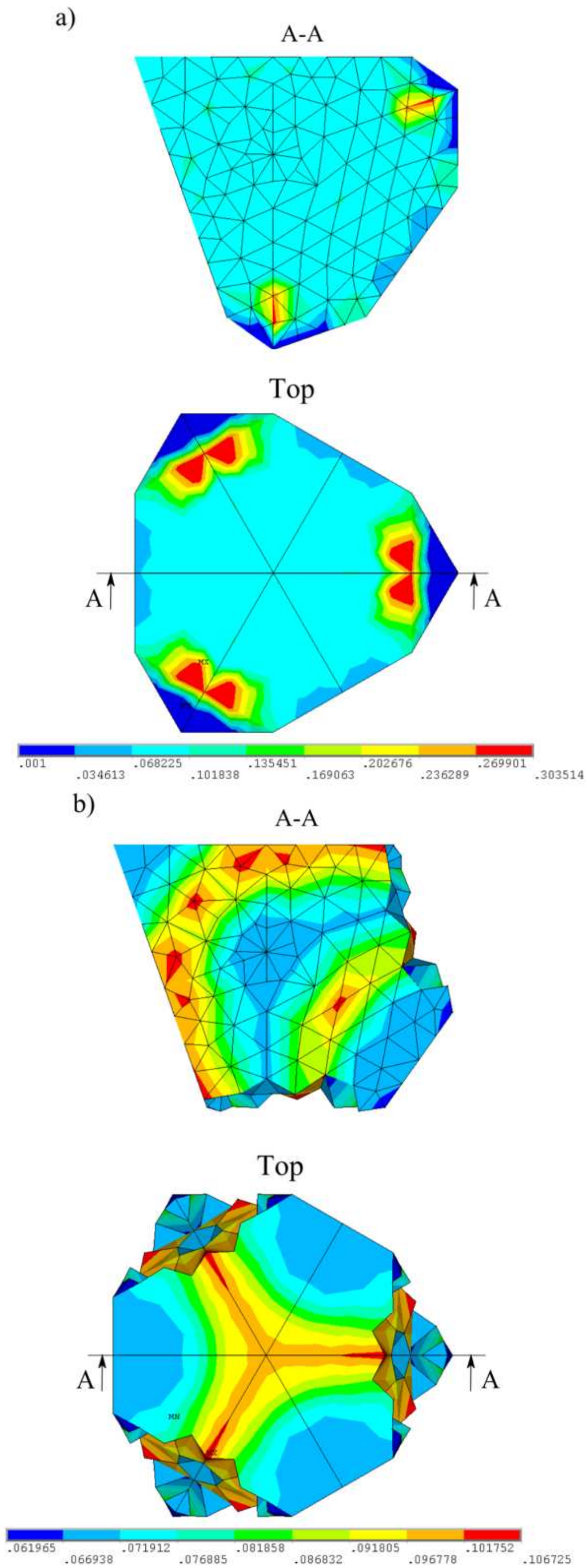


Fig.9. Pseudodensities of the design volume of the superelement, topologically optimized according to compliance energy criterion: total volume (a) and volume having high density regions cut out (b)

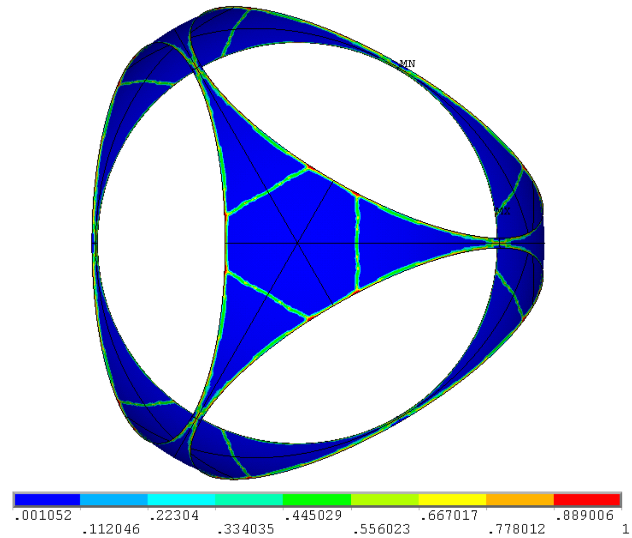


Fig.10. Pseudodensities of the shell volume of the superelement (bottom view), topologically optimized according to compliance energy criterion (the shell volume corresponds to optimized *mu* or mass - displacement product value, obtained with constant thickness distribution function and rigid outer edges definition)

Variable Thickness Parametric Optimization Results

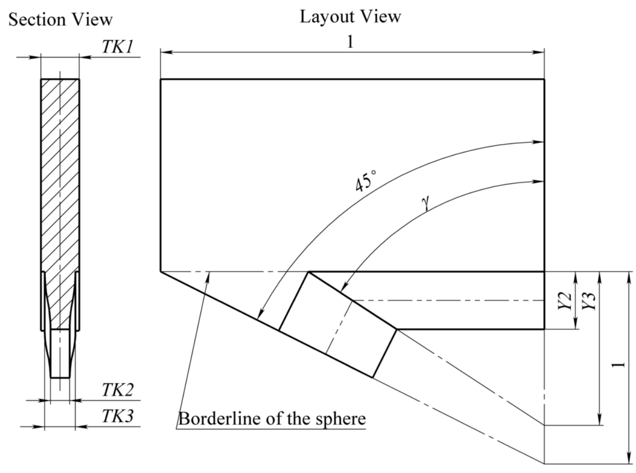


Fig.11. Normalized piecewise defined thickness distribution for the finite symmetry element of the superelement, where $GC = \gamma/45°$, $Y2C = Y2/Y3$

Numerical results (Table 7) reveal that both criteria have converged to geometries which are close to the mass limit, *mu* being the closest. Compared to the geometries of the initial results (Table 4), it can be seen that the radiuses r_1 and r_2 have become larger, resulting in smaller diameters of the central sphere. The most notable difference between the obtained geometries (Fig. 12) is the shape of the cuts in centres of the spherical sections, which are approximately hexagonal for the criterion *mu* and triangular for the criterion $mw_i\tilde{\sigma}_i$. The hexagonal cuts should account for the higher stiffness for the criterion *mu*, whereas the triangular cuts should account for a lower overall level of stresses for the criterion $mw_i\tilde{\sigma}_i$.

The equivalent stress distributions for the obtained geometries (Fig. 13) demonstrate that the overall stress distribution is more uniform for the criterion $mw_i\tilde{\sigma}_i$, whereas for the criterion *mu* the maximum local stress value is about three times smaller than for that of criterion $mw_i\tilde{\sigma}_i$.

TABLE 7

Values of shape parameters, output parameters and objective functions, corresponding to optimal values of minimum mass - maximum static stiffness criteria

		Optimization Criteria	
		mu	$mw_i\tilde{\sigma}_i$
Shape parameters	r_1 (mm)	6.28	51.1
	r_2 (mm)	58.8	37.0
	d_2 (mm)	141.9	117.3
	α (deg)	32.0	40.3
	TK1 (mm)	3.2	3.4
	TK1 (mm)	7.3	5.1
	TK1 (mm)	7.1	0.4
	Y2C	0.43	0.33
	Y3	0.40	0.69
	GC	0.57	0.90
	l^a (mm)	3.9	2.4
	$d_1{}^a$ (mm)	184.2	166.6
Output par-s	*m* (g)	247	231
	U (μJ)	1.13	1.80
	u (mm)	5.50E-04	9.54E-04
	$\tilde{\sigma}_{max}$ (kPa)	67.5	187.9
Obj f-n	*mu* (g·mm)	**0.136**	0.220
	$mw_i\tilde{\sigma}_i$ (g·kPa)	7525	**6914**

[a] dependent parameters

The sensitivity analysis (Table 8) shows that for the criterion *mu* the greatest change is due to the parameters related to the overall shape, especially radius r_2, which, being the largest between r_1 and r_2, is also implicitly related to the size of the hexagonal cut. For the criterion $mw_i\tilde{\sigma}_i$ the greatest influence is exerted by the parameters related to the shape of the triangular cut, namely, *GC* and *Y3*, followed by diameter d_2, which is greatly responsible for the overall shape.

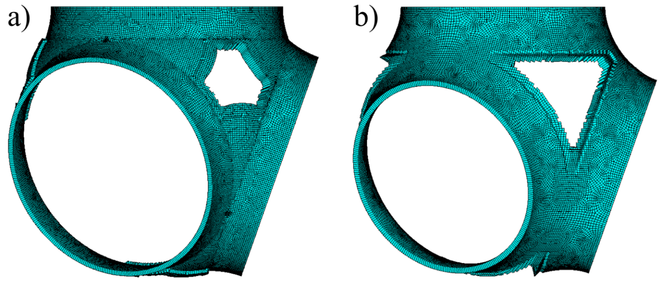


Fig.12. Superelement's overall geometry (front view) corresponding to optimized value of minimum mass - maximum static stiffness criterion *mu* or mass - displacement product (a), and minimum mass - maximum static stiffness criterion $mw_i\tilde{\sigma}_i$ or mass - weighted equivalent stress sum product (b)

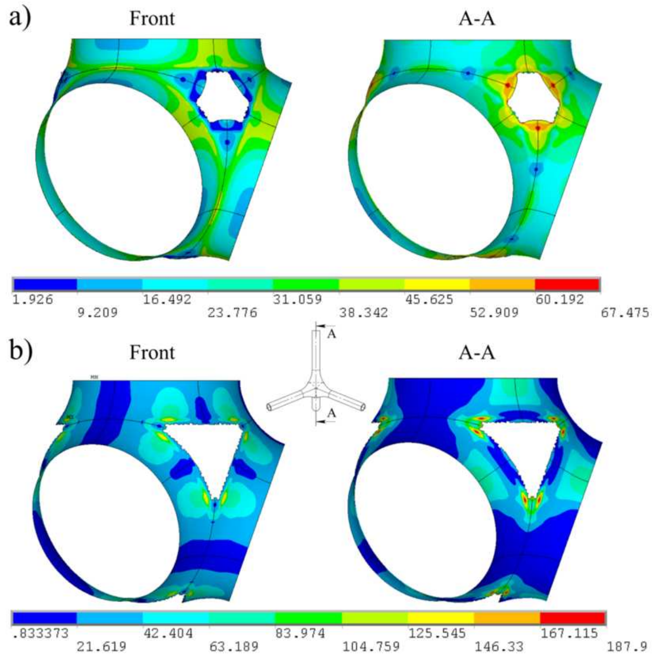


Fig.13. Equivalent stress distribution (kPa) for superelement's geometry corresponding to optimized minimum mass - maximum static stiffness criteria *mu* or mass - displacement product (a) and $mw_i\tilde{\sigma}_i$ or mass - weighted equivalent stress sum product values (b)

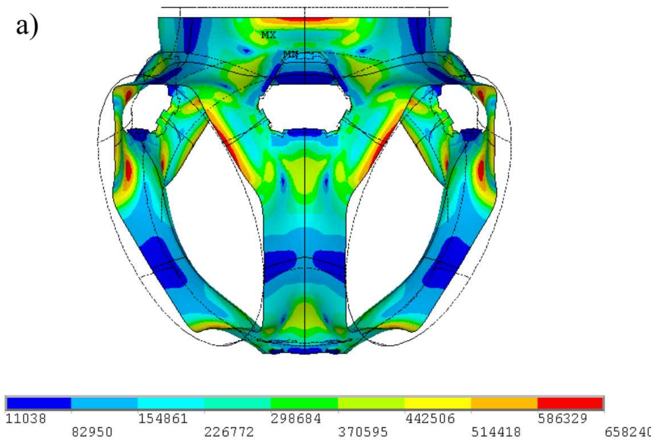
Additional nonlinear buckling analysis, which included stress stiffening effects, was carried out for both obtained geometries (Fig. 14). Accordingly it can be stated that stability constraints were not relevant for the study. It should be added that for the criterion $mw_i\tilde{\sigma}_i$ buckling was preceded by contact appearance situation.

TABLE 8

Change of the minimum mass - maximum static stiffness criteria values due to +1 % change in design variable values with respect to the range of each considered variable

	$\Delta mw_i\tilde{\sigma}_i$ (g·kPa)	Δmu (g·mm)
r_1	-9.2	3.44E-05
r_2	-22.7	-1.25E-03
d_2	126.2	3.32E-04
α	-4.9	-1.94E-04
TK1	1.7	-1.50E-04
TK2	-0.7	-6.53E-05
TK3	0.3	-5.13E-05
Y2C	37.1	1.65E-04
Y3	183.2	8.25E-05
GC	189.7	-9.49E-05

a)



b)

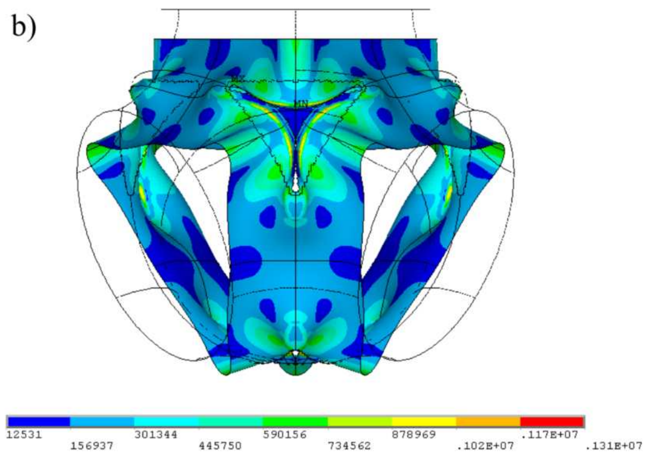


Fig.14. Equivalent stress distribution (kPa) for superelement in near-buckling state (left view of the deformed view shown) corresponding to optimized minimum mass - maximum static stiffness criterion *mu* or mass - displacement product (***F**kr**/F*** = 3013) (a) and $mw_i\tilde{\sigma}_i$ or mass - weighted equivalent stress sum product values (***F**kr**/F*** = 1287) (b)

The observed general tendency of the obtained geometries could be described as convergence towards maximum static stiffness optimal structures, which could be explained by referring to the correlation values between mass *m* and the criteria and between the displacement *u* and the criteria (Table 3), which are negative in the former and positive in the latter case. The similarity of the criteria is also confirmed by the high correlation between them – 0.92 (Table 2).

CONCLUSIONS AND OUTLOOK

- Close-to-optimal geometries for the superelement of spatial hexagonal lattice have been obtained according to criteria $mw_i\tilde{\sigma}_i$ and *mu* under the given conditions.
- The obtained geometries approximately correspond to maximum static stiffness optimal structures, constrained by upper mass limit.
- The results can be considered as Pareto-optimal [4]: the final choice of parameters is application dependent.
- The criterion $mw_i\tilde{\sigma}_i$ can be recommended only in cases when overall stress level is of priority compared to the overall stiffness.
- Possible improvements of the results in combination with additional load cases include implementation of:
  – alternative thickness distribution functions and shell shapes;
  – alternative optimization criteria, based on response parameter correlations;
  – deadweight for the calculation model;
  – other topological optimization methods, including level set methods.

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Osvalds Verners, Modris Dobelis. Telpiska režģa konstrukcijas četrzaru superelementa struktūroptimizācija
Pētījuma mērķis bija minimālas masas un maksimāla statiskā stinguma optimālas formas noteikšana heksagonala tipa režģa superelementam. Papildus tika pētīta arī korelācija starp statisko un dinamisko superelementa stingumu. Pašsvara samazināšanas nolūkos superelements tika ģeometriski modelēts kā čaulas konstrukcija.
Optimizācija, kas bija balstīta uz superelementa formas parametrizāciju, ietvēra ģeometrisku čaulas modeļa izveidi atbilstoši sākotnējiem apsvērumiem un atbilstošu optimizācijas kritēriju definēšanu. Optimalitātes kritēriji tika balstīti uz korelācijām starp izvēlētiem superelementa mehāniskiem raksturlielumiem. Čaulas biezuma sadalījuma noteikšanai papildus tika pielietota topoloģijas optimizācijas metode.
Optimizācijas metodikas uzlabošanai papildus slodžu kombināciju gadījumā ir jāņem vērā sekojošie faktori: čaulas biezuma sadalījuma un formas funkcija, papildus optimizācijas kritēriji, svara ierobežojumi aprēķina modelim, dažādu citu topoloģisko optimizācijas metožu izmantošana.
Izvēlētie optimalitātes kritēriji deva līdzīgas ģeometrijas, kas atbilst noteiktam ģeometriskam robežstāvoklim, kas noteikts ar maksimālas masas ierobežojumu. Tas ļāva secināt, ka izvēlētie kritēriji ir devuši ģeometrijas, kas atbilst maksimāla stinguma optimālām konstrukcijām.

Освалдс Вернерс, Модрис Добелис. Оптимизация формы суперэлемента гексагональной решеточной структуры
Цель исследования - определение минимальной массы и оптимальной формы максимальной статической жесткости для суперэлемента решетки шестиугольного типа. Дополнительно была исследована корреляция между статической и динамической жесткости суперэлемента. С целью снижения собственного веса суперэлемент был геометрически смоделирован в виде оболочки.
Оптимизация, которая была основана на параметризации формы суперэлемента, включала создание геометрической модели оболочек в соответствии с первоначальными предположениями и определение соответствующих критериев оптимальности. Критерии оптимальности были основаны на корреляции между выбранными механическими параметрами суперэлементов.
С целью совершенствования методики оптимизации в случае дополнительных комбинаций нагрузок, предлагается учитывать следующие факторы: функция распределения толщины оболочки и ее формы, дополнительные критерии оптимизации, ограничения по весу, использование других топологических методов оптимизации.
Выбранные критерии оптимальности обусловливали аналогичные геометрии, которые отвечают определенным геометрическим предельным состояниям при ограничении максимальной массы. Это позволило сделать вывод, что выбранные критерии дали геометрию, соответствующую максимальной статической жесткости оптимальной конструкции.